Std. - 11 (Semester-2)
Maths (050) (E)
Sample Question Paper

Time : 3 Hrs. Total Marks : 100

Instructions :

(1) Answer all the questions.

(2) Write your answers according to instructions given below with the questions.

(3) Start each Section from a new page and write answers of all questions in order.

SECTION - A

- **Given below 1 to 15 multiple choice questions. Each carrying one mark. Write the serial number [(A) OR (B) OR (C) OR (D)] in your answer-book of the alternative which you feel is the correct answer of the question.** **[15]**

1. "$P(n) = n^2 - n + 41$ is a prime is true for $n \in$ _____ .

   (A) $\{41m / m \in N\}$ (B) $\{82m / m \in N\}$

   (C) $\{123m / m \in N\}$ (D) $\{m / m^3 - 6m^2 + 11m - 6 = 0\}$

2. If $|Z + [1.81]| \leq 5$ then maximum value of $\sqrt[3]{|Z+4|}$ is _____ $(Z \in C)$

   (A) [2.35] (B) $\sqrt[3]{27}$ (C) $\sqrt{64}$ (D) $2\sqrt{2}$

3. Coefficient of x^{-4} in the expansion of $\left(x - \frac{1}{x^2}\right)^{17}$ is :

   (A) $\binom{17}{7}$ (B) $\binom{16}{7} - \binom{16}{6}$

   (C) $-\left\{\binom{16}{7} + \binom{16}{6}\right\}$ (D) $\binom{17}{6}$

4. The value of $\left(\frac{\operatorname{cosec}10^\circ - \sec 10^\circ}{\operatorname{cosec}10^\circ + \sec 10^\circ}\right) - \left(\frac{\sin 10^\circ - \cos 10^\circ}{\sin 10^\circ + \cos 10^\circ}\right)$ = ...

   (A) $2\tan 55^\circ$ (B) 0

   (C) $2\tan 35^\circ$ (D) $2\cot 35^\circ$

5. If $\tan\theta = -0.75$, $\frac{5\pi}{2} < \theta < 3\pi$ then $p(2\theta)$ is in the :

   (A) II[nd] quadrant (B) III[rd] and IV[th] quadrant

   (C) Only in the III[rd] quadrant (D) Only in the IV[th] quadrant

6. Value of $\sin\frac{\pi}{10} + \cos\frac{\pi}{5} =$ _____

(A) $\frac{\sqrt{5}-1}{4}$ (B) $\sqrt{2.25}$

(C) $\sqrt{1.25}$ (D) $\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}-1}{4}\right)$

7. The general solution of the equation $\cos^{50}x - \sin^{50}x = 1$ is _____

(A) $\left\{k\pi + \frac{\pi}{2} / k \in Z\right\}$ (B) $\left\{\frac{k\pi}{4} / k \in Z\right\} - \left\{k\pi + \frac{\pi}{2} / k \in Z\right\}$

(C) $\left\{\frac{k\pi}{4} / k \in Z\right\} - \{2k\pi / k \in Z\}$ (D) ϕ

8. The perimeter of ΔABC is 6 times the arithmetic mean of the sines of its angles. If a is 1 then A is.....

(A) 60° (B) 30° (C) 90° (D) 15°

9. $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 +$ ..... (upto 100 terms) = ...

(A) 12,500 (B) 12,650 (C) 12,600 (D) 12,550

10. The eccentricity for the Standard form of ellipse whose length of minor axis is equal to the distance between foci is _____ .

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{2}{\sqrt{3}}$

11. A vector which is in the directions of (3, 6, 2) and has magnitude 49 is :

(A) $\left(\frac{3}{7}, \frac{6}{7}, \frac{2}{7}\right)$ (B) (12, 24, 8)

(C) (21, 42, 14) (D) (147, 294, 98)

12. If P(1, 2, 1), Q(2, 3, 2), R(2, 1, 3), S(3, 2, 4) then _____ .

(A) $\overleftrightarrow{PQ} \cap \overleftrightarrow{RS}$ is single-ton

(B) $\overleftrightarrow{PQ} \cap \overleftrightarrow{RS} = \left\{A(x, y, z) / A \in \overleftrightarrow{QP} \ \& \ A \in \overleftrightarrow{RS}\right\}$

(C) $\overleftrightarrow{PQ} \perp \overleftrightarrow{RS}$

(D) $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$

13. $\lim_{x \to 5} \frac{x^m - 5^m}{x - 5} = 500$ then value of m is :

(A) -4 (B) 5 (C) 4 (D) -5

14. If $\lim_{x \to 2} \frac{x^2 - (a+2)x + a}{x - 2} = 2$ then value of a is .....

(A) 0 (B) 1 (C) -1 (D) 2

15. If $f(x) = \cos^2 x$ then $f'(\pi/6) =$ _____ .

(A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{4}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $\frac{1}{4}$

SECTION – B

- **Answer the following 16 to 27 questions as directed in the question. Each carrying one mark.** **[12]**

16. If α is complex number such that $\alpha^2 + \alpha + 1 = 0$ then show that $\alpha^{31} = \alpha$.

17. "The inequality $n! > 2^{n-1}$ is true for nonempty proper subset of N"

Above statement for natural variable n is true or false ? Why ?

18. If $\frac{\sin(\alpha + \beta)}{\cos(\alpha - \beta)} = \frac{1 - m}{1 + m}$, then prove that $\tan\left(\frac{\pi}{4} - \alpha\right) \tan\left(\frac{\pi}{4} - \beta\right) = m$. $(m \neq -1)$

19. If $\sum_{i=1}^{3} \sin\theta_i = 3$, then find the value of $\sum_{i=1}^{3} (\cos\theta_i + \sin\theta_i)$.

20. If $\cos A + \cos B = a_1$ અને $\sin A + \sin B = a_2$ (where $a_1, a_2 \neq 0$) then prove that $\sin(A + B) = \frac{2a_1 a_2}{a_1^2 + a_2^2}$.

21. If $\{a_n\}$ be a G.P. such that $4a_4 - a_6 = 0$ and $a_2 + a_5 = 216$, then find the value of a_1.

22. Find the distance of the point (1, 2, 5) from X - axis.

23. Find $\lim_{x \to \pi/4} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$

24. Find $\lim_{x \to 0} \frac{(1 - \cos 2x) \cdot \sin 5x}{x^2 \sin 3x}$

25. Find $\lim_{x \to 0} \frac{\tan x^\circ - \sin x^\circ}{x^3}$

OR

25. Find a and b if $\lim_{x \to 0} \frac{x(1 + a\cos x) - b \sin x}{x^3} = 1$

26. State the number of points in the interval (0, 2) at which $f(x) = |4x - 1| + |3x - 1| + |x - 1| + |x|$ is not differentiable.

OR

26. If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$, $g'(a) = 2$, then find the value of $\lim_{x \to a} \frac{f(x)\,g(x) - f(a)g(a)}{x^3 - a^3}$, $(a \neq 0)$.

27. Find derivative of cotx at $x = \frac{5\pi}{4}$ from first priniciple.

OR

27. Find $\frac{d}{dx}\left\{\frac{\sec x - \tan x + 3}{1 + 3\sec x + 3\tan x}\right\}$, $(x \neq (2k + 1)\frac{\pi}{2}, k \in Z)$

SECTION - C

- **Answer the following 28 to 37 questions as directed in the questions. Each carrying two marks.** [20]

28. Using principle of mathematical Induction prove that $41^n - 1$ is divisible by 40.

29. Prove that : $\frac{\sec^2 129^\circ - \operatorname{cosec}^2 31^\circ}{\sec 51^\circ - \operatorname{cosec} 31^\circ} = \operatorname{cosec} 39^\circ - \sec 121^\circ$

OR

29. Prove that : $\operatorname{cosec}\theta + \operatorname{cosec} 2\theta + \operatorname{cosec} 4\theta + \cot 4\theta = \cot\frac{\theta}{2}$.

30. Find all real X such that $\frac{3 + 2i \sin x}{1 - 2i \sin x}$ is real. Also find the number.

31. For $x = 0.5$ the sum of all the terms in the expansion of $\left(1 + \frac{1}{x}\right)^n$ is 6561 then find r and T_{r+1} for which T_{r+1} is greatest term in the expansion.

32. If $A = \tan 6^\circ \tan 42^\circ$ and $B = \cot 66^\circ \cot 78^\circ$, then prove that $\cos(A - B) = 1$.

33. The equation $p\sin x + \cos 2x = 2p - 7$ possesses non-empty solution set for which real values of p ?

34. Find the length of a focal chord of the parabola $y^2 = 16x$, which make an angle $\frac{\pi}{12}$ with the axis of the parabola.

35. If C is the centre of the ellipse $9x^2 + 16y^2 = 144$ and S is one focus. Then find the ratio of CS to the length of semi major axis.

OR

35. If $(a\sec\theta, b\tan\theta)$ and $(a\sec\phi, b\tan\phi)$ are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then prove that $\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{e-1}{e+1}$.

36. Given that A(0, 7, 10), B(-1, 6, 6), C(-4, 9, 6) determine the type of Δ ABC.

37. Find $\lim_{x\to -1} \frac{\sqrt{x^5+1} - \sqrt{x^3+1}}{\sqrt{x^7+1}}$ if it exists.

OR

37. Find $\lim_{x\to \pi} \frac{\sqrt{17+\cos x} - 4}{(x-\pi)^2}$

SECTION - D

- **Answer the following 38 to 44 questions, each carrying three marks as directed in the questions.** **[21]**

38. Solve : $\sqrt{3}\sin x = \sqrt{2} + \cos x$.

39. In a ΔABC, $a = 2b$ and $|A - B| = \pi/3$. Find C.

40. If $\sec(x + y)$, $\sec x$, $\sec(x - y)$ are in A.P., then prove that $\cos x = \pm\sqrt{2}\cos\frac{y}{2}$, (Where $\cos x \neq 1$ & $\cos y \neq 1$).

OR

40. Find the sum of the series : $1^2 - 2^2 + 3^2 - 4^2 \ldots\ldots\ldots + 29^2 - 30^2$.

41. An arch is in the form a semi - ellipse. It is 10m wide and 4 m high at the centre. Find the height of the arch at a point 2 m from one end.

42. Determine whether the points A(0, 0, 0), B(1, 0, 0), C(0, 1, 0), D(0, 0, 1) are vertices of a quadrilateral or not ? If they form a quadrilateral, then determine its type.

43. Find $\lim_{x\to \pi/2} \frac{2x\sin x - \pi}{\cos^2 x + \cos x}$

44. If $f(x) = (lx^2 + mx + \cos x)(n + p\cot x)$, (where $lmnp \neq 0$, and $x \neq \frac{k\pi}{4}$, $K \in Z$), then find $f'(x)$.

SECTION - E

- **Answer the following 45 to 47 questions each carrying four marks as directed in the questions.** [12]

45. Prove : $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$, $n \in N$

OR

45. If $a_1 = 1$, $a_2 = 11$ and $a_n = 2a_{n-1} + 3a_{n-2}$, $n \geq 3$ then prove that $n \in N$ $a_n = 2(-1)^n + 3^n$ for $n \in N$.

46. Find all the complex numbers $Z \in C - R$, Satisfying the condition $\bar{Z} = Z^2$

47. If $y = \frac{x \sin x}{1 + \cos x}$ then find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

SECTION - F

- **Answer the following 48 to 51 questions, each carrying five marks as directed in the questions.** [20]

48. Prove that $(2 + \sqrt{3})^7 + (2 - \sqrt{3})^7 = 10084$ hence deduce that $10083 < (2 + \sqrt{3})^7 < 10084$ and $0 < (2 - \sqrt{3})^7 < 1$.

49. Prove that $a\cot\left(\frac{2\pi}{3} - x\right) - b\tan\left(\frac{2\pi}{3} + x\right) = 0 \Rightarrow \sec 2x = 2\left(\frac{a-b}{a+b}\right)$. (where $a \neq -b$)

OR

49. If α and β are roots of $a\cos\theta + b\sin\theta = c$, then expressed the values of $\cos\alpha + \cos\beta$ and $\cos\alpha \cdot \cos\beta$ in terms of a, b and c.

50. Prove $\cos x \cdot \cos 2x \cdot \cos 2^2x \cdot \cos 2^3x \ldots\ldots\ldots \cos 2^{n-1}x = \frac{\sin 2^n x}{2^n \sin x}$ and use it to find the value of $\cos\frac{2\pi}{15} \cdot \cos\frac{4\pi}{15} \cdot \cos\frac{8\pi}{15} \cdot \cos\frac{14\pi}{15}$

51. Sum to n terms the series, $1.4 + 2.5 + 3.6 + \ldots\ldots\ldots\ldots$. and hence obtain the sum of first 100 terms.

OR

51. Product of first five consecutive terms of G.P. is 243. If the sum of the second and the fourth terms is 12.75, find these first five terms of this G.P.