

**SOLUTIONS & ANSWERS FOR KERALA ENGINEERING
ENTRANCE EXAMINATION-2011 – PAPER II
VERSION – B1**

[MATHEMATICS]

1. Ans: $\frac{1}{2}(1 + \sqrt{1 + 4 \log_2 x})$

Sol: $y = 2^{x(x-1)}$
 $\log_2 y = x(x-1)$
 $\Rightarrow x^2 - x - \log_2 y = 0$
 $x = \frac{1 \pm \sqrt{1 + 4 \log_2 y}}{2}$
 $\therefore f^{-1}(x) = \frac{1}{2}(1 + \sqrt{1 + 4 \log_2 x})$

8. Ans: $-\frac{1}{2}$

Sol: $\ln\left(\frac{2+i}{ai-1} \times \frac{(-1-ai)}{(-1-ai)}\right)$
 $= \frac{-2a-1}{1+a^2} = 0$
 $\therefore a = -\frac{1}{2}$

2. Ans: 6

Sol: $n(A \cap B) \cap A = n(A' \cup B' \cap A)$
 $= n(A' \cap A) + n(B' \cap A)$
 $= n(A - B) = 8 - 2$
 $= 6$

9. Ans: $\frac{\pi}{2}$

Sol: $\left(\frac{i}{2} - \frac{2}{i}\right) = \frac{i}{2} + 2i = \frac{5i}{2}$
 $\therefore \arg\left(\frac{i}{2} - \frac{2}{i}\right) = \frac{\pi}{2}$

3. Ans: $\sin x^2 + \cos x^2$

Sol: $f(x) = \sin x + \cos x$; $g(x) = x^2$
 $f \circ g(x) = \sin x^2 + \cos x^2$

10. Ans: $-|z_1 - z_2|^2$

Sol: $Z_1 = 3 + 4i$; $Z_2 = -1 + 2i$
 $|z_1 + z_2|^2 - 2(|z_1|^2 + |z_2|^2)$
 $= |2 + 6i|^2 - 2(25 + 5)$
 $= 40 - 60 = -20$
 From the options we get
 $-|z_1 - z_2|^2 = -20$

4. Ans: 32

Sol: $n(A) = 5$
 $n(P(A)) = 2^5 = 32$

11. Ans: 0

Sol: $(z_1 + z_2) \leq |z_1| + |z_2|$
 Equality holds when $z_1, z_2, z_1 + z_2$ are collinear with the origin
 $\therefore \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 = 0$

5. Ans: $-3 < x < 3$

Sol: $f(x) = \frac{1}{\sqrt{9-x^2}}$
 $\Rightarrow 9 - x^2 > 0$
 $\therefore -3 < x < 3$

6. Ans: $\frac{\pi}{2}$

Sol: Period of the function $|\sin 2x| + |\cos x|$
 Period of $|\sin 2x| = \frac{\pi}{2}$; Period of
 $|\cos 8x| = \frac{\pi}{4}$
 \therefore The required answer is $\frac{\pi}{2}$.

12. Ans: $6, \frac{2}{3}$

Sol: For equal roots,
 $(2 + m)^2 - 4(m^2 - 4m + 4) = 0$
 $4 + m^2 + 4m - 4m^2 + 16m - 16 = 0$
 $3m^2 - 20m + 12 = 0$
 $m = \frac{20 \pm \sqrt{400 - 144}}{6} = \frac{20 \pm 16}{6}$
 $m = 6, \frac{2}{3}$

7. Ans: 0

Sol: $i^1 - i^2 + i^3 - i^4 + \dots + i^{100}$
 This is a GP with $a = i$ and $r = -i$
 $S_{100} = \frac{i(1 - (-i)^{100})}{1 + i} = 0$

13. Ans: 2

Sol: $b^2 + 64$ must be the square of an even number
 $\Rightarrow b = 6, -6$.
 No: of values = 2.

$$\therefore a + \frac{600}{a} = 50 \Rightarrow a = 30, 20$$

$$\text{For } a = 30 \Rightarrow d = \frac{1}{10}$$

$$a = 20 \Rightarrow d = \frac{1}{10}$$

14. Ans: (-i)

Sol: Sum of roots = 1
 Product of roots = (1 - i)
 The other root is (-i)

$$\text{In either cases } |S_1 - S_{101}| = 100 \left| \frac{1}{10} \right| = 10$$

15. Ans: (0, 1)

Sol: For roots with opposite signs, product must be negative
 $\therefore \frac{a^2 - a}{3} < 0 \Rightarrow a^2 - a < 0$
 a lies in (0, 1)

20. Ans: $n - 2 + \frac{1}{n - 2}$

Sol: $a_1 = 0 \Rightarrow a_2 = d$ (Common difference)

\therefore The given expression

$$= \left(2 + \frac{3}{2} + \frac{4}{3} + \dots + \frac{n-1}{n-2} \right)$$

$$- \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-3} \right)$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-2}$$

$$= n - 3 + \frac{n-1}{n-2} = n - 3 + \frac{n-2+1}{n-2}$$

$$= n - 2 + \frac{1}{n-2}$$

16. Ans: 4

Sol: For $x < 0$, $x^2 + 3x + 2 = 0 \Rightarrow$ two real roots
 For $x > 0$, $x^2 - 3x + 2 = 0 \Rightarrow$ two real roots
 \therefore Four real roots

21. Ans: 66

17. Ans: $\frac{a-b}{a+b}$

Sol: $(x^2 - bx)(m+1) = (m-1)(ax-c)$
 $(m+1)x^2 - x[bm+b+am-a] + c(m-1) = 0$
 Sum of roots = 0
 $\therefore m(a+b) + (b-a) = 0$
 $m = \frac{a-b}{a+b}$

Sol: $1^2 - 2^2 = 3(-1)$

$$3^2 - 4^2 = 7(-1)$$

$$9^2 - 10^2 = 19(-1)$$

\therefore Required sum

$$= -1(3 + 7 + 11 + 5 + 19) + 11^2$$

$$= -55 + 121 = 66$$

18. Ans: 2 : 1

Sol: $a + 8d = 0$
 $\frac{a+28d}{a+18d} = \frac{(a+8d)+20d}{(a+8d)+10d} = \frac{20}{10} = 2$

22. Ans: 6

19. Ans: 10

Sol: $\frac{1}{S_1 S_2} = \left(\frac{1}{S_1} - \frac{1}{S_2} \right) \frac{1}{d}$ where d in the common difference

\therefore The given expression

$$= \left(\frac{1}{S_1} - \frac{1}{S_{101}} \right) \frac{1}{d} = \frac{1}{6}$$

$$\frac{S_{101} - S_1}{S_1 S_{101}} = \frac{d}{6} \longrightarrow (1)$$

$$S_1 + S_{101} = 50 \Rightarrow a + a + 100d = 50$$

$$a + 50d = 25 \longrightarrow (2)$$

$$(1) \Rightarrow \frac{100d}{a(S_{101})} = \frac{d}{6} \Rightarrow S_{101} = \frac{600}{a}$$

Using (2) $a + S_{101} = 50$

$$\text{Sol: } S_{2n} = 3.S_n \Leftrightarrow \frac{\frac{2n}{2}(2a + (2n-1)d)}{\frac{n}{2}(2a + (n-1)d)}$$

$$\Rightarrow 2a - (n+1)d = 0. \Rightarrow 2a = (n+1)d$$

$$k = \frac{S_{3n}}{S_n} = \frac{3n(2a + (3n-1)d)}{\frac{n}{2}(2a + (n-1)d)}$$

$$= \frac{3(4nd)}{(2nd)} = 6$$

23. Ans: 8045

Sol: $9 - a = 2b = 3a - b - 9$

$$\therefore a + 2b = 9$$

$$3a - 3b = 9 \Rightarrow a - b = 3$$

$$\therefore 3b = 6 \Rightarrow b = 2 \Rightarrow a = 5;$$

Common Difference = 4

$$\therefore 2011^{\text{th}} \text{ term} = 5 + 2010 \times 4 = 8045$$

24. Ans: 49

Sol: $(n+2)(n+1) = 2550 = 51 \times 50$
 $\Rightarrow n = 49$

25. Ans: 3

Sol: $nC_{r-1} = 28 \quad nC_r = 56 \quad nC_{r+1} = 70$

$$\frac{n^1}{(r-1)^1(n-r+1)^1} = 28 \frac{n^1}{r^1(n-r)^1} = 56$$

$$\frac{n^1}{(r+1)^1(n-r-1)^1} = 70$$

$$\frac{70}{56} = \frac{r^1(n-r)^1}{(r+1)(n-r-1)^1} = \frac{n-r}{r+1}$$

$$\frac{56}{28} = \frac{n-r+1}{r} = 2 \Rightarrow n = 3r - 1$$

$$\therefore \frac{5}{4} = \frac{3r-1-r}{r+1} = \frac{2r-1}{r+1}$$

$$5r+5 = 8r-4 \Rightarrow 9 = 3r \Rightarrow r = 3$$

26. Ans: 192

Sol: Number of numbers = $5!$ + numbers of 4 digit numbers beginning b, 7 or 8
 $= 5! + 3 \times 4P_3$
 $= 120 + 72$
 $= 192$

27. Ans: $\frac{3}{2}$

Sol: Put $x = 1$

$$\left(1 - \frac{1}{3}\right)^{199} \times \left(1 + \frac{1}{2}\right)^{200}$$

$$\left(\frac{2}{3}\right)^{199} \times \left(\frac{3}{2}\right)^{200} = \frac{3}{2}$$

28. Ans: $\frac{3}{2}, 4$

Sol: $na = 6, \frac{n(n-1)}{2}a^2 = \frac{27}{2} \Rightarrow (n-1)a = \frac{9}{2}$

$$\therefore 6 - a = \frac{9}{2} \Rightarrow a = \frac{3}{2}, n = 4$$

29. Ans: $\frac{1}{(n+1)(n+2)}$

Sol: $x(1-x)^n = C_0x - C_1x^2 + C_2x^3 - \dots + (-1)^n C_n x^{n+1}$

$$\int_0^1 x(1-x)^n dx = \left[\frac{C_0x^2}{2} - \frac{C_1x^3}{3} + \frac{C_2x^4}{4} - \dots + (-1)^n C_n \frac{x^{n+2}}{n+2} \right]_0^1$$

$$\therefore \frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \frac{C_3}{5} + \dots + (-1)^n \frac{C_n}{n+2}$$

$$= \int_0^1 x(1-x)^n dx$$

$$= \int_0^1 (x^n - x^{n+1}) dx$$

$$= \frac{1}{n+1} - \frac{1}{n+2} = \frac{1}{(n+1)(n+2)}$$

30. Ans: -2, -1

Sol: $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$$a + 4 + 2b = 0$$

$$2a + 2 - 2b = 0$$

$$\Rightarrow a = -2 \text{ and } b = -1$$

31. Ans: -3

Sol: $A^{-1} = \frac{1}{2x} \begin{bmatrix} x & -1 \\ 0 & 2 \end{bmatrix}$

$$\therefore \text{comparing } \frac{-1}{2x} = \frac{1}{6}$$

$$\therefore x = -3$$

32. Ans: $A^{10} = \begin{bmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & \cos 10\alpha \end{bmatrix}$

Sol: By induction $A^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

$$\therefore A^{10} = \begin{bmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & \cos 10\alpha \end{bmatrix}$$

33. Ans: $7A^8$

Sol: $A^2 = I$

$$\therefore A^2 + 2A^4 + 4A^6 = 7I = 7A^8$$

34. Ans: -2

Sol: $A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix} = I$

$$\Rightarrow x = 0$$

$$\therefore x^2 + x - 2 = -2$$

35. Ans: $\frac{c-b}{a+b+c}$

Sol: Adding $2(a+b+c)(x+y+z) - (a+b+c)(x+y+z) = 0$

$$\text{ie., } x+y+z = 0 \text{ (}\Theta a+b+c \neq 0\text{)}$$

$$\therefore \text{for (1)}$$

$$(b+c)(-x) - ax = b-c$$

$$\therefore x = -\frac{b-c}{a+b+c} = \frac{c-b}{a+b+c}$$

36. Ans: $(-\frac{2}{3}, 8)$

Sol: $4x^2 - 12x + 9 < x^2 + 10x + 25$

$$3x^2 - 22x - 16 < 0$$

$$3x^2 - 24x + 2x - 16 < 0$$

$$3x(x-8) + 2(x+8) < 0$$

$$(3x + 2)(x - 8) < 0$$

$$\Rightarrow x \in \left(-\frac{2}{3}, 8\right)$$

$$\Rightarrow x = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

37. Ans: $(-\infty, 2) \cup [7, \infty)$.

Sol: $\frac{x+3}{x-2} - 2 \leq 0$
 $\frac{7-x}{x-2} \leq 0$
 $\therefore x \neq 2$ and $(x-2)(x-7) \geq 0$
 $\therefore x \in (-\infty, 2) \cup [7, \infty)$.

38. Ans: Roses are not red or the sun is a star

Sol: Obvious

39. Ans: $\sim q \vee \sim p$

Sol: $p \rightarrow \sim q = \sim p \vee \sim q$
 $= \sim q \vee \sim p$

40. Ans: $(\sim p \wedge q) \vee \sim q$

Sol: $\sim [(p \vee \sim q) \wedge q] = \sim (p \vee \sim q) \vee \sim q$
 $= (\sim p \wedge q) \vee \sim q$

41. Ans: 0

Sol: $\cos 20^\circ - \cos 80^\circ - \cos 40^\circ$
 $= \cos 20^\circ - 2 \cdot \frac{1}{2} \cdot \cos 20^\circ = 0$

42. Ans: 1

Sol: $\frac{\cos^2 \frac{\pi}{4} - \sin^2 \theta}{\sin^2 \frac{\pi}{4} - \sin^2 \theta} = 1$

43. Ans: 0

Sol: $\sin(\theta + \alpha) \cos \alpha - \cos(\theta + \alpha) \sin \alpha$
 $= 3 [\sin(\theta + \alpha) \cos \alpha + \cos(\theta + \alpha) \sin \alpha]$
 $2 \sin(\theta + \alpha) \cos \alpha = -4 \cos(\theta + \alpha) \sin \alpha$
 $\therefore \tan(\theta + \alpha) + 2 \tan \alpha = 0$

44. Ans: $\cot \beta$

Sol: $\frac{2 \cos \frac{\alpha+\gamma}{2} \cdot \sin \frac{\alpha-\gamma}{2}}{-2 \sin \frac{\alpha+\gamma}{2} \cdot \sin \frac{\gamma-\alpha}{2}}$
 $= \cot \beta$ since $\frac{\alpha+\gamma}{2} = \beta$.

45. Ans: $\frac{\sqrt{3}}{2}$

Sol: $3 \sin^{-1} x = \pi$

46. Ans: $\sin^4 \theta$.

Sol: $\frac{1}{8} (2 - 4 \cos 2\theta + 2 \cos^2 2\theta)$
 $\frac{1}{4} (1 - 2 \cos 2\theta + \cos^2 2\theta)$
 $\frac{1}{4} (1 - \cos 2\theta)^2 = \frac{1}{4} \times 4 \sin^4 \theta = \sin^4 \theta$.

47. Ans: $-\frac{31}{8}$

Sol: $8 \cos^2 2\theta - 65 \cos 2\theta + 8 = 0$
 $8 \cos^2 2\theta - 64 \cos 2\theta - \cos 2\theta + 8 = 0$
 $8 \cos 2\theta (\cos 2\theta - 8) - (\cos 2\theta - 8) = 0$
 $(8 \cos 2\theta - 1)(\cos 2\theta - 8) = 0$
 $\therefore \cos 2\theta = \frac{1}{8}$
 $4 \cos 4\theta = -4(1 - 2 \cos^2 2\theta)$
 $= -4 \left(1 - \frac{2}{64}\right)$
 $= -\frac{31}{8}$

48. Ans: $\frac{3\pi}{4}$

Sol: it is $\pi + \tan^{-1} \frac{2+3}{1-6}$
 $= \frac{3\pi}{4}$

49. Ans: $2 \leq k \leq 6$

Sol: $k \sin x + 1 - 2 \sin^2 x = 2k - 7$
 $2 \sin^2 x - k \sin x + 2k - 8 = 0$
 $\sin x = 2$ is an inadmissible root
The other root is $\frac{k-4}{2}$
 $\therefore |k-4| \leq 2$
 $-2 \leq k-4 \leq 2$
 $2 \leq k \leq 6$

50. Ans: $2 \left| a \sin \frac{\alpha-\beta}{2} \right|$

Sol: $|a| \sqrt{4 \sin^2 \frac{\alpha+\beta}{2} \cdot \sin^2 \frac{\alpha-\beta}{2} + 4 \cos^2 \frac{\alpha+\beta}{2} \cdot \sin^2 \frac{\alpha-\beta}{2}}$
 $= 2 \left| a \sin \frac{\alpha-\beta}{2} \right|$

51. Ans: 5

Sol: $AC = BD = \sqrt{3^2 + 4^2} = 5$

52. Ans: -2

Sol: $y - 2 = \frac{2-1}{1+\frac{1}{2}} = (x-1)$
 $= \frac{2}{3} (x-1)$

\therefore x intercept = $-3 + 1 = -2$

53. Ans: below the x-axis at distance of $\frac{3}{2}$

Sol: $ax + 2by + 3b + \lambda (b \times -2ay - 3a) = 0$

When $\lambda b + a = 0 \Rightarrow \lambda = -\frac{a}{b}$

Y-intercept = $-\frac{3b - 3\lambda a}{2b - 2\lambda a}$
 $= -\frac{3(b^2 + a^2)}{2(b^2 - c^2)} = \frac{-3}{2}$

54. Ans: $\frac{1}{p^2} + \frac{1}{q^2}$

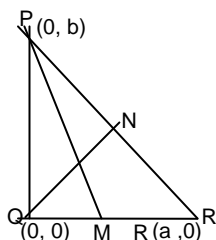
Sol: obviously

55. Ans: $x - y = 3$

Sol: slope :1, pt (2, -1)
 line is $y + 1 = x - 2$
 $\Rightarrow x - y - 3 = 0$

56. Ans: $a^2 = 2b^2$

Sol:



Slope of PM = $\frac{-2b}{a}$

Slope of QN = $\frac{b}{a}$

$\therefore -\frac{2b}{a} \cdot \frac{b}{a} = -1$

$\Rightarrow 2b^2 = a^2$

57. Ans: 0

Sol: Slope of the line parallel to x-axis

\therefore slope = 0

58. Ans: $\sqrt{\mu - \lambda}$

Sol: Length of the tangent from (x, y) to second

circle = $\sqrt{x_1^2 + y_1^2 + 2fy_1 + \mu}$

But $x_1^2 + y_1^2 + 2fy_1 = -\lambda$

$\therefore = \sqrt{\mu - \lambda}$

59. Ans: 20

Sol: Point (4, -3) lies outside the circle

\therefore Radius of circle = 6

Distance from centre (-2, 5) to (4, -3)
 is 10

\therefore Maximum distance = $10 + 6 = 16$

Minimum distance = $10 - 6 = 4$

\therefore sum = $16 + 4 = 20$

60. Question wrong

61. Ans: $x = -\frac{1}{2} + \cos \theta$ $y = \frac{-\sqrt{3}}{2} + \sin \theta$

Sol: Given equation is

$\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{\sqrt{3}}{2}\right)^2 = 1$

\therefore parametric equation is

$x = -\frac{1}{2} + \cos \theta$ $y = \frac{-\sqrt{3}}{2} + \sin \theta$

62. Ans: $8\sqrt{3}$

Sol: $y = \frac{1}{\sqrt{3}}x$, $y^2 = 4x$

$\frac{x^2}{3} = 4x$

$\therefore x = 12$ ($x \neq 0$)

\therefore point is $(12, 4\sqrt{3})$

\therefore side = $\sqrt{144 + 48}$

= $\sqrt{192}$

= $8\sqrt{3}$

63. Ans: $8x - 5 = 0$

Sol: $y^2 = 4\left(\frac{5}{8}\right)x$

Equation is $x = \frac{5}{8}$ ie $8x - 5 = 0$

64. Ans: 2

Sol: clearly the point $(\pm 2, 0)$ are the foci

"2a" = 8, "2ae" = 4

$\therefore a^2 - b^2 = 4$

$a^2 = 16$

$b^2 = 12$

ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$

$$\frac{x^2}{16} + \frac{9}{12} = 1$$

$$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2.$$

65. Ans: $\left(-\frac{7}{2}, 1\right)$

Sol: $y^2 + 6x - 2y + 13 = 0$

$$(y - 1)^2 = -6x - 12$$

$$= 6(-x - 2)$$

$$Y^2 = -4\left(\frac{3}{2}\right)(X)$$

focus is $\left(-\frac{3}{2}, 0\right)$

ie $\left(-\frac{7}{2}, 1\right)$.

66. Ans: $2\sqrt{5}$

Sol: $y = x^2 - 4x + 3$

$$y + 1 = (x - 2)^2$$

vertex is $(2, -1)$

$$x^2 = 9 - (y - 3)^2$$

centre is $(0, 3)$

$$\text{distance} = \sqrt{4 + 16}$$

$$= 2\sqrt{5}.$$

67. Ans: $|a|^4 b$.

Sol: $\bar{a} \times \{a \times \{a \times (a \times b)\}\}$

$$= a \times \{a \times \{(a \cdot b)a - |a|^2 b\}\}$$

$$= a \times \{-|a|^2 a \times b\} \quad (a \cdot b) = 0$$

$$= -|a|^2 a \times (a \times b)$$

$$= -|a|^2 \{(a \cdot b)a - |a|^2 b\}$$

$$= |a|^4 b.$$

68. Ans: -6

Sol: $\sqrt{64 + a^2} = 10$

$$a^2 = 100 - 64 = 36$$

$$\therefore a = \pm 6$$

But $8i + aj$ has same direction $-4i - 3j$

$$\therefore a = -6.$$

69. Ans: -5

Sol: $\bar{a} \cdot m\bar{b} = 120$

$$\Rightarrow m(-24) = 120 \Rightarrow m = -5.$$

70. Ans: 90°

Sol: clearly $\bar{a}, \bar{b}, \bar{c}$ from triangle

\therefore angle between

$$\bar{a} \text{ and } \bar{b} = 180 - (65 + 25) = 90^\circ.$$

71. Ans: $2\hat{i} + 3\hat{j} + \hat{k}$

Sol: Vertex $A = 2i + 6j + 4k$

Centroid = $2i + 4j + 2k$

D divides AG externally in the ratio 3 : 1

$$\therefore D \text{ is } \frac{3(2i + 4j + 2k) - (2i + 6j + 4k)}{3 - 1}$$

$$= 2i + 3j + k.$$

72. Ans: 3

Sol: $\frac{2 - 2a - 1}{\sqrt{6}} = \frac{-5}{\sqrt{6}}$

$$\Rightarrow a = 3.$$

73. Ans: $\frac{1}{4}[(\sqrt{6} + \sqrt{2})\hat{i} + (\sqrt{6} - \sqrt{2})\hat{j}]$

Sol: $\frac{x + y}{\sqrt{2}} = \frac{\sqrt{3}}{2}$

$$\frac{x - y}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$x + y = \frac{\sqrt{6}}{2}$$

$$x - y = \frac{\sqrt{2}}{2}$$

$$x = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$y = \frac{\sqrt{6} - \sqrt{2}}{4}.$$

74. Ans: 0°

Sol: $(2i + 3j + 4k)(i + 2j - 2k) = 2 + 6 - 8 = 0$

\therefore line is perpendicular to the normal to Plane

\therefore line is parallel to the plane

Angle is 0 .

75. Ans: $(4, 0, -1)$

Sol: $1 + 3\lambda = 4 + 2\mu$

$$1 - \lambda = 0 \Rightarrow \lambda = 1, \mu = 0$$

And the point is $(4, 0, -1)$

76. Ans: $6x + 3y + 2z - 6 = 0$

Sol: Obviously

77. Ans: 17

Sol: Length = $\sqrt{9^2 + 12^2 + 8^2}$
 $= \sqrt{15^2 + 8^2} = 17.$

78. Ans: $\frac{x-1}{1} = \frac{y}{-2} = \frac{z+2}{5}$

Sol: Obviously

79. Ans: $3x - 2y + 4z = 11$

Sol: $3(x-1) - 2(y-2) + 4(z-3) = 0$
 $3x - 2y + 4z - 11 = 0.$

80. Ans: 0

Sol: $\begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & 0 \\ k & 2 & 1 \end{vmatrix} = 0$
 $\Rightarrow k = 0.$

81. Ans: perpendicular to x-axis

Sol: Direction ratios of the line : (0, 1, 2)
 Direction ratios of X axis : (1, 0, 0)
 \therefore perpendicular to X axis.

82. Ans: 25

Sol: Total = $9 \times 15 = 135$
 $135 + x = 160$
 $x = 160 - 135$
 $= 25.$

83. Ans: $\frac{50}{101}$

Sol: $\frac{99 \times 100}{2} + x = 100x$
 $99 \times 50 + x = 100x \times 100$
 $9999x = 99 \times 50$
 $x = \frac{99 \times 50}{9999} = \frac{50}{101}.$

84. Ans: 24

Sol: $\frac{x}{5}, \frac{x}{4}, \frac{x}{3}, \frac{x}{2}, x$
 $\frac{x}{3} = 8$
 $x = 24.$

85. Ans: 27.1

Sol: 10×2.71

86. Ans: [-3, -2]

Sol: $-1 \leq \log_2(x^2 + 5x + 8) \leq 1$
 $\Rightarrow 2^{-1} \leq x^2 + 5x + 8 \leq 2$

Case 1 :

$x^2 + 5x + 8 \geq \frac{1}{2}$
 $\Rightarrow \left(x + \frac{5}{2}\right)^2 + \frac{7}{4} \geq \frac{1}{2}$

It is true $\forall x \in \mathbb{R}.$

Case 2

$x^2 + 5x + 8 \leq 2$
 $\Rightarrow x^2 + 5x + 6 \leq 0$
 $\Rightarrow (x+2)(x+3) \leq 0$
 $\Rightarrow x \in [-3, -2]$

87. Ans: 20

Sol: $\lim_{x \rightarrow 0} \frac{(1+2x)^{10} - 1}{x} = 20(1+2x)^9 = 20.$

88. Ans: $[2\log_e 2, \infty)$

Sol: $f(x) = \log_e(3x^2 + 4)$
 $y = \log_e(3x^2 + 4)$
 $3x^2 + 4 = e^y$
 $3x^2 = e^y - 4$
 $x^2 = \frac{e^y - 4}{3}$
 $x = \sqrt{\frac{e^y - 4}{3}}$
 $e^y - 4 \geq 0$
 $e^y \geq 4$
 $y \geq \log_e 4 = 2 \log_e 2.$

89. Ans: $\frac{100}{77} \times 2^{23}$

Sol: $\lim_{x \rightarrow 2} \frac{x^{100} - 2^{100}}{x^{77} - 2^{77}} = \frac{100x^{99}}{77x^{76}}$
 $= \frac{100 \times 2^{99}}{77 \times 2^{76}}$
 $= \frac{100}{77} \times 2^{99-76}$
 $= \frac{100}{77} \times 2^{23}.$

90. Ans: $\frac{1}{4}$

Sol: $\lim_{k \rightarrow \infty} \frac{K^2(k+1)^2}{4K^4} = \lim_{k \rightarrow \infty} \frac{k^4 \left(1 + \frac{1}{k}\right)^2}{4k^4}$
 $= \frac{1}{4}.$

91. Ans: $-\frac{1}{2}$

Sol: $y = \frac{1}{\operatorname{cosec}^2 \cot^{-1} \sqrt{\frac{1+x}{1-x}}}$
 $= \frac{1}{1 + \cot^2 \cot^{-1} \sqrt{\frac{1+x}{1-x}}} = \frac{1}{1 + \frac{1+x}{1-x}}$
 $= \frac{1-x}{1-x+1+x} = \frac{1-x}{2}$
 $\frac{dy}{dx} = \frac{-1}{2}$

92. Ans: $\frac{1}{3}$

Sol: $x = 3\sin^{-1}t$
 $\frac{dx}{dt} = \frac{3 \times 1}{\sqrt{1-t^2}}$
 $y = \sin^{-1}t \quad \frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}}$
 $\frac{dy}{dx} = \frac{1}{3}$

93. Ans: 274

Sol: At $x=0$
 $\frac{dy}{dx} = (x+1)(x+2)(x+3)(x+4) \rightarrow 24$
 $+ (x+1)(x+2)(x+3)(x+5) \rightarrow 30$
 $+ (x+1)(x+2)(x+4)(x+5) \rightarrow 40$
 $+ (x+1)(x+3)(x+4)(x+5) \rightarrow 60$
 $+ (x+2)(x+3)(x+4)(x+5) \rightarrow 120$
 $\hline = 274$

94. Ans: $-\frac{1}{2}$

Sol: $y = \cot^{-1} \tan\left(\frac{x}{2}\right) = \cot^{-1} \cot\left(\frac{\pi}{2} - \frac{x}{2}\right)$
 $= \frac{\pi}{2} - \frac{x}{2}$
 $\frac{dy}{dx} = \frac{-1}{2}$

95. Ans: 2

Sol: $y = (\sin^{-1}x)^2$
 $\frac{dy}{dx} = 2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}}$
 $\sqrt{1-x^2} \frac{dy}{dx} = 2\sin^{-1}x$

$$\sqrt{1-x^2} \frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{-2x}{2\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$$

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 2$$

96. Ans: -1

Sol: $x^y y^x = 16$
 $y \log x + x \log y = \log 16$
 $\frac{y}{x} + \log x \frac{dy}{dx} + \log y + \frac{x}{y} \frac{dy}{dx} = 0$

$$\frac{dy}{dx} \left(\log x + \frac{x}{y} \right) = \frac{y}{x} - \log y$$

at (2, 2) $\frac{dy}{dx} (\log 2 + 1) = -1 - \log 2$

$$\frac{dy}{dx} = -1$$

97. Ans: $2\cos(2y) - 5$

Sol: $\sin 2y = x + 5y$
 $\Rightarrow x = \sin 2y - 5y$
 $\Rightarrow \frac{dx}{dy} = 2\cos 2y - 5$

98. Ans: 416

Sol: $R(x) = 13x^2 + 26x + 15$
 $\frac{d}{dx} R(x) = 26x + 26$
 Marginal revenue at $x = 15$
 $= 26 \times 15 + 26$
 $= 390 + 26 = 416$

99. Ans: $(0, 1) \cup (2, \infty)$

Sol: $f(x) = x^2(x-2)^2$
 $f'(x) = x^2 \cdot 2(x-2) + (x-2)^2 \times 2x$
 $= 2x(x-2)(x+x-2)$
 $= 2x(x-2)(2x-2)$
 $= 4x(x-2)(x-1)$
 > 0
 $(0, 1) \cup (2, \infty)$

100. Ans: (6,7)

Sol: $\frac{dy}{dx} = \frac{4}{2\sqrt{4x+1}}$
 $= \frac{2}{\sqrt{4x+1}}$
 $\frac{2}{\sqrt{4x+1}} = \frac{2}{5}$
 $\Rightarrow 4x+1 = 25$
 $\Rightarrow 4x = 24$
 $\Rightarrow x = 6$
 $\Rightarrow y = 2 + \sqrt{24+1}$
 $= 2+5$
 $= 7$

∴ The required point is (6,7)

101. Ans: $y = \frac{1}{4}$

Sol: $y = \frac{1}{(x^2 + 2x + 5)^2}$
 $\frac{dy}{dx} = -\frac{1(2x+2)}{(x^2 + 2x + 5)^2}$
 $= 0 \Rightarrow x = -1$
 $y = \frac{1}{(-1)^2 + 2(-1) + 5}$
 $= \frac{1}{4}$

102. Ans: $9x + y - 6 = 0$

Sol: $x = \frac{t-1}{t+1}$ $y = \frac{t+1}{t-1}$
 $\Rightarrow \frac{dx}{dt} = \frac{(t+1) - (t-1)}{(t+1)^2} = \frac{2}{(t+1)^2}$
 $\& \frac{dy}{dt} = \frac{(t-1)(t+1)}{(t-1)^2} = \frac{12}{(t-1)^2}$
 $\Rightarrow \frac{dy}{dx} = \frac{-2}{(t-1)^2} \cdot \frac{(t+1)^2}{2}$
 $= -\left[\frac{t+1}{t-1}\right]^2$
 $\Rightarrow \frac{dy}{dx} \Big|_{t=2} = -\left[\frac{3}{1}\right]^2 = -9$
 $x = \frac{1}{3}$ & $y = 3$ at $t = 2$
 \therefore Equation of tangent at $t=2$ is given by,
 $y - 3 = -9\left(x - \frac{1}{3}\right) = -3(3x - 1)$ or
 $9x + y - 6 = 0$

103. Ans: $(-6, 3)$

Sol: All the points in the given options are points of the conic.
 Distance of the point $(-6, 3)$ to the line $3x + 2y + 1 = 0$ is minimum.

104. Ans: $\frac{25}{4}$

Sol: $\frac{1}{2\sqrt{x}} = \frac{3-2}{5}$
 $\rightarrow x = \frac{25}{4}$

105. Ans: $2 \tan^{-1} \sqrt{x} + C$

Sol: $\sqrt{x} = t$
 $\Rightarrow 2 \int \frac{dt}{t^2 + 1}$
 $= 2 \tan^{-1} \sqrt{x} + C$

106. Ans: $-\frac{\log x}{x} - \frac{1}{x} + C$

Sol: $\cos x = t$
 $\Rightarrow \int te^{-t} dt$
 $= \frac{te^{-t}}{-1} - \int \frac{e^{-t}}{-1} dt$
 $= -te^{-t} - e^{-t} + C$
 $= -\frac{\log x}{x} - \frac{1}{x} + C.$

107. Ans: $\tan x$

Sol: $\frac{f(x)}{\log \cos x} = \frac{-1}{\log \cos x} \times \frac{-\sin x}{\cos x}$
 $= \frac{\tan x}{\log \cos x}$
 $\Rightarrow f(x) = \tan x$

108. Ans: $x - \sqrt{1-x^2} \sin^{-1} x + C$

Sol: $\sin^{-1} x = t \Rightarrow$
 $I = \int t \sin t dt$
 $= -t \cos t - \int -\cos t dt$
 $= -t \cos t + \sin t + C$
 $= -\sqrt{1-x^2} \sin^{-1} x + x + C$

109. Ans: $-\frac{3}{2}x + \frac{35}{36} \log |9e^{2x} - 4| + C$

Sol: $\frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} = \frac{4e^{2x} + 6}{9e^{2x} - 4}$
 $4e^{2x} + 6 = A(18e^{2x}) + B(9e^{2x} - 4)$
 $\Rightarrow -4B = 6$ or $B = -\frac{3}{2}$
 and $18A + 9B = 4$
 $\Rightarrow 18A - \frac{27}{2} = 4$
 $\Rightarrow A = \frac{1}{18} \left[4 + \frac{27}{2} \right] = \frac{35}{36}$
 $\therefore \int \frac{(4e^x + 6e^{-x}) dx}{9e^x - 4e^{-x}} = \frac{35}{36} \int \frac{18e^{2x}}{9e^{2x} - 4} dx + \frac{-3}{2} \int dx$
 $= \frac{35}{36} \log |9e^{2x} - 4| - \frac{3}{2}x + C.$

110. Ans: $\sin^{-1}x + \sqrt{1-x^2} + C$

Sol:
$$\int \frac{(1-x)dx}{\sqrt{1-x^2}} = \int \frac{dx}{\sqrt{1-x^2}} + \frac{1}{2} \int \frac{-2xdx}{\sqrt{1-x^2}}$$

$$= \sin^{-1}x + \sqrt{1-x^2} + C.$$

111. Ans: $\frac{x}{2} + \frac{1}{2} \log|\cos x + \sin x| + C$

Sol:
$$\int \frac{\cos x dx}{\cos x + \sin x}$$

$$= \frac{1}{2} \left[\int dx + \int \frac{\cos x - \sin x}{\cos x + \sin x} \right]$$

$$= \frac{x}{2} + \frac{1}{2} \log|\cos x + \sin x| + C$$

112. Ans: -1

Sol:
$$-\int_a^0 (x+1)dx = -\frac{1}{2}$$

$$\left(\frac{x^2}{2} + x\right)_a^0 = \frac{1}{2}$$

$$\Rightarrow -\frac{a^2}{2} - a = \frac{1}{2}$$

$$\Rightarrow a^2 + 2a + 1 = 0$$

$$\Rightarrow (a+1)^2 = 0$$

$$\Rightarrow a = -1$$

113. Ans: $\frac{1}{42}$

Sol:
$$\int_0^1 x(1-x)^5 dx = \int_0^1 (1-x)x^5 dx$$

$$= \int_0^1 (x^5 - x^6) dx = \left[\frac{x^6}{6} - \frac{x^7}{7} \right]_0^1$$

$$= \frac{1}{6} - \frac{1}{7} = \frac{1}{42}.$$

114. Ans: 3

Sol:
$$\int_0^2 |x-2| dx = \int_0^2 (2-x) dx = \left[2x - \frac{x^2}{2} \right]_0^2$$

$$= 4 - \frac{4}{2} = 2$$

$$\int_0^2 [x] dx = \int_0^1 [x] dx + \int_1^2 [x] dx = 0 + \int_1^2 1 dx = 1$$

$$\therefore \int_0^2 (|x-2| + [x]) dx = 2 + 1 = 3.$$

115. Ans: $\frac{1}{25} - \frac{6e^{-5}}{25}$

Sol:
$$\int_0^1 xe^{-5x} dx = \left(\frac{xe^{-5x}}{-5} \right)_0^1 - \int_0^1 \frac{e^{-5x}}{-5} dx$$

$$= -\frac{1}{5}e^{-5} + \frac{1}{5} \left(\frac{e^{-5x}}{-5} \right)_0^1$$

$$= -\frac{e^{-5}}{5} - \frac{1}{25} [e^{-5} - 1]$$

$$= \frac{1}{25} - e^{-5} \left[\frac{1}{5} + \frac{1}{25} \right]$$

$$= \frac{1}{25} - e^{-5} \left[\frac{5+1}{25} \right]$$

$$= \frac{1}{25} - \frac{6e^{-5}}{25}.$$

116. Ans: 4

Sol: Area = $4 \int_0^{\frac{\pi}{2}} \sin x dx = 4[-\cos x]_0^{\frac{\pi}{2}}$
 $= 4.$

117. Ans: order 1; degree 3

Sol: $y^2 = 2c(x + \sqrt{c})$ ----- (1)
 $\Rightarrow y^2 = 2cx + 2c\sqrt{c}$
 $\Rightarrow 2y \frac{dy}{dx} = 2c \Rightarrow c = y \frac{dy}{dx} = yy_1$
 Substituting in (1)
 $y^2 = 2yy_1 [x + \sqrt{yy_1}]$
 $\Rightarrow y^2 = 2xyy_1 + 2(yy_1)^{3/2}$
 $\Rightarrow [y^2 - 2xyy_1]^2 = 4(yy_1)^3$
 \therefore order = 1, degree = 3.

118. Ans: $\sqrt{1+x^2}$

Sol: $(1+x^2) \frac{dy}{dx} + xy = x$
 $\Rightarrow \frac{dy}{dx} + \left(\frac{x}{1+x^2} \right) y = \frac{x}{(1+x^2)}$
 I.F = $e^{\int \frac{x}{1+x^2} dx} = e^{\frac{1}{2} \log(1+x^2)} = \sqrt{1+x^2}.$

119. Ans: $x^2y + 1 = 3x$

Sol: $x \frac{dy}{dx} + y = \frac{1}{x^2}$
 $\Rightarrow \frac{d}{dx}(xy) = \frac{1}{x^2}$
 $\Rightarrow xy = \int \frac{1}{x^2} dx = -\frac{1}{x} + C$
Substituting $x = 1, y = 2$
 $\Rightarrow 2 = \frac{-1}{1} + C \Rightarrow C = 3$
 $\therefore xy = \frac{-1}{x} + 3$
 $\Rightarrow x^2y + 1 = 3x$

120. Ans: $e^{-y} = e^{-x} - e^x - x^2 + C$

Sol: $\frac{dy}{dx} = e^y [e^x + e^{-x} + 2x]$
 $\int e^{-y} dy = \int (e^x + e^{-x} + 2x)$
 $\Rightarrow -e^{-y} = e^x - e^{-x} + x^2 + C$
 $\Rightarrow e^{-y} = e^{-x} - e^x - x^2 + C.$



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