14. State and prove Euler's formulae on connected plane graphs. Deduce that

    (i) All planar embeddings of a given connected planar graph have the same number of faces.

    (ii) $\Sigma \leq 3\upsilon - 6$ in a simple planar graph with $\upsilon \geq 3$.

    (iii) $K_{3,3}$ is non - planar. (2+6+4+4+4)

15. (a) Prove that each vertex in a diconnected tournament with $\upsilon \geq 3$ is contained in a directed K cycle, $3 \leq k \leq \upsilon$. (15)

    (b) Define indegree and outdegree of a vertex in a digraph. Prove that

$$\sum_{v \in V} \overline{d}(v) = \sum_{v \in V} d^{+}(v) = \sum$$

(1+1+3)

Register Number :

Name of the Candidate :

1 9 7 9

M.Sc. DEGREE EXAMINATION, 2010

(MATHEMATICS)

(SECOND YEAR)

(PAPER - VII)

230. GRAPH THEORY

(Including Lateral Entry)

May] [Time : 3 Hours

Maximum : 100 Marks

PART - A (8 × 5 = 40)

Answer any EIGHT questions.
All questions carry equal marks.

1. Show that in any set of two or more people, there are always two with exactly the same number of friends inside the group.

2. Show that in a tree, any two vertices are joined by a unique path.

Turn Over

3. Show that if G is simple graph with at least three vertices with $\partial \geq \nu/2$, then G is Hamiltonian.

4. Show that a matching M in G is a maximum matching if and only if, G contains no M augmenting path.

5. Define Ramsey numbers $\Upsilon(m,n)$ and show that $\Upsilon(m,n) = \Upsilon(n,m)$.

6. If G is bipartite, show that $\chi' = \Delta$.

7. Define *k*- critical graph. If G is *k*-critical, show that $\partial \geq k-1$.

8. With usual notations, prove that

$$\Pi_k(G) = \Pi_k(G-e) - \Pi_k-(G \cdot e).$$

9. Show that a digraph D contains a directed path of length $\chi - 1$.

10. Show that every tournament is either diconnected or can be transformed into a diconnected tournament by the reorientation of just one arc.

PART - B $(3 \times 20 = 60)$

Answer any THREE questions.
All questions carry equal marks.

11. (a) Show that a graph on at least three vertices is 2 - connected *iff* any two vertices of G are connected by at least two internally disjoint paths. (10)

    (b) Show that a graph is bipartite if it contains no odd cycles. (10)

12. (a) Show that G has a perfect matching if $o(G-S) \leq |S|$ for all $S \subset V$. (15)

    (b) Show that a connected graph is Eulerian if it has no vertices of odd degree. (5)

13. (a) Define α, β, α' and β'. Prove that $\alpha + \beta = V = \alpha' + \beta'$. (12)

    (b) Let G be a connected graph that is not an odd cycle. Show that G has a 2-edge colouring in which both colours are represented at each vertex of degree at least two. (8)