Discovery Experiment: ***Oscillations and Resonance in an RLC Circuit***

Having completed this experiment you will have

- Used circuits containing inductors, capacitors and resistors
- Practised using an oscilloscope
- Practised building simple circuits
- Understood how a radio works

Preparatory task** [1 mark]**:

Read the entire script; familiarise yourself with the objectives of this practical and the skills required; write a brief plan and description in your lab book of what you will be doing; and complete the calculation below. If you are not confident in having acquired the skills you will be using today, make use of the web resource where you can gain further practice.

The circuit shown in figure 1 has a time varying voltage passing through it. It is clear that

$$V_1 - V_L = V_R\,, \tag{1}$$

if V_1 denotes the voltage across the terminals of the power supply (i.e., the "electromotive force", or "emf", of the power supply). Let us assume that the current is $I = I_0 \sin(\omega t)$. Then the voltage across the resistor is given by Ohm's law as

$$V_R = IR = I_0 \sin(\omega t) R\,, \tag{2}$$

and the voltage across the inductor is given by Lenz's law as

$$V_L = L\frac{dI}{dt} = LI_0\omega\cos(\omega t). \tag{3}$$

Therefore, we expect that the voltage drop across the inductor is π/2 out of phase with that across the resistor. The maximum of the voltage drop across the inductor, V_L^{MAX} is related to the maximum voltage drop across the resistor, V_R^{MAX}. Note that these maxima do not occur at the same time. Show, by eliminating I_0, that V_L^{MAX} and V_R^{MAX} are related by the equation

$$V_L^{MAX} = \left(\frac{\omega L}{R}\right)V_R^{MAX} = \left(\frac{2\pi fL}{R}\right)V_R^{MAX}\,. \tag{4}$$

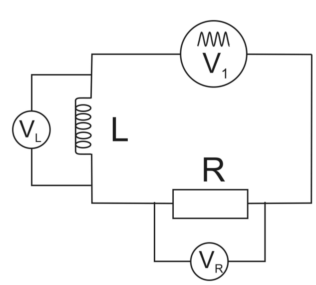

Figure 1: The RL circuit considered in the preliminary task.

1. What it's about

This practical investigates "RLC circuits" (also known as "LCR circuits"), namely circuits containing resistors (R), inductors (L) and capacitors (C). It builds on your previous experiences from the skills sessions where you looked at circuits with resistors and capacitors and measured voltages using an oscilloscope. In this practical we are primarily interested in driven ac RLC circuits. You will investigate the response of such circuits to different frequencies, and see what happens as the circuit enters resonance. You will investigate damping and oscillations which arise from the interaction between a capacitor and an inductor. By the end of the practical you will see how resonance effects are used to tune radios.

2. Preparation

A current passing through a coil produces a magnetic field - this is the principle of electromagnetic coils. If this current *changes as a function of time* then so does the magnetic field. The latter in turn produces an electromotive force (emf), namely a potential difference between the two ends of the coil. Lenz's law states that the emf opposes the change that has induced it. The effect is therefore known as back emf. The measure of the ability of coil to produce a back emf is known as its self-inductance, L, and is defined as:

$$EMF = L\frac{dI}{dt}. \tag{5}$$

3. Ready to Start

The components used in this experiment are set out on a circuit box. There is a fixed capacitor of 0.1 ±10% μF, a variable capacitor of 10-270 ±10% pF, and two resistors, 100 ±2% kΩ and 2.2 ±2%.Ω. The inductor is made of 70 turns of copper wire wound around an iron core.

Task 1:

Build the simple circuit shown in figure 2 using the circuit box. Use the 100 kΩ resistor for R. Adjust the function generator to give a 300 kHz, 5V sine wave. Connect the Y_A input to channel I and Y_B to channel II. Ensure that the dual button is pressed so that both traces are displayed and adjust the timebase so you can see both traces. The back emf of the inductor is displayed on channel I. Channel II shows the output voltage of the oscillator. Because the voltage drop across the inductor is much smaller than that across the resistor, the voltage displayed on channel II is, to an excellent approximation, that dropped across the resistor.

Which of the two signals should you trigger the oscilloscope on?

Make a quick sketch of the two waves in your lab book. Annotate you sketch to show the amplitudes of the voltages.

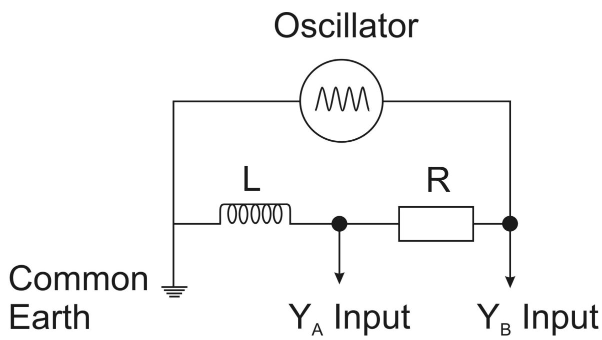

Figure 2: The circuit considered in tasks 1 and 2.

4. Measurements

4.1 An RL circuit

Task 2 *[2 marks]:*

Measure V_L over a range of frequencies (10 kHz – 400 kHz, with five points below 100 kHz and five points above 100 kHz) whilst keeping the amplitude of the supply voltage constant. (Keeping the amplitude of the oscillator constant means that V_R remains unchanged.) Investigate what happens at high frequencies: is there any deviation from linearity?

Use Excel to plot a graph of V_L^{MAX} against frequency (f). Determine the value of the inductance, L, and its error from the slope of the graph. (The SI unit of inductance is the henry.) Comment on whether deviation from linearity at the highest frequencies might affect your measurement of the inductance.

Was the signal dropped across the inductor difficult to measure at any particular frequencies? Is your assumption that the circuit is a pure RL circuit justified?

YOU SHOULD HAVE REACHED THIS STAGE WITHIN 1 HOUR

4.2 Oscillations in an RLC circuit

In practice it is almost impossible to construct a simple RL circuit. At high frequencies the capacitance of the interconnecting wires becomes significant. Thus we can only ever make RLC circuits, which under certain conditions can be approximated to an RL circuit.

Suppose the circuit shown in figure 3 has been pushed into electrical oscillations. This circuit is similar to that considered in figure 1, but here there is no external voltage (i.e., V_I=0). Hence, we can write $V_C = -V_L$. (Note that V_C and V_L are measured going round the circuit; therefore if V_C is measured from left to right, V_L is measured from right to left.) These voltages and the charge of the capacitor, q, vary in time.

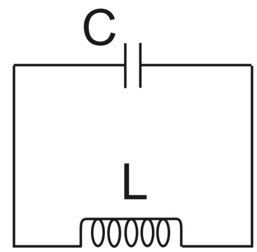

Figure 3: An LC circuit.

The instantaneous voltage across the capacitor, V_C, is given by $V_C = q/C$, and that across the inductor by equation (2), namely by $V_L = L\, dI/dt$. As the current flowing through the circuit is related to the charge of the capacitor by $I = dq/dt$, we arrive at the following equations:

$$L\frac{d}{dt}(I) = L\frac{d}{dt}\left(\frac{dq}{dt}\right) = L\frac{d^2q}{dt^2} = -V_C. \qquad (6)$$

From this we deduce the equation

$$\frac{d^2q}{dt^2} = -\frac{q}{LC} \qquad (7)$$

which you may recognize from the vibrations and waves course as an equation describing simple harmonic motion with a period $T_0 = 2\pi\sqrt{LC}$. The "natural frequency" of the circuit is defined as $f_0 = 1/T_0$.

Task 3:

Add the 0.1±10% μF capacitor across the inductor (as in the circuit shown in figure 4). Start the natural oscillations with a 1 kHz ***square*** *wave. Display the two voltage traces on the oscilloscope as before. Make a quick sketch of the two displays in your lab book.*

How many high frequency oscillations occur within one half cycle of the square wave? (This number may not be an integer. Give an estimate of the error.)

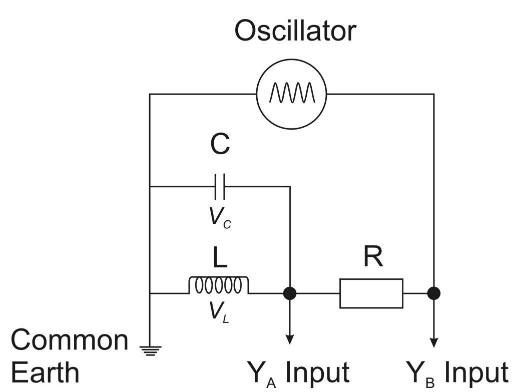

Figure 4: The circuit considered in tasks 3 – 6.

Task 4 *[1 mark]****:***

Calculate the period of these high frequency oscillations knowing that the number you recorded in task 3 occurred in 0.5 ms (for a 1 kHz square wave).

The inverse of this period gives the "resonant frequency of the circuit". What value do you measure this to be, and what is its error?

The inductance is also related to this resonant frequency through the equation

$$L = \frac{1}{\left(4\pi^2 f_o^2 C\right)} . \tag{8}$$

Knowing C and its error, what value do you obtain for the inductance L and its error?

How does this value compare with that you recorded in task 2? Are the two values consistent with each other? What is your best value for L and its error?

YOU SHOULD HAVE REACHED THIS STAGE WITHIN 1¾ HOURS

In task 3, you made sketches of the voltage traces. You should have noticed that the high frequency oscillations were slightly damped. This is because of the internal resistance of the coil. This modifies equations (6) to include an additional term which describes this damping:

$$L\frac{d^2q}{dt^2} = -\frac{q}{C} - r\frac{dq}{dt} , \tag{9}$$

where r is the resistance of the coil. If the oscillations last a long time (i.e., the damping is light), the circuit is said to have a high quality factor (or Q factor). This is a dimensionless constant defined by the equation

$$Q = \frac{2\pi f_0 L}{r} = \frac{\omega_0 L}{r} . \tag{10}$$

One can show that

$$Q = \frac{\pi n_{1/2}}{\ln(2)} , \tag{11}$$

where $n_{1/2}$ is the number of oscillations number at which the initial voltage has fallen to half its value. (This number may not be an integer.)

***Task 5** [1 mark]:*

Adjust the voltage on the function generator such that the amplitude of the first oscillation has a voltage that can be easily read from the oscilloscope. Find the oscillation number where the amplitude has fallen by half. (As noted above, this number may not be an integer.) What value of Q do you get using equation (11)? What is your error?

Using your experimentally determined value of Q, what do you get for the magnitude and error of the internal resistance r of the inductor using equation (10)? Does your value for the internal resistance of the coil seem reasonable?(Justify your answer by an order of magnitude estimate.)

YOU SHOULD HAVE REACHED THIS STAGE WITHIN 2¼ HOURS

4.3 Resonance in an RLC circuit

At 1 kHz, the circuit was set into oscillations. The amplitude of the voltage oscillations is related to the amount of power being absorbed by the system. This is a maximum at the resonant frequency of the circuit. The maximum is known as a resonance. You calculated the value of this resonant frequency in task 4.

***Task 6** [1 mark]:*

Reset the function generator to give a 5 V pk-pk sine wave. Vary the frequency of the applied sine wave in the vicinity of your predicted resonant frequency. Find the resonant frequency of your circuit, f_0. What is the best method of finding f_0? Write down the value you obtained and an estimate of its uncertainty, and compare it with your experimentally determined value from task 4.

Investigate the response of the circuit as the frequency is varied through the resonant frequency. What happens to the relative phase between the two signals as you tune the frequency through the resonance?

5. Exploration: Tuning a radio

The coil wound on the ferrite rod is commonly used as an aerial in an AM radio receiver. The radio signals excite the RLC circuit, which as we have seen has a maximum close to the resonant frequency. In the above experiments, the circuit impedance has been relatively small, but we can increase it by reducing the capacitance *C* from μF to pF. The circuit will now have a much higher natural frequency. Tuning a radio means matching the RLC circuit to have its resonant frequency at the same frequency of the radio station we wish to listen to. For most of the experiments we did above the properties of the circuit are fixed, and the frequency of the applied sine wave was varied; in contrast, for a radio the frequency of the incident electromagnetic waves are fixed, and we tune the resonant frequency of the circuit with a variable capacitor.

Modify the previous circuits and connect A and B using the marked dashed lines on the box. Instead of the fixed 0.1 μF capacitor, add the variable capacitor across the inductor. Disconnect the function generator.

Switch the radio on, and connect an external aerial. Adjust the resonant frequency by tuning the variable capacitor and listen to your favourite AM radio stations!

6. To conclude

You will cover simple harmonic motion and resonance in the Vibration and Waves Lecture course. More details on RL and RLC circuits can be found in R. Wolfson, *Essential University Physics, International Ed.* (Pearson Addison-Wesley, San Francisco, 2007), chapters 27 and 28.

For your report: Please ensure that all your measurements include appropriate error estimates in your write-up of this experiment. Include a thorough analysis of the systematic and the random errors which might affect your results. Also, make sure that you present your data clearly and remember to adhere to the 5 rules for quoting your results. Include error bars on your graph(s).