Code No: RR10102 **Set No.1**

I B.Tech. Regular Examinations, June -2005
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Bio-Technology)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Test the convergence of the following series $\sum_{n=1}^{\infty} \frac{1.3.5.....(2n+1)}{2.5.8....(3n+2)}$
   (b) Test the following series for absolute /conditional convergence $\sum_{n=1}^{\infty} (-1)^n \frac{\log n}{n^2}$
   (c) Expand sin x in powers of $\left(x - \Pi/2\right)$ up to the term containing $\left(x - \Pi/2\right)^4$
2. (a) Expand f(x,y) = e^ylog(1+x) in powers of x and y.
   (b) Show that the evolute of $x = a\left(Cos\theta + \log\tan\frac{\theta}{2}\right)$, y = a Sinθ is the catenary $y = a\,Cos\,h\,\frac{x}{a}$
3. (a) Trace the curve $9ay^2 = (x - 2a)(x - 5a)^2$.
   (b) Find the volume of the solid generated by revolving the lemniscates $r^2 = a^2$ Cos 2θ about the line $\theta = \frac{\pi}{2}$.
4. (a) Form the differential equation by eliminating the arbitrary constant: $x^2+y^2 = c$.
   (b) Solve the differential equation: $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1 + x)\, e^x \sec y$.
   (c) Find the orthogonal trajectories of the coaxial circles $x^2 + y^2 + 2\lambda y + c = 2$, λ being a parameters.
5. (a) Solve the differential equation: $(D^4 - 5D^2 + 4)y = 10\, cosx$.
   (b) Solve the differential equation: $(x^2D^2 - x^3D + 1)y = \frac{\log x \sin(\log x)+1}{x}$
6. (a) State and prove second shifting theorem.
   (b) Find the inverse Laplace Transformation of $\left[\frac{s+3}{(s^2+6s+13)^2}\right]$
   (c) Evaluate $\int\int\int z^2 dxdydz$ taken over the volume bounded by $x^2 + y^2 = a^2$, $x^2 + y^2 = z$ and $z = 0$.

7. (a) Show that $\nabla \times \left[\frac{\overline{A} \times \overline{r}}{r^3}\right] = -\frac{A}{r^3} + \left[3\,\overline{r}\,\frac{(\overline{A}\,.\,\overline{r})}{r^5}\right]$

   (b) If $\overline{F} = y\,i + x(1-2z)\,j - xy\,k$ evaluate $\int\limits_C \nabla \times \overline{F}\,.\,N\,ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ above the xy plane.

8. Verify stokes theorem for $\mathbf{F}=(x^2+y^2)\,\mathbf{i}-2xy\mathbf{j}$ taken around the rectangle bounded by the lines x $=\pm$a, y=0, y=6.

⋆⋆⋆⋆⋆

Code No: RR10102 **Set No.2**

I B.Tech. Regular Examinations, June -2005
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Bio-Technology)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Test the convergence of the series
$\frac{x}{1} + \frac{1.x^3}{2.3} + \frac{1.3x^5}{2.4.5} + \frac{1.3.5x^7}{2.4.6.7} + \ldots(x > 0)$
(b) Examine whether the following series is absolutely convergent or conditionally convergent $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots.(x > 0)$
(c) Write the Maclaurins series with Lagrange's form of remainder for f(x) = cosx.

2. (a) Locate the stationary points and examine their nature of the following functions:
$u = x^4 + y^4 - 2x^2 + 4xy - 2y^2, (x > 0, y > 0)$
(b) Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where $a^2 + b^2 = k^2$, k being a constant.

3. Trace the lemniscate of Bernouli : $r^2 = a^2 \cos 2\theta$. Prove that the volume of revolution about the initial line is $\frac{\pi a^3}{6\sqrt{2}} \left[3 \log(\sqrt{2} + 1) - \sqrt{2}\right]$

4. (a) Form the differential equation by eliminating the arbitrary constant
$y = 1 + c\sqrt{1 - x^2}$
(b) Solve the differential equation:
$$[\cos x \tan y + \cos(x+y)]\, dx + [\sin x \sec^2 y + \cos(x+y)]\, dy = 0.$$
(c) In a certain chemical reaction the rate of conversion of a substance at time t is proportional to the quantity of the substance still untransformed at that instant. At the end of one hour 60 grams remain and at the end of four hours 21 grams. How many grams of the first substance was there initially?

5. (a) Solve the differential equation: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\,Cosh x$ given that y(0) = 0, y′(0) = 1.
(b) Solve the differential equation: $(3x + 2)^2 \frac{d^2y}{dx^2} + 3(3x + 2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

6. (a) Define unit step function and find the Laplace Transform of unit step function.

(b) Find the inverse Laplace Transformation of $\left[\frac{s^2+s-2}{s(s+3)(s-2)}\right]$

(c) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$

7. (a) Evaluate $\nabla^2 \log r$ where $r = \sqrt{x^2+y^2+z^2}$

   (b) Find constants a, b, c so that the vector **A** =(x+2y+az)**i** +(bx-3y-z)**j**+(4x+cy+2z)**k** is irrotational. Also find φ such that $\mathbf{A} = \nabla\phi$.

8. (a) Apply Green's theorem to prove that the area enclosed by a plane curve is $\frac{1}{2}\oint_C$ (x d y - y d x). Hence find the area of an ellipse whose semi major and minor axes are of lengths a and b.

   (b) Evaluate $\iint_S (y^2z^2\,i + z^2x^2\,j + z^2y^2\,k)\,.\,N\,ds$ where S is the part of the unit sphere above the xy- plane.

⋆⋆⋆⋆⋆

Code No: RR10102 **Set No.3**

I B.Tech. Regular Examinations, June -2005
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Bio-Technology)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Test the convergence of the following series $\sum_{n=1}^{\infty} \frac{1.3.5.....(2n+1)}{2.5.8....(3n+2)}$
   (b) Test the following series for absolute /conditional convergence $\sum_{n=1}^{\infty} (-1)^n \frac{\log n}{n^2}$
   (c) Expand sin x in powers of $\left(x - \Pi/2\right)$ up to the term containing $\left(x - \Pi/2\right)^4$
2. (a) Locate the stationary points and examine their nature of the following functions:
   u = $x^4 + y^4 - 2x^2 + 4xy - 2y^2$, (x > 0, y > 0)
   (b) Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where $a^2 + b^2 = k^2$, k being a constant.
3. (a) Trace the curve r = a + b Cos θ. (a > b).
   (b) Find the surface area got by rotating the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$about the minor axis.
4. (a) Form the differential equation by eliminating the arbitrary constant x tan(y/x) = c.
   (b) Solve the differential equation: $\frac{dy}{dx}$ + y cos x = y^3sin 2x.
   (c) Find the orthogonal trajectories of the family of circles $x^2 + y^2 = ax$.
5. (a) Solve the differential equation: $(D^2 + 1)y = e^{-x} + x^3 + e^x \sin x$.
   (b) Solve the differential equation: $(D^2 + 1)y$ = x sinx by variation of parameters method.
6. (a) State and prove second shifting theorem.
   (b) Find the inverse Laplace Transformation of $\left[\frac{s+3}{(s^2+6s+13)^2}\right]$

(c) Evaluate $\int\int\int z^2 dxdydz$ taken over the volume bounded by $x^2 + y^2 = a^2$, $x^2 + y^2 = z$ and $z = 0$.

7. (a) Find the directional derivative of $\varphi = x^2yz + 4xz^2$ at (1, -2, -1) in the direction 2**i**-**j**-2**k**

   (b) Find **A** x $\nabla\phi$ if **A**$= yz^2i - 3xz^2j + 2xyzk$ and $\phi = xyz$

8. Verify divergence theorem for $\mathbf{F} = x^3\ \mathbf{i} + y^3\mathbf{j} + z^3\ \mathbf{k}$ taken over the surface of the sphere $x^2+y^2+z^2= a^2$.

⋆ ⋆ ⋆ ⋆ ⋆

Code No: RR10102 **Set No.4**

I B.Tech. Regular Examinations, June -2005
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Bio-Technology)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
⋆⋆⋆⋆⋆

1. (a) Test the following series for convergence or divergence.
   $\frac{1}{2} + \frac{\sqrt{2}}{3} + \frac{\sqrt{3}}{8} + \ldots\ldots + \frac{\sqrt{n}}{n^2-1}$
   (b) Test whether the following series is absolutely convergent.
   $\sum_{1}^{\infty} (-1)^{n+1} \left(\sqrt{n+1} - \sqrt{n}\right)$
   (c) Verify Lagrange's mean value theorem f(x) = log_e^x in [1,e].
2. (a) Find Taylor's expansion of f(x,y) = $\cot^{-1}$xy in powers of (x+0.5) and (y-2) up to second degree terms.
   (b) Show that the evolute of the curve x = a(Cos $\theta + \theta$ Sin θ), y = a(Sin $\theta + \theta$Cos θ) is a circle.
3. (a) Trace the Folium of Decartes : $x^3 + y^3 = 3axy$.
   (b) Determine the volume of the solid generated by revolving the limacon r = a + b cosθ (a>b) about the initial line.
4. (a) Form the differential equation by eliminating the arbitrary constant : log y/x = cx.
   (b) Solve the differential equation: dr + (2r cot θ + sin 2θ) dθ = 0.
   (c) The number N of bacteria in a culture groups at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after 1 ½ hour.
5. (a) Solve the differential equation: $(D^2 + 1)y = e^{-x} + x^3 + e^x$ sin x.
   (b) Solve the differential equation: $(D^2 + 1)y$ = x sinx by variation of parameters method.
6. (a) Find L [t e^{3t} Sin 2t]
   (b) Find $L^{-1}\left[\frac{s+3}{(s^2-10s+29)}\right]$

(c) Evaluate $\int_0^{\pi/4}\int_0^{a\,Sin\theta} \frac{r\,dr\,d\theta}{\sqrt{a^2-r^2}}$

7. (a) Find grad ϕ where $\phi = (x^2+y^2+z^2)e^{-\sqrt{x^2+y^2+z^2}}$.

   (b) Find the work done in moving a particle in the force field $\mathbf{F}=3x^2i + (2xz - y)j + zk$ along the space curve $x = 2t^2, y = t, z = 4t^2 - t$ from t=0 to t=1.

8. Verify divergence theorem for $2x^2y\mathbf{i} - y^2\mathbf{j} + 4xz^2\mathbf{k}$ taken over the region of first octant of the cylinder $y^2+z^2 = 9$ and x = 2.

⋆⋆⋆⋆⋆