J 1278

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2006.

Third Semester

MA 231 — MATHEMATICS — III

(Common to all branches except Biomedical Engineering, Civil Engineering and Computer Based Constructions, Fashion Technology, Industrial Bio-Technology, Textile Chemistry)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form partial differential equation by eliminating the arbitrary function from $z = f(xy)$.

2. Write down the complete solution of $z = px + qy + c\sqrt{1 + p^2 + q^2}$.

3. Find a_n in expanding e^{-x} as Fourier series in $(-\pi, \pi)$.

4. State Parseval's Identity of Fourier series.

5. A tightly stretched string of length 2 L is fastened at both ends. The mid point of the string is displaced to a distance 'b' and released from rest in this position. Write the Initial Conditions.

6. In one dimensional heat equation $u_t = \alpha^2 u_{xx}$. What does α^2 stands for?

7. State initial and final value theorems.

8. Define convolution and convolution theorem of Laplace transforms.

9. If $F\{f(x)\} = \bar{f}(s)$ then give the value of $F\{f(ax)\}$.

10. Find Fourier transform of $f(x) \begin{array}{ll} = 1 & |x| \le 1 \\ = 0 & |x| > 1. \end{array}$

PART B — ($5 \times 16 = 80$ marks)

11. (i) Solve $(x-2z)p+(2z-y)q = y-x$

(ii) Solve $\{D^2+4DD'-5D'^2\}z = \sin(2x+3y)$.

12. (a) (i) Expand $f(x) = x^2 - x$ as Fourier series in $(-\pi,\pi)$.

(ii) Find Half Range cosine series given

$$f(x) = x \quad 0 \le x \le 1$$
$$= 2-x \quad 1 \le x \le 2.$$

Or

(b) Find the Fourier series of period 2π as far as second harmonic given.

$x°$:	0°	30°	60°	90°	120°	150°	180°
y :	2.34	3.01	3.69	4.15	3.69	2.2	0.83
$x°$:	210	240°	270°	300	330°	360°	
y :	0.51	0.88	1.09	1.19	1.64	2.34	

13. (a) An elastic string of length $2l$ fixed at both ends is disturbed from its position at equilibrium position by imparting to each point an initial velocity of magnitude $k(2lx - x^2)$. Find the displacement function $y(x,t)$.

Or

(b) An infinitely long plate in the form of an area is enclosed between the lines $y=0$ $y=\pi$ for positive values of x. The temperature is zero along the edges $y=0$ $y=\pi$ and the edge at infinity. If the edge $x=0$ is kept at temperature K_y find the steady-state temperature distribution in the plate.

14. (a) (i) Find $L\left\{e^{-t}\int_0^t \frac{\sin t}{t}\,dt\right\}$.

(ii) Find $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4))(s^2+9)}\right\}$.

Or

(b) (i) Using convolution theorem $L^{-1}\left\{\dfrac{s}{(s^2+a^2)^2}\right\}$

(ii) Solve $\dfrac{dy}{dt}+x = \cos t$

$\dfrac{dx}{dt}+y=\sin t \quad x(0)=2 \quad y(0)=0$.

15. (a) (i) Find Fourier transform of $e^{-a^2x^2}$. Hence prove $e^{-\frac{x^2}{2}}$ is self reciprocal.

(ii) Find Fourier Sine and Cosine transform of x^{n-1}.

Or

(b) (i) Using Parseval's Identity for Fourier cosine transform of e^{-ax} evaluate $\int_0^{\infty}\dfrac{dx}{(a^2+x^2)^2}$.

(ii) Find Fourier Sine transform of e^{-ax} $(a>0)$. Hence find $F_s\{xe^{-ax}\}$.
