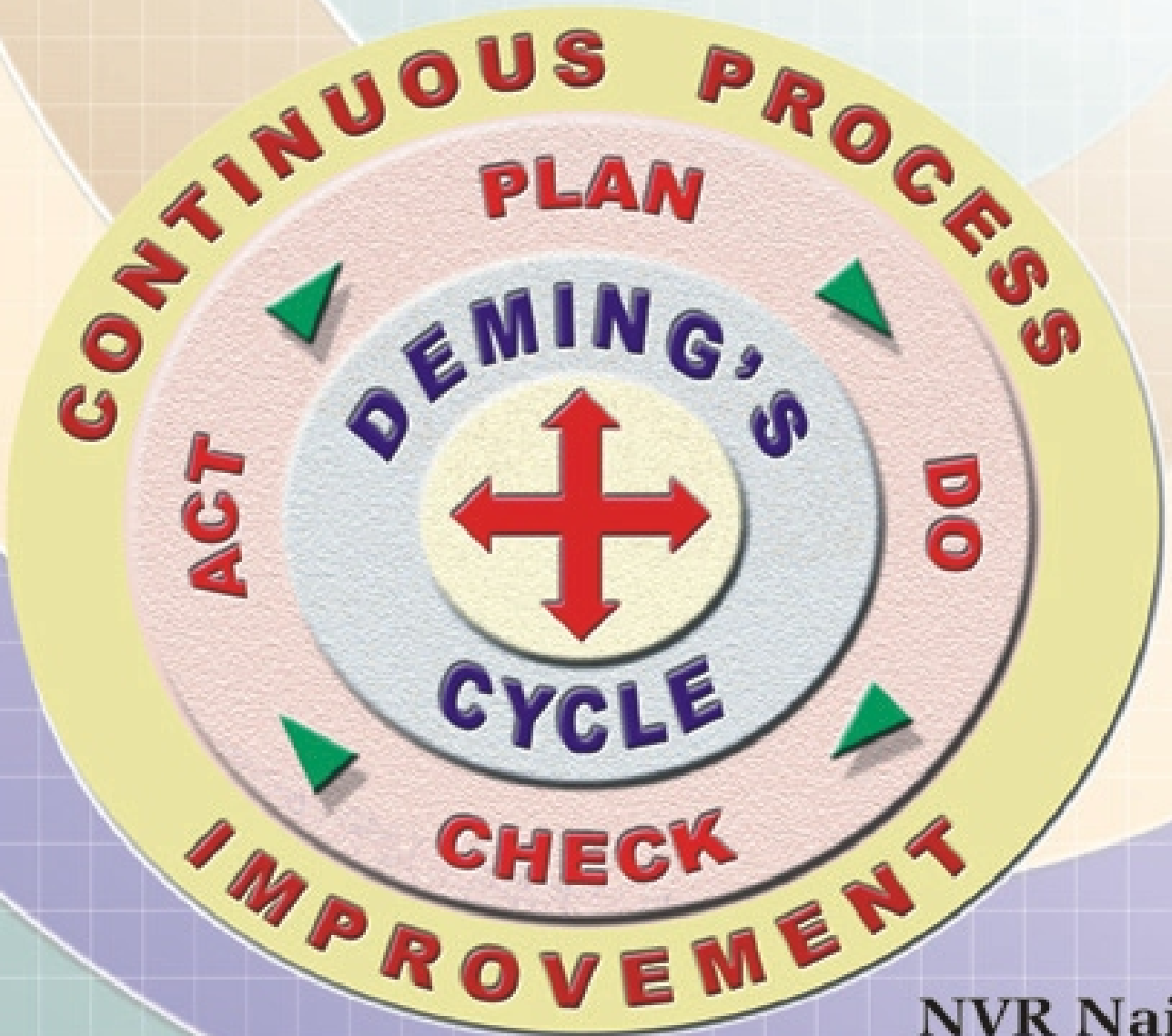
NEW AGE

# Total Quality Management




NVR Naidu
KM Babu
G Rajendra

NEW AGE INTERNATIONAL PUBLISHERS

# Total Quality Management

# Total Quality Management

**NVR Naidu**
*Professor in Industrial Engineering and Management, MS Ramiah Institute of Technology, Bangalore*

**KM Babu**
*Vice Principal and Prof. and Head of Industrial Engineering and Management Department, BMS College of Engineering, Bangalore*

**G Rajendra**
*Assistant Professor in the department of Industrial Engineering and Management, Dr. Ambedkar Institute of Technology, Bangalore*

PUBLISHING FOR ONE WORLD

**NEW AGE INTERNATIONAL (P) LIMITED, PUBLISHERS**

New Delhi • Bangalore • Chennai • Cochin • Guwahati • Hyderabad
Jalandhar • Kolkata • Lucknow • Mumbai • Ranchi

Visit us at **www.newagepublishers.com**

Published by New Age International (P) Ltd., Publishers



**ISBN (13) : 978-81-224-2534-5**

**PUBLISHING FOR ONE WORLD**

**NEW AGE INTERNATIONAL (P) LIMITED, PUBLISHERS**
4835/24, Ansari Road, Daryaganj, New Delhi - 110002
Visit us at **www.newagepublishers.com**

*This book is dedicated*
*to lotus feet of*
*Lord* ***VENKATESHWARA***
*and to our*
*Beloved Parents*

# PREFACE

Total Quality Management has now emerged as the most spectacular aspect of Management. This book covers fundamental concepts of Total Quality Management, its Principles & Practices and Tools and Techniques. Sufficient theory is presented to ensure that the reader has a sound understanding of the subject.

This book will serve the instructional needs of Engineering and Management students in technical institutions and universities. Apart from this, it also provides needs of various business organizations, healthcare and manufacturing industries.

This book covers a brief introduction of probability distributions followed by leadership theory and tools and techniques of total quality managemett. Other chapters discuss Control Charts for Variables and Attributes, Statistical Process Control (SPC), Quality Systems, Bench Marking, Quality Function Deployment (QFD), Quality by Design, Experimental Design, Taguchi's Quality Engineering Products, Failure Mode and Effect Analysis (FMEA), Total Productive Maintenance, ISO 9000 and Management Tools.

This book also contains a number of solved problems for ease of grasping and appended with a good number of exercise problems at the end of each chapter. Statistical Quality Control Tables are also provided in order to facilitate ease of viewing the tables while using the book.

The authors wish to express their sincere thanks to the Principals and Managements of their respective colleges. Further, they would like to thank Mr. Saumya Gupta, Managing Director and Mr. V.R. Babu, New Age International (P) Limited, Publishers for their commitment and encouragement in bringing out this book in time with good quality.

Bangalore

Dr. N.V.R. Naidu
Dr. K.M. Babu
Sri G. Rajendra

**This page intentionally left blank**

**This page intentionally left blank**

**This page intentionally left blank**

**This page intentionally left blank**

**This page intentionally left blank**

# CONTENTS

# 1 Quality and Quality Control

## INSPECTION

Inspection is the most common method of attaining standardisation, uniformity and quality of workmanship. It is the cost art of controlling the product quality after comparison with the established standards and specifications. It is the function of quality control. If the said item does not fall within the zone of acceptability it will be rejected and corrective measure will be applied to see that the items in future conform to specified standards.

Inspection is an indispensable tool of modern manufacturing process. It helps to control quality, reduces manufacturing costs, eliminate scrap losses and assignable causes of defective work.

### Objectives of Inspection

(1) To collect information regarding the performance of the product with established standards for the use of engineering production, purchasing and quality control etc.

(2) To sort out poor quality of manufactured product and thus to maintain standards.

(3) To establish and increase the reputation by protecting customers from receiving poor quality products.

(4) Detect source of weakness and failure in the finished products and thus check the work of designer.

### Purpose of Inspection

(1) To distinguish good lots from bad lots

(2) To distinguish good pieces from bad pieces.

(3) To determine if the process is changing.

(4) To determine if the process is approaching the specification limits.

(5) To rate quality of product.

(6) To rate accuracy of inspectors.

(7) To measure the precision of the measuring instrument.

(8) To secure products – design information.

(9) To measure process capability.

### Stages of Inspection

(1) Inspection of incoming material

(2) Inspection of production process

(3) Inspection of finished goods.

(1) **Inspection of incoming materials.** It is also called receiving inspection. It consists of inspecting and checking of all the purchased raw materials and parts that are supplied before they are taken on to stock or used in actual manufacturing. Inspection may take place either at supplier's end or at manufacturer's gate. If the incoming materials are large in quantity and involve huge transportation cost it is economical to inspect them at the place of vendor or supplier.

(2) **Inspection of production process.** The work of inspection is done while the production process is simultaneously going on. Inspection is done at various work centres of men and machines and at the critical production points. This had the advantage of preventing wastage of time and money on defective units and preventing delays in assembly.

(3) **Inspection of finished goods.** This is the last stage when finished goods are inspected and carried out before marketing to see that poor quality product may be either rejected or sold at reduced price.

### Inspection Procedures

There are three ways of doing inspection. They are Floor inspection, Centralised inspection and Combined inspection.

### Floor Inspection

It suggests the checking of materials in process at the machine or in the production time by patrolling inspectors. These inspectors moves from machine to machine and from one to the other work centres. Inspectors have to be highly skilled. This method of inspection minimise the material handling, does not disrupt the line layout of machinery and quickly locate the defect and readily offers field and correction.

### Advantages

(1) Encourage co-operation of inspector and foreman.

(2) Random checking may be more successful than batch checking.

(3) Does not delay in production.

(4) Saves time and expense of having to more batches of work for inspection.

(5) Inspectors may see and be able to report on reason of faculty work.

### Disadvantages

(1) Difficult in inspection due to vibration.

(2) Possibility of biased inspection because of worker.

(3) Pressure on inspector.

(4) High cost of inspection because of numerous sets of inspections and skilled inspectors.

### Suitability

(1) Heavy products are produced.

(2) Different work centres are integrated in continuous line layout.

### Centralised Inspection

Materials in process may be inspected and checked at centralised inspection centre which are located at one or more places in the manufacturing industry.

**Advantages**

(1) Better quality checkup.

(2) Closed supervision.

(3) Absence of workers pressure.

(4) Orderly production flow and low inspection cost.

**Disadvantages**

(1) More material handling.

(2) Delays of inspection room causes wastage of time.

(3) Work of production control increases.

(4) Due to non-detection of machining errors in time, there may be more spoilage of work.

**Suitability**

(1) Incoming materials inspection.

(2) Finished product inspection.

(3) Departmental inspection.

(4) High precision products of delicate products.

(5) Small and less expensive products.

**Combined Inspection**

Combination of two methods what ever may be the method of inspection, whether floor or central. The main objective is to locate and prevent defect which may not repeat itself in subsequent operation to see whether any corrective measure is required and finally to maintained quality economically.

**Methods of Inspection**

There are two methods of inspection. They are 100% inspection and Sampling inspection.

**100% Inspection**

This type will involve careful inspection in detail of quality at each strategic point or stage of manufacture where the test involved is non-destructive and every piece is separately inspected. It requires more number of inspectors and hence it is a costly method. There is no sampling error. This is subjected to inspection error arising out of fatigue, negligence, difficulty of supervision etc. Hence complete accuracy of influence is seldomly attained.

It is suitable only when a small number of pieces are there or a very high degree of quality is required. Example : Jet engines, Aircraft, Medical and Scientific equipment.

**Sampling Inspection**

In this method randomly selected samples are inspected. Samples taken from different batches of products are representatives. If the sample prove defective. The entire concerned is to be rejected or recovered. Sampling inspection is cheaper and quicker. It requires less number of Inspectors. Its subjected to sampling errors but the magnitude of

sampling error can be estimated. In the case of destructive test, random or sampling inspection is desirable. This type of inspection governs wide currency due to the introduction of automatic machines or equipments which are less susceptible to chance variable and hence require less inspection, suitable for inspection of products which have less precision importance and are less costly.

**Example :** Electrical bulbs, radio bulbs, washing machine etc.

Destructive tests conducted for the products whose endurance or ultimate strength properties are required.

**Example :** Flexible strength, resistance capacity, compressibility etc.

**Drawbacks of Inspection**

(1) Inspection adds to the cost of the product but not for its value.

(2) It is partially subjective, often the inspector has to judge whether a product passes or not.

**Example :** Inspector discovering a slight burnish on a surface must decide whether it is bad enough to justify rejection even with micrometers a tight or loose fit change measurement by say 0.0006 inches. The inspectors design is important as he enforces quality standards.

(3) Fatigue and Monotony may affect any inspection judgement.

(4) Inspection merely separates good and bad items. It is no way to prevent the production of bad items.

**Quality**

Different meaning could be attached to the word Quality under different circumstances. The word Quality does not mean the Quality of manufactured product only. It may refer to the Quality of the process (*i.e.*, men, material, machines) and even that of management. Where the quality of manufactured product referred as or defined as "Quality of product as the degree in which it fulfills the requirement of the customer. It is not absolute but it judged or realised by comparing it with some standards".

It is usually determined by some characteristics namely design, size, material, chemical composition, mechanical functioning workmanship, finish and other properties. In the final analysis the Quality standards for the products are established by the customer.

**Example :** Gear used in sugarcane extracting machine through not of the same material and without possessing good finish, tolerance and accuracy as that of gear used in the hand stock of a teeth may be considered of good quality if it work satisfactory in the juice extracting machine.

Quality begins with the design of a product in accordance with the customer specification further it involves the established measurement standards, the use of proper material, selection of suitable manufacturing process and the necessary tooling to manufacture the product, the performance of the necessary manufacturing operations and the inspection of the product to check the manufacturing operations and the inspection of the product to check on performance with the specifications. Quality characteristics can be classified as follows :

(1) Quality of design

(2) Quality of conformance with specifications

(3) Quality of performance.

**Control**

The process through which the standards are established and met with standards is called control. This process consists of observing our activity performance, comparing the performance with some standard and then taking action if the observed performance is significantly to different from the standards.

The control process involves a universal sequence of steps as follows :

(1) Choose the control subject.

(2) Choose a unit of measure.

(3) Set a standard value *i.e.,* specify the quality characteristics

(4) Choose a sensing device which can measure.

(5) Measure actual performance.

(6) Interpret the difference between actual and standard.

(7) Taking action, if any, on the difference.

**Quality Control**

Quality control can be defined as that Industrial Management technique by means of which product of uniform acceptable quality is manufactured.

**Factors Affecting Quality**

(1) Men, Materials and Machines

(2) Manufacturing conditions

(3) Market research in demand of purchases

(4) Money in capability to invest

(5) Management policy for quality level

(6) Production methods and product design

(7) Packing and transportation

(8) After sales service

**Objectives of Quality Control**

(1) To decide about the standard of Quality of a product that is easily acceptable to the customer.

(2) To check the variation during manufacturing.

(3) To prevent the poor quality products reaching to customer.

**Statistical Quality Control (SQC)**

A Quality control system performs inspection, testing and analysis to conclude whether the quality of each product is as per laid quality standard or not. It's called "Statistical Quality Control" when statistical techniques are employed to control quality or to solve quality control problem. SQC makes inspection more reliable and at the same time less costly. It controls the quality levels of the outgoing products.

SQC should be viewed as a kit of tools which may influence related to the function of specification, production or inspection.

A successful SQC programme is expected to yield the following results :

(1) Improvement of quality.
(2) Reduction of scrap and rework.
(3) Efficient use of men and machines.
(4) Economy in use of materials.
(5) Removing production bottle-necks.
(6) Decreased inspection costs.
(7) Reduction in cost/unit.
(8) Scientific evaluation of tolerance.
(9) Scientific evaluation of quality and production.
(10) Quality consciousness at all levels.
(11) Reduction in customer complaints.

### Tools of SQC

The principle tools of SQC are as follows :

(1) Frequency distribution.
(2) Control charts for measurement and attribute data.
(3) Acceptance sampling techniques.
(4) Regression and correlation analysis.
(5) Tests of significance.
(6) Design of experiments.

## QUALITY CHARACTERISTICS

### Quality of Design

Quality design is a technical term. It can be regarded as a composite of 3 separate terms or steps in a common progression of activities.

(*i*) Identification of what constitutes fitness for use to the user (Quality of market research).

(*ii*) Choice of concept of product or service to be responsible to the identified needs of the user (Quality of concept).

(*iii*) Translation of the chosen product concept into a detailed set of specifications which is faithfully executed, will then meet the user's need (Quality of specification).

The total progression composed of these three activities is called "Quality of Design" and it may be said to consist of Quality of market research: Quality of concept and Quality of specification.

**Example :** All automobiles provide the user with the service of transportation. The various models differ as to size, comfort, appearance, performance, economy, status conferred etc. These differences are in turn the results of intended or designed differences in the size, styling, materials, tolerances, test programs etc. Higher quality of design can be attained only at an increase in costs.

**Quality of Comformance**

The design must reflect the needs of fitness for use, and the products must also confirm to the design. The extent to which the product does confirm to the design is called "Quality of conformance". This extent of conformance is determined by variables as :

(*i*) Choice of process *i.e.*, whether they are able to hold the tolerances.

(*ii*) Training of the supervision and the work force.

(*iii*) Degree of adherence to the program of inspect, test, audit etc. motivation for quality.

Higher quality of conformance can be attained with an accompanying reduction in cost.

**Example :** Two scooters both are produced at the same level of time but one may be 100% according to the drawing and specification of the same design; the second scooter may be 90% according to the drawing and specification and probably a few dimensions may be different from those of drawing. Therefore quality of conformance of 1st scooter is better than the 2nd scooter even though both are of same design.

**Quality Costs**

Quality costs are the incurring in introducting quality and benefits. This is done by identifying and defining the following categories of costs which are associated with making, finding, repairing or avoiding (preventing) defects.

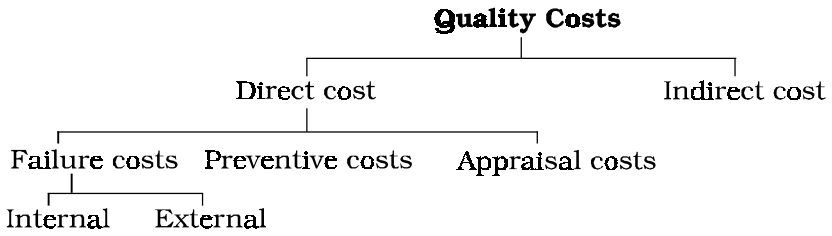



**Fig. 1.1.** Hierarcy of quality cost or Brakdown of quality cost.

**(A) Failure costs**

**Internal failure costs.** These are costs which would disappear if no defects exit in the product prior to shipment to the customer. They include.

Scrap : The net loss in labour and material resulting from defectives which cannot economically be repaired or used.

Rework : The cost of correcting defectives to make them fit for use.

Retest : The cost of inspection and retest of products that have undergone rework or other revision.

Down time : The cost of idle facilities resulting from defects. (**Example :** Aircraft idle due to unreliability, printing press down due to paper break).

Yield losses : The cost of process yield lower that might be attainable by improved controls. Includes "overfill" of containers (going to customers) due to variability in filling and measuring equipment.

**External failure costs.** These costs would also disappear if there were no defects. They are disguished from the internal failure costs by the fact that the defects are found at the shipment to the customer. They include :

| | |
|---|---|
| Complaint adjustment | : All costs of investigation and adjustment of justified complaints attributable to defective product or installation. |
| Returned material | : All costs associated with receipts and returned from the field. |
| Warranty charges | : All costs involved in service to customers under warranty contracts. |
| Allowances | : Costs of concessions made to customers due to substandard products being accepted by the customer as is include loss in income due to down grading products for sale as seconds. |

**(B) Appraisal Costs**

These are costs incurred to discover the conditions of the products, mainly during the "first come through" costs include.

*Incoming material inspection* : The cost of determining the quality of vendor made products, whether by inspection on receipt or at source or by surveillance method.

*Inspection and test* : The cost of checking the conformance of the product throughout its progression, in the factory, including final aceptance and check of packing and shipping includes life, environmental and reliability tests. Also includes testing done at customer's premises prior to giving up the product to the customer.

*Maintaining accuracy of test equipment* : Includes the cost of operating the system that keeps the measuring instruments and equipment in calibration.

*Materials and services consumed* : Includes costs of product consumed through destructive tests, materials consumed and services where significant.

*Evaluation of stock* : Include the costs of testing products in field storage or in stock to evaluate degradation.

**(C) Prevention Costs**

These costs are incurred to keep future and appraisal costs at a minimum. It includes :

*Quality Planning* : This includes the broad array of activities which collectively create quality plan, the inspection plan, reliability plan, data system and numerac specialised plans. It includes also preparation of the manuals and procedures needed to communicate these plans to all concerned.

*New Product review* : Includes preparation of bid proposals evaluation of new design, preparation of test and experiment programs and other quality activities associated with the launching of new designs.

*Training* : The costs of preparing training programs for attaining and improving quality performance includes the cost of conducting formal training programs as well.

*Process control* : Includes that part of process control which is conducted to achieve fitness for use as distinguished from achieving productivity, safety *etc*.

*Quality data acquisition and analysis* : This is the work of running the quality of data systems to acquire continuing data on quality performance. It includes analysis of these data to identify the quality troubles, to sound alarms *etc.*

*Quality reporting* : Includes the work of summaring and publishing quality information to the middle and upper management.

*Improvement Projects* : Includes the work of structuring and carrying out programs for break through to new levels of performance *i.e.*, defective prevention programs, motivation programs *etc.*

**Total Quality Control**

Total Quality Control defined as an effective system for intergrating the quality development, quality maintainance and quality improvement efforts of the various groups in an organisation so as to enable production and service at the most economical level which allow for full customer satisfaction.

It may be classified as a ''Management Tool'' for many industries outstanding improvement in product quality design and reduction in operating costs and losses.

Product quality is defined as "The composite product of engineering and manufacture that determine the degree to which the product in use will meet the expectations of the customer".

"Control" represents a tool with four steps :

Setting up of quality standards.

Appraising conformance to these standards

Acting when these standards are exceeded.

Planning for improvements in these standards.

Quality control emerges as a based function based on the collection analysis and interpretations of data on all aspects of the enterprise.

Total quality control is an aid for good engineering designs, good manufacturing methods and conscious inspection activity that have always been required for the production of high quality articles.

Quality of any product is effected at many stages of the industrial cycle :

*Marketing* : Evaluates the level of Quality which customers want for which they are willing to pay.

*Engineering* : Reduces this marketing evaluations to exact specification.

*Purchasing* : Chooses, contracts with and retains vendors for parts and materials.

*Manufacturing Engineering* : Select the jigs, tools and processes for production.

*Manufacturing Supervision and shop operators* : Exert a major quality influence during parts making, sub assembly and final assembly.

*Mechanical Inspection and function Test* : Check conformance to specifications.

*Shipping* : Influences the calibre of packaging and transportation.

*Installation* : Helps ensure proper operations by installing the product according to proper instructions and maintaining it through product service.

In other words, the determination of both quality and quality costs actually takes place throughout the entire industrial cycle.

Quality control is responsible for quality assurance at optimum quality costs. The benefits resulting from Total Quality Control programmes are :

Improvements in product quality and design

Reduction in operating costs and losses

Reduction in production line bottle necks

Improvement in employee morale

Improved inspection methods

Setting time standards for labour

Definite schedule for preventive maintainance

Availability of purposeful data for use in co-advertising

Furnishing of actual basis for cost accounting for standard and for scrap, rework and inspection.

# 2 Probability Distributions

## INTRODUCTION

The statistician is basically concerned with the presentation and interpretation of chance outcomes that occur in a planned study or scientific investigation.

The probability distribution is reprsented graphically, in tabular form and by means of a formula. This presentation shows the behaviour of a random variable. Many random variable associated with statistical. Experiments can be described by probability distribution.

For example : All random variables representing the number of successes in $n$ independent trials of an experiment, where the probability of a success is constant for all $n$ trials, have the same general type of behaviour and can be represented by a single formula. The mean or variance of a given probability distribution is defined to be the mean or variance of any random variable having that distribution.

Care should be taken in choosing the probability that correctly describes the observations generated by the experiment.

### Terminology

*Game* : A game is any trail which can yield more than one result.

*Outcome* : Outcome refers to the individual result of a game.

*Event* : An event refers to a collection of outcomes.

### Types of Events

(*a*) *Mutually exclusive events* : A set of events called mutually exclusive if they can not occur simultaneously.

(*b*) *Independent events* : A set of events are called independent, if the occurance of one does not depend on the occurance of the others.

(*c*) *Collectively exhaustive events* : A set of events are said to be collectively exhaustive if they together exhaust all possible results.

(*d*) *Complementary events* : Two events are complementary if they are mutually exclusive and collectively exhaustive.

## PROBABILITY

The Probability of occurance of an event is defined as the ratio of number of outcomes-favourable to the occurance of the event to the number of possible outcomes of the event.

Let 'A' be an event and the probability of the event 'A' is denoted by P(A), read as, probability of A, and is given by

$$P(A) = \frac{\text{Number of favourable outcomes to A}}{\text{Total number of possible outcomes}}.$$

**Properties of Probability**

(1) It is a non-negative number which lies between zero and one.

(2) A probability tending to one implies certainity of occurance of an event.

(3) Probability tending to zero implies the uncertainity of occurence of an event.

(4) The sum of probabilities of a set of mutually exclusive events are always equal to one.

(5) The sum of the probability of complementary events are always equal to one.

Definition of Probability function : $f(x)$ or $P(x)$ associated with every random variable is a real function with positive values known as probability function. The probability function gives the magnitude of probability with which a random variable takes a given value. Probability function can be either descrete or continuous.

**Properties of Probability Functions**

1. $f(x) \geq 0$ for all values of $x$, where '$x$' is a random variable.
2. $\Sigma f(x) = 1$ for all discrete distributions.
3. $\int f(x)\,dx = 1$ for all continuous distributions.

**Types of Probability Distributions**

The Probability distributions can be classified into two. They are Discrete Probability Distribution and Continuous Probability Distribution.

**Discrete Probability Distributions**

| | |
|---|---|
| Hypergeometric distribution | Binomial distribution |
| Possion distribution | Multinominal distribution |

**Continuous Probability Distributions**

| | |
|---|---|
| Normal distribution | Exponential distribution |
| Uniform distribution | Weibull distribution |

**Discrete Probability Distributions**

**Hypergeometric Probability Distribution**

This distribution occurs when the universe (lot) is finite and a random sample is taken from the lot without replacement. As per this distribution, when a sample of size '$n$' is drawn from a population of size 'N' which contains two types of articles. One is defective and the other is non-defective. This distribution is applicable for small populations only.

The probability of occurrence of '$r$' defectives is given by

$$P(r) = \frac{{}^{NP}C_r \; {}^{(N-NP)}C_{n-r}}{{}^{N}C_n}$$

where N = lot size $p$ = fraction defective of a lot, $n$ = sample size, $r$ = occurance of defectives

Properties :

(1) It is derived from the basic principles of algebra

(2) It is a discrete probability distribution

(3) The calculation of this distribution is difficult although it gives theoretically correct result.

(4) The Binomial distribution will give approximate results to the hypergeometric distribution when lot size is very large in comparision with the simple size ($N \geq 10n$).

**SOLVED PROBLEMS**

**Problem 1.** *A lot 12 balls are located in a container has 4 defectives. What is probability of occurences of (a) Exactly one defective (b) Atleast one defective (c) Less than one defective (d) Less than 2 defectives (e) Greater than one defective (f) None are defectives in a sample of size 5.*

**Solution.** Given data : N = 12 ; $d$ or $Np$ = 4 ; $n$ = 5

Since lot size is known the suitable distribution to this problem is hypergeometric distribution. The probability distribution function is

$$P(r) = \frac{{}^{NP}C_r \; {}^{(N-NP)}C_{n-r}}{{}^{N}C_n}$$

Computation of probabilities :

$$P(0) = \frac{{}^{4}C_0 \; {}^{8}C_{5-0}}{{}^{12}C_5} = 0.071$$

$$P(1) = \frac{{}^{4}C_1 \; {}^{8}C_{5-1}}{{}^{12}C_5} = 0.354$$

(*a*) *The probability of occurrence of exactly one defective*

The required probability = $P(r = 1) = P(1) = 0.354$

(*b*) *Atleast one defective*

The required probability = $P(r >= 1) = P(1) + P(2) + P(3) + P(4)$

Since we know that $P(0) + P(1) + P(2) + P(3) + P(4) = 1$ (because there are 4 defectives) therefore $P(r >= 1)$ can be written as $1 - P(0)$

$= P(r >= 1) = 1 - P(0) = 1 - 0.071 = 0.929$

(*c*) *Less than one defective*

The required probability = $P(r < 1) = P(0) = 0.071$

(*d*) *Less than 2 defective*

The required probability = $P(r < 2) = P(0) + P(1) = 0.071 + 0.354 = 0.425$

(*e*) *Greater than one defective*

The required probability = $P(r > 1) = P(2) + P(3) + P(4)$

This can be written as $P(r <= 1) = 1 - \{P(0) + P(1)\}$

$= 1 - \{0.071 + 0.354\} = 0.575$

(*f* ) *None are defective*

The required probability = P($r$ = 0) = P(0) = 0.071.

**Problem 2.** *A random sample of 5 items is drawn from a lot of 30 items containing exactly 3 defectives. Determine the probability that the sample will contain (a) No defective ; (b) Exactly 2 defectives ; (c) Less than 2 defective ; and (d) More than 1 defective ; using the exact method.*

**Solution.** Given data :

Lot size, N = 30 ; Sample size, $n$ = 5 ; and Defective, $d$ or N$p$ = 3

Since the lot size is known the suitable distribution to the above problem is hypergeometric distribution. As per hypergeometric

$$P(r) = \frac{^{NP}C_r - ^{(N-NP)}C_{n-r}}{^{N}C_n}$$

Computation of probabilities :

$$P(0) = \frac{^{3}C_0 \; ^{27}C_{5-0}}{^{30}C_5} = 0.566$$

$$P(1) = \frac{^{3}C_1 \; ^{27}C_{5-1}}{^{30}C_5} = 0.369$$

$$P(2) = \frac{^{3}C_2 \; ^{27}C_{5-2}}{^{30}C_5} = 0.062$$

(*a*) *No defective* :

The required probability = P($r$ = 0) = 0.566

(*b*) *Exactly 2 defectives*

The required probability = P($r$ = 2) = 0.062

(*c*) *Less than 2 defectives*

The required probability = P($r$ < 2) = P(0) + P(1) = 0.566 + 0.369 = 0.935

(*d*) *More than 1 defective*

The required probability = P($r$ > 1) = 1 – {P(0) + P(1)}

= 1 – {0.566 + 0.369} = 0.065.

## EXERCISE PROBLEMS

**1.** A random sample is to be selected from a lot of 15 articles, 4 of which are non-confirming. What is the probability that the sample will contain less than 3 non-containing items.

**2.** A lot of 200 capacitors is known to have 20% defective. If a sample of 50 have to be inspected from a lot what is probability of finding (*a*) Exactly one defective ; (*b*) More than 3 defectives ; and (*c*) Less than 3 defectives.

**3.** From a lot of 15 missiles 5 are selected at random and fired. If the lot certains 4 defective missiles that will not fire. What is probability ability that (*a*) All will 5 fire ; and (*b*) Atmost 2 will not fire.

**4.** To avoid detection at customs, a travellers has planed 6 narcotic tablets in a bottle containing 9 vitamin pills that are similar in appearance. If the customs official selects the 3 of tablets at random for analysis what is the probability that the traveller will be arrested from illegal possesion of narcotic pills.

**5.** When a certain quality characteristic of a manufacturing product falls below its (LSL) the product is designated as class I defective. When product falls above (USL). The product is designated as defective class II. A sample of 5 articles is taken from a lot of 15 articles that contains 2 defectives of class I and three defectives of class II. What is the probability that the sample will contain no defective of class I and exactly one defective of class II.

**6.** A random sample of 20 is to be selected from a lot containing 200 articles, 12 of which are defective. Determine probability that the sample will contain Less than 2 defectives.

**7.** A lot of 20 radio sets containing 3 defective sets. If 5 sets are selected at random what is probability that (*a*) Less than 3 defectives (*b*) Exactly 2 defectives (*c*) Greater than one defective (*d*) Less than or equal to one defective and (*e*) Greater than 2 defective.

## BINOMIAL DISTRIBUTION

This is discrete probability distribution. It is applicable when the lot size is very large or infinite or atleast 10 times the sample size. This distribution is applicable for attributes such as defective or non-defective, pass or fail etc. The probability of occurrence of defectives in a random sample is given by

$$P(r) = {}^{n}C_{r}\, p^{r}\, (q)^{n-r}$$

where $n$ = sample size

$r$ = occurence of defectives

$p$ = fraction defective of a lot

$q$ or $(1 - p)$ = success

The mean of this distribution is given by $\mu = np$

$$\text{Standard deviation} = \sqrt{npq}$$

$$\text{Variance} = npq$$

This distribution will give an approximate results to hypergeometric distribution when the lot size is very large when compared with sample size, that is, $N \geq 10n$. It will also give approximate results to normal curve for the practical situation with same mean and standard deviation.

This property can be utilized for the construction of '*p*' chart to check whether process is in a state of statistical control or not.

## SOLVED PROBLEMS

**Problem 1.** *Samples of 20 pieces each are drawn at random from very large lots of components. Determine the probability that such a sample will contain : (a) More than one defective ; (b) Less than 2 defectives (c) Less than or equal to 2 defectives ; (d) Atmost 2 defectives ; (e) Atleast 2 defective ; and (f) Exactly 2 defecitves. The lot is 20% defective. Use the method which gives theoretically correct results.*

**Solution.** Given data :

$n = 20$ ; $p = 0.2$ ; $q = 1 - 0.2 = 0.8$

Since lot size is infinite and unknown, the theoretically correct distribution to the given problem is binomial. As per binomial

$$P(r) = {}^{n}C_r \, p^r \, q^{n-r}$$

Computation of probabilities :

$P(0) = {}^{20}C_0 \, (0.2)^0 \, (1 - 0.2)^{20-0} = 0.0115$

$P(1) = {}^{20}C_1 \, (0.2)^1 \, (1 - 0.2)^{20-1} = 0.0576$

$P(2) = {}^{20}C_2 \, (0.2)^2 \, (1 - 0.2)^{20-2} = 0.1368$

(*a*) *More than one defective*

The required probability = $P(r > 1) = P(2) + P(3) + .....$

This can be written as $= 1 - P(r \leq 1)$

$= 1 - P(0) + P(1)$

$= 1 - (0.0115 + 0.0576)$

$= 0.931$

(*b*) *Less than two defectives*

The required probability = $P(r < 2) = P(0) + P(1)$

$= 0.0115 + 0.0576 = 0.0691$

(*c*) *Less than or equal to 2 defective*

The required probability = $P(r \leq 2) = P(0) + P(1) + P(2)$

$= 0.0115 + 0.0576 + 0.1368 = 0.2059$

(*d*) *Atmost 2 defectives* (same as '*c*')

The required probability = $P(r \leq 2) = 0.2059$

(*e*) *Atleast 2 defectives*

The required probability = $P(r \geq 2) = P(2) + P(3) + .....$

This can be written as $1 - P(r < 2\ ) = 1 - \{P(0) + P(1)\}$

$= 1 - (0.0691) = 0.9309$

(*f*) *Exactly 2 defectives*

The required probability = $P(r = 2) = P(2) = 0.1368$.

**Problem 2.** *A factory finds that on an average 20% of the bolts produced by a given machine will be defective for a certain specified requirements. If 30 bolts are selected at random from a days production of a machine find the probability that : (a) Exactly 3 defectives ; (b) 2 or more defectives ; (c) More than 2 defectives ; and (d) Less than 2 defectives. Compute the results by method which is theoretically correct.*

**Solution.** Given data : $n = 30$ ; $p = 0.2$ ; $q = 0.8$

Since lot size is unknown the suitable distribution to this problem is binomial distribution. As per binomial

$$P(r) = {}^{n}C_r \, p^r \, q^{n-r}$$

Computation of probabilities :

$$P(0) = {}^{30}C_0\,(0.2)^0\,(1-0.2)^{30-0} = 0.0012$$
$$P(1) = {}^{30}C_1\,(0.2)^1\,(1-0.2)^{30-1} = 0.0093$$
$$P(2) = {}^{30}C_2\,(0.2)^2\,(1-0.2)^{30-2} = 0.0336$$
$$P(3) = {}^{30}C_3\,(0.2)^3\,(1-0.2)^{30-3} = 0.0785$$

(*a*) *Exactly 3 defectives*

The required probability = $P(r = 3) = P(3) = 0.07853$

(*b*) *2 or more defectives*

The required probability = $P(r \geq 2) = P(2) + P(3) + \ldots$

This can be written as $= 1 - P(r < 2) = 1 - \{P(0) + P(1)\}$

$= 1 - (0.00124 + 0.0093) = 0.9894$

(*c*) *More than 2 defectives*

The required probability = $P(r > 2) = P(3) + P(4) + \ldots$

$= 1 - P(r \leq 2) = 1 - \{P(0) + P(1) + P(2)\}$

$= 1 - (0.00124 + 0.0093 + 0.0366) = 0.9558.$

**Problem 3.** *If an unbiased coin is tossed 6 times, what is the probability that only 2 heads will occur.*

**Solution.** Given data : $n = 6$ ; $p = 0.5$ ; $q = 0.5$

The required probability = $P(r = 2) = P(2)$

$= {}^{6}C_2\,(0.5)^4\,(0.5)^4 = 0.2343.$

**Problem 4.** *The probability that a radio manufactured by a company will be defective is 1/10. If 15 of such radio's are inspected, find the probability that (a) Exactly 3 defectives ; (b) Atleast 1 defective ; and (c) None will be defective.*

**Solution.** Given data : $n = 15$ ; $p = 0.1$ ; and $q = 0.9$

Since lot size is unknown theoretically correct distribution is binomial.

Exactly 3 defectives = P(3)

$P(3) = {}^{15}C_3\,(0.1)^3\,(0.9)^{12} = 0.1285$

*Atleast one defective*

The required probability $= P(r \geq 1) = P(1) + P(2) + \ldots.$

$= 1 - P\,(r < 1)$

This can be written as $= 1 - P(0) = 1 - \{{}^{15}C_0\,(0.1)^0\,.\,(0.9)^{15}\}$

$= 1 - \{1 \times 0.2058\} = 0.7941$

*None will be defective*

The required probability = P(0) = 0.2059.

**Problem 5.** *The probability that a bulb produced by a factory will fuse after 100 days is 0.05. Find the probability out of 5 such bulbs (a) None ; (b) Not more than one ; (c) Greater than one and (d) Atleast one ; will fuse after 400 days of use.*

**Solution.** Given data : $P = 0.05$ ; $n = 5$ ; and $q = 0.95$

(*a*) None will fuse is $P(r = 0) = 0.7738$

(*b*) Not more than one will fuse is $P(r < 1) = 0.9974$

(*c*) More than one will fuse is $P(r > 1) = 1 - \{P(0) + P(1)\} = 0.0226$

(*d*) Atleast one will fuse is $P(r > 1) = 1 - P(0) = 0.2262.$

**Problem 6.** *If 10 fair coins are tossed what is the probability that (a) Exactly 3 heads ; and (b) Not more than 3 heads.*

**Solution.** Given data: $n = 10$ ; $p = 0.5$ ; $q = 0.5$

(*a*) Exactly 3 heads is $P(3) = {}^{10}C_3 (0.5)^3 (0.5)^7 = 0.1172$

(*b*) Not more than 3 heads is $P(0) + P(1) + P(2) + P(3) = 0.1719$.

**Problem 7.** *A random sample of 25 articles is taken from a stream of products housing 20% defectives. What is the probability that sample will contain exactly 5 defectives.*

**Solution.** Given data : $n = 25$ ; $p = 0.2$ ; and $q = 0.8$

Exactly 5 defectives is $P(5) = {}^{25}C_5 (0.8)^{20} (0.2)^5 = 0.1960$.

## EXERCISE PROBLEMS

**1.** A company is interested in evaluating its current inspection procedure of shipment of 50 identical items. The procedure is to take a sample of 5 and pass, the shipment if not more than 2 core found to be defective. What the probability of passing 20% defective shipment as per this procedure.

**2.** A traffic control engineer reports that 75% of the vehicles passing through a check point are from out of the state. What is the probability that less than 4 of the next 9 vehicles are from within the state.

**3.** A dice is thrown 3 times. If getting a six is considered as success, find the probability that (*a*) 3 successes ; and (*b*) Atleast 2 successes.

**4.** A lot of 2000 capacitors is known to have 20% defective. If a sample of 40 are inspected randomly from a lot, what is the probability of finding ; (*a*) Exactly 10 defectives ; (*b*) Greater than 4 defectives ; (*c*) Less than 3 defectives.

**5.** 5 percent products produced is in a manufacturing process turns out to be defective. Find the probability that in a sample of 10 units chosen at random will have exactly 2 defectives : (*a*) Using binomial distribution ; (*b*) Poisson approximation to the binomial ; (*c*) Normal approximation to the binomial.

### POISSON DISTRIBUTION

This is applicable to many situations that are under observation per unit of time. Example Number of machine break down per day or number of power cuts per day etc.

This is also applicable to the situation that involve, the no. of observations per unit of amount. Example Number of defects per casting, number of mistakes per page etc.

The probability of occurrence of defectives in a random sample is given by

$$P(r) = \frac{e^{-np} (np)^r}{r!}$$

where $n$ = sample size

$p$ = fraction defective of a lot

$r$ = occurrence of defectives.

For this distribution Mean = $np$ or $\lambda$ or $\mu$. Standard deviation = $\sigma$ and Variance $\sigma^2 = \mu$.

This will be applicable for all the countable type data. This distribution will give approximates results to binomial or hypergeometric distribution when $P \leq 0.1$ and $n > 10$.

**SOLVED PROBLEMS**

**Problem 1.** *If a random variable is posssion distributed with a mean of 5.2 determine : (a) Standard deviation ; (b) Variance ; (c) The probability that a random variable will have a value of 3 or less ; (d) The probability that a random variable will have a value of more than 6.*

**Solution.** Given data : Mean, $\mu = 5.2$

(*a*) Standard deviation, $\sigma = \sqrt{5.2} = 2.283$

(*b*) Variance, $\sigma^2 = 5.2$

Computation of probabilities

$$P(r) = \frac{e^{-np}(np)^r}{r!}$$

$$P(0) = \frac{e^{-5.2}(5.2)^0}{0!} = 0.0055$$

$$P(1) = \frac{e^{-5.2}(5.2)^1}{1!} = 0.0286$$

Similarly P(2) = 0.0745 ; P(3) = 0.1292 ; P(4) = 0.1680 ;
P(5) = 0.1764 ; P(6) = 0.1514

(*c*) The probability that a random variable will have a value of 3 or less

= P(0) + P(1) + P(2) + P(3)

= 0.0055 + 0.028 + 0.0745 + 0.1292 = 0.2378

(*d*) The probability that a random variable will have a value of more than 6

$P(r > 6) = 1 - \{P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)\}$

= 1 – (0.2378 + 0.1680 + 0.1764 + 0.1514)

= 0.2664.

**Problem 2.** *An acceptance plan calls for the inspection of the sample of 115 articles, out of a lot of 3000. If there are 6 or less non confirming articles in the sample the lot is accepted with 7 or more the lot is rejected. If the lot contains 5% non confirming items what is the probability that : (a) It will be accepted ; and (b) Rejected solve using possion as an approximation.*

**Solution.** Given data : $n = 115$

Probability of non-confirming items = $P(r \leq 6)$

Lot is accepted for a mean of 5.75, the associated probabilities as per poisson's distribution are as follows : P(0) = 0.0031, P(1) = 0.0183, P(2) = 0.526, P(3) = 0.1008, P(4) = 0.1449, P(5) = 0.1667, P(6) = 1677.

Therefore $P(r \leq 6) = 0.6541$

(*b*) Probability that the lot is rejected is $P(r \leq 7)$ *i.e.*,

$= 1 - \{P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)\}$

= 1 – 0.6541

= 0.3459.

## EXERCISE PROBLEMS

**1.** The probability that a casting produced by a certain foundry has blow holes of 0.004. Find the probability that less than 4 of the next 2000 items produced by the foundry has the blow holes.

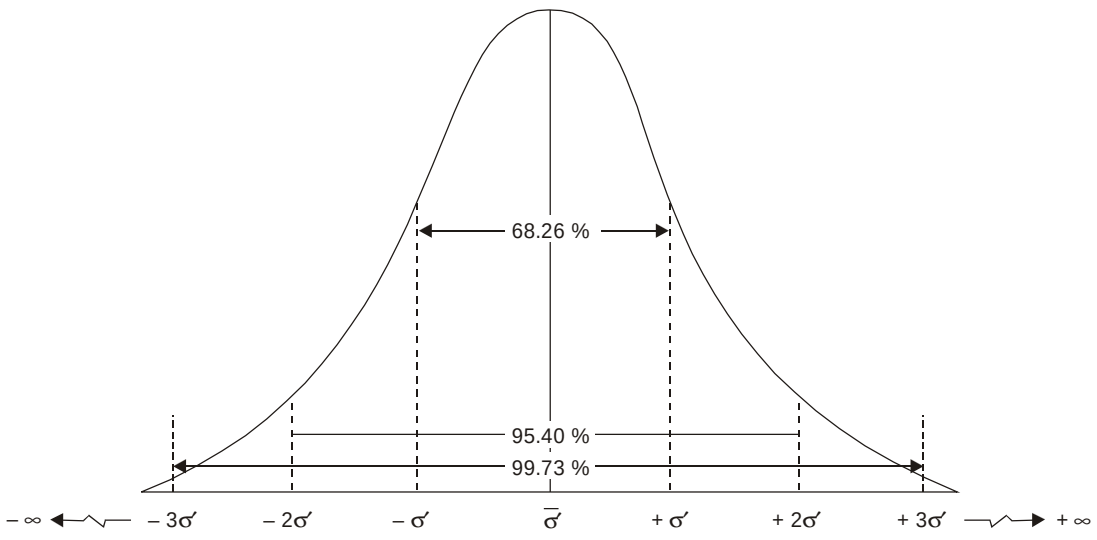
**2.** If an inventory study it was determined that average the demand for a particular item at a warehouse were made 5 items per day. What is the probability that on a given day this term is required : (*a*) More than 3 times ; (*b*) Not at all.

**3.** A manufacturer know from experience that five out of 1000 shells is defective. What is the probability that in a batch of 600 : (*a*) Exactly 4 defective ; and (*b*) More than 4 defectives $n = 600$.

**4.** A Secretary makes 2 errors per page on the average. What is the probability that on the next page she makes : (*a*) 4 or more errors ; and (*b*) No error.

### NORMAL PROBABILITY DISTRIBUTION

The variation of the pattern obtained for a set of quality characteristic data that has been produced by a process subjected to a chance cause alone is known as normal distribution.




### The Properties or Characteristics of Normal Distribution

(1) Mean, Median and mode are identical

(2) It is a bell shaped curve

(3) It is symmetric about the mean

(4) Curve starts from – ∞ to + ∞.

(5) It represents populations of infinite size.

(6) It is defined by two parameters namely mean and standard deviation.

(7) The distribution is unimodal.

(8) General form of curve is given by Probability Density Function (*pdf*)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{\sigma^2}}$$

$$Z = \frac{(x-\mu)}{\sigma}$$

where Z = standardised normal value

$x$ = individual limit

$\mu$ = mean

$\sigma$ = standard deviation.

**Applications**

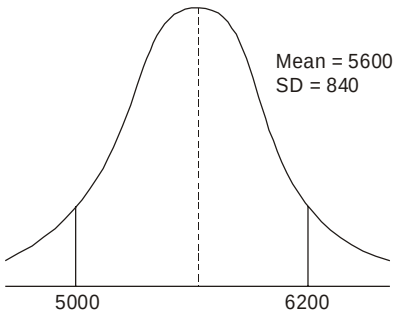
(1) Main application is 99.73% area covered between + 3$\sigma$ or – 3$\sigma$ limits is the base for the control charts.

(2) It is possible to find the percentage of the data which are less than the particular value, greater than the particular value and between the two specified limits.

**Solved Problems**

**Problem 1.** *The mean value of the modulus of rupture of a large number of test specimens has been found to be 5600 psi. If the standard deviation is 840 psi [paid square per inch] and distribution is approximated to be normal : (a) What percentage of specimens will have modulus of rupture within 5000 and 6200 psi ; (b) What percentage of it will be above 4000 ; and (c) What percentage of it will be below 3500 ?*

**Solution.**




(*a*) Mean, $\mu$ = 5600 psi

Standard deviation, $\sigma$ = 840 psi

Lower Limit = 5600

Upper Limit = 6200

The percentage of specimens falling below 5000

For Z = – 0.7142 the probability from nomal table is 23.89%.

The percentage of specimen from nomal table is 0.7611 that is 76.1%

Therefore the percentage of the specimens falling between 5000 and 6200

= 76.11 – 23.89% = 52.22%

(*b*) Percentage of specimens falling below 4000 is

Z = 4000 – 5600/840 = Z = – 0.190

Probability from normal tables is 0.028 *i.e.,* 2.8%

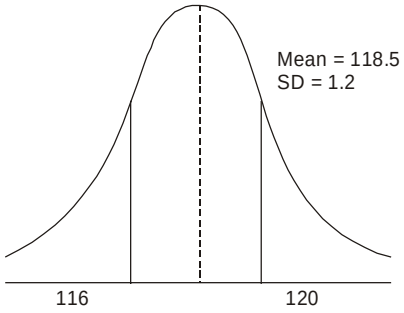
Therefore the probability above 4000 = 100 – 2.87 = 97.13%

(*c*) Z = 3500 – 5600/840 = – 2.5

The probability from tables = 0.0062

Therefore the percentage of specimen falling below 3500 = 0.62%.

**Problem 2.** *A large no. of test of line voltage to the home residences show a mean of 118.5 volts and standard devation of 1.2 volts : (a) Determine percentage of data lying between 116 and 120 volts ; (b) It is desired to have 12% line voltages below 115 volts. How should the mean voltage be adjusted.*




**Solution.** Mean, $\mu = 118.5$ volts

SD, $\sigma = 1.2$ volts

The percentage of specimen lying below 116 volts

$Z = x - \mu/\sigma = 116 - 118.5/1.2 = -2.0833$

Probability from the tables is 0.0188 = 1.88%

The percentage of specimen lying below 120 volts

$Z = 120 - 118.5/1.2 = 1.25$

Probability from table is 0.8944 = 89.44%

Therefore the percentage of specimen falling between 116 and 120 volts

$= 89.44 - 1.88 = 87.56\%$

(*b*) $Z = 115 - \mu/1.2$

In probability 0.12 the Z value obtained from tables is – 1.170

Therefore,

$-1.170 = 115 - \mu/1.2$

$-1.404 = 115 - \mu$

$\mu = 116.404$

Mean required = 116.404

The adjustment is 116.404 – 115 = 1.404 voltage.

**Problem 3.** *The diameter is piston ring in normally distributed with mean 10 cm and standard of 0.03 cm : (a) What percentage of rings will have the diameter exceeding 10.075 cm and (b) Below what value of diameter will 15% piston rings fall.*

**Solution.** $\mu = 10$ cm $\sigma = 0.03$

The percentage of piston rings which have dimension greater than 10.075 is

$$Z = \frac{x - \mu}{\sigma} = \frac{10.075 - 10}{0.03} = 2.5$$

The values of Z from normal tables = 0.9938 = 99.38%

Therefore 100 – 99.38 = 0.62% of items are falling greater than 10.075 cms.

(*b*) 15% = 0.15

The Z value for probability 0.15 from normal tables = – 1.04

$$-1.04 = \frac{x - 10}{0.03}$$

$0.0312 = x - 10$

$x = 9.9688$ cms.

**Problem 4.** *A manufacturing plant wants to maintain 1.5% of the product below the weight specification of 0.567 kg. Data is normally distributed and standard deviation is 0.01 kg what is the mean weight might required.*

**Solution.** Given : $\sigma = 0.01$ kg

The Z value for probability 0.015 from normal tables = – 2.16

We know that $Z = \frac{x - \mu}{\sigma}$

$$-2.16 = \frac{0.567 - \mu}{0.01}$$

$$-0.0216 = 0.567 - \mu$$

$$\mu = 0.567 + 0.0216 = 0.5886$$

Therefore the mean weight required is 0.5886 μ.

**Problem 5.** *The an eye life of a certain type of small motor is 10 years with a variance of 4 years. Manufacturer replaces free all motors that fail were under the guarantee. If he is willing to replace only 5% of the motors that fail. Low long a guarantee should be offered. Assume life of motor follow normal distribution.*

**Solution.** Given data : $\mu = 10$ years

$\sigma = 2$ years

5% = 0.05

The value of Z from probability tables Z = 1.64

$$-1.64 = \frac{x - 10}{2}$$

$$-3.28 = x - 10$$

$$x = -3.28 + 10 = 6.72 \text{ years.}$$

## EXERCISE PROBLEMS

**1.** In a precision grinding of a complicated part is more economical to rework. Than top scrap. It is desired to establish a rework percentage of 1.25. Assume data follows Normal distributions. The standard deviation is 0.01 upper specification limits is 25.38. Find the mean with this estimated mean what % of the terms are falling below 25.1 mm.

**2.** If weights of 300 students are normally distributed. With a mean of 68 kg and variance of 9 kg how many students have their weights : (*a*) Greater than 72 kg ; (*b*) Less than 64 kg ; and (*c*) Between 64 and 72 kg.

**3.** A transistor radio operator on 3 size, 1.5 volts battery so that nominally it operates on 4.5 volts. Suppose actual voltage of single new battery is nominally distributed with mean 1.5 volts and standard deviation of 0.2 volts, the radio will not operate properly at the outset if voltage falls outside the range 4 to 5 volts. What is the probability that radio will not operate properly.

**4.** The calorifie value of a certain value follows a normal distribution with mean of 11,000 K calorie and standard deviation of 200K calories. Find the probability that calorific value, the test shows : (*a*) Less than 10,870 ; (*b*) Greater than 11,340 ; and (*c*) Between 10,900 and 11,460 K calories.

**5.** The mean value of moduler of repture of large no. of test specimens has been found to be 415 kg sqcm. If standard deviation is 60kg : (*a*) What percentage of the specimen

will have moduler of rupture between 350 and 450 ; (*b*) What percentage of it will be above 300 ; and (*c*) The percentage of it will be below 250.

**6.** Above in a mars produced part having a mean diameter of 2.498 cms with standard deviation 0.013 cms : (*a*) Determine the percentage of scrap ; (*b*) Determine the percentage of rework ; and (*c*) Determine the percentage of acceptable items meeting the specifications of 2.50 ± 0.025.

**12.** Location pins for work holding devies are ground to a deviater of 12.5 mm with a tolerance of 0.05 mm. If the process is centred at 12.5 mm and dispersion 0.02 mm : (*a*) What percentage of product (s.d.) must be scrap ; (*b*) What percentage of product can be rework ; (*c*) How can one process centre be charged in order to eliminate the scrap ; and (*d*) What is rework percentage now ?

**13.** The mean value of moduler of rupture of large no. of test specimens has been found to be 5600 psi. and the variance is 1,15,600. Find the lowest and upper value's of the modulus of rupture so that 95% of the specimens fall within these values.

## NORMAL APPROXIMATION TO THE BINOMIAL

**Problem 1.** *A manufacturing process produces items with 25% defect. What is the probability that sample of 50 will contain : (a) Exactly 15 defectives ; (b) Less than or equal to 11 defectives ; and (c) 15 or more defectives.*

**Solution.** Given data : $n = 50$ ; $P = 0.25$ ; $q = 0.75$

Theoretical the correct distribution is binomial. As per binomial $P(r) = {}^{n}C_{r}\, p^{r} q^{n-r}$ ;

The mean is given by $\mu = np$

$$= 50 \times 0.25 = 12.5$$

Standard deviation is given by $\sigma = 3.062$

(*a*) Exactly 15 defective means probability that defectives less than 14.5 defective is

$$Z = \frac{x - \mu}{\sigma} = \frac{14.5 - 12.5}{3.0618} = 0.65$$

The probability from tables = 0.7422 = 74.22%

Less than 15.5 defectives

$$Z = \frac{15.5 - 12.5}{3.061} = 0.97 = 97\%$$

Probability from normal tables = 0.8340 = 83.40%

Therefore exactly 15 defectives is 83.40 – 74.22 = 9.18%

– 9.18% has exactly 15 defectives.

(*b*) Less than or equal to 11 defective is

$$Z = \frac{11.5 - 12.5}{3.0618} = -0.3314$$

Probability is 37.45%

(*c*) More than 15 or 15 defectives is

$$Z = \frac{14.5 - 12.5}{3.0618}$$

Probability is 74.22%

for more than 15 is 100 – 74.22 = 25.78%.

**Problem 2.** *A manufacturing process produces 100 items with 10% defective (a) what is probability that the sample will contain exactly 8 defectives ; (b) 14 or more defectives ; (c) Less than or equal to 6 defectives use the normal approximation to the possion distribution.*

**Solution.** Given data : $\mu = np$ ; $\sigma^2 = \mu$

$$\mu = 100 \times \frac{10}{100} = 10$$

$$\sigma = 3.1622$$

Exactly 8 defectives :

Less than 7.5 defectives

$$Z = \frac{x - \mu}{\sigma} = \frac{7.5 - 10}{3.1622} = -0.790$$

Probability from table is 0.2148 = 12.48%

Less than 8.5 defectives

$$Z = \frac{8.5 - 10}{3.1622} = -0.47$$

Probability from table is 0.3192 = 31.92%

Therefore for exactly 8 defectives is 31.92% – 21.48 = 10.44%

(*b*) For 14 or more defectives

$$Z = \frac{13.5 - 10}{3.1622} = 1.10$$

Probability from table is 0.8643 = 86.43%

For 14 or more is 100 – 86.43 = 13.57%

(*c*) For less than 6 defectives

$$Z = \frac{x - \mu}{\sigma} = \frac{6.5 - 10}{3.1622} = -1.10$$

Value from tables is 0.1357 = 13.57%.

## EXERCISE PROBLEMS

**1.** If the weights of 400 students are nomally distributed with mean 68 kg and variance of 9 kg. How many students have their weights : (*a*) Less than or equal to 64 kg ; (*b*) Greater than or equal to 72 kg ; and (*c*) Exactly 64 kg. Using nomally approximation to binomial.

# 3 An Overview of Total Quality Management

## EVOLUTION OF QUALITY

The requirement for Quality Control dates back to the time when human race wanted to replicate an object. The desire to control quality is as old as human's ability to produce things the forerunning attempts to control quality resulted in rather crude replicas of original objects. These replicas were produced in a way that could easily be discerned by the nacked eye.

As time passed, human's developed the competence to duplicate objects so that they become indistinguishable from one another. The drawback for this was that the assembly with any alternation or adjustment was not possible.

Eli Whitney conceived the idea of perfect interchangeability of parts. He emphasised that if proper raw material, methods and equipment are used and if workmen exercised the right amount of course, items can be produced somewhat in an identical manner. In 1799, he contracted to supply rifles to the army. Mr. Whitney was partially successful in getting each workman to make one part of the exact specification he could still do only selective assembly. But did establish the fact that production time can be reduced.

Perhaps this was the germs of man production. It was not until the early 1800s that man began to realise the necessity of tolerance in parts. The interchangeability in industrial activity resulted in many problems on measurements. A Swedish engineer named Johansson conceived the idea of a hard metal block that could be machined and polished to exact dimension, which can be used as points of reference. These blocks were referred to as 'Jo' blocks. In the middle of the $17^{th}$ century, Pascal, the French philosopher and mathematician become quite talker by the games of chance. He formulated that theory of probability in association with Pierre Fermat. During the 1800s, considerable progress was made in the development of the sampling theory.

Modern quality control or statistical quality control (SQC) as we know it today started with invention of quality control chart by Walter A Shewhart of Bell Telephone Labs, USA in 1930s. Dr. Shewhart proposed the statistical methods could be effectively used for examining whether the items produced by any process were of uniform quality or not. The real impetus for the application of these methods on a massive scale resulted form the economic pressure for more efficient utilisation of equipment and resources during world war II Dr. Shewhart wrote a book economic control of quality of manufactured products, which was published in 1931. The objective explicitly put-forth in the title was "Economic Control".

The influence of the US military services on the adoption of sampling acceptance techniques was well established. World War II was the catalyst that made the control charts applicable in the US. By applying quality control, the US was able to produce military requirements inexpensively and in high volumes. The wartime standards published in those days was known as "Z–1 standards".

**British Standards**

England also developed quality control at relatively early date, the application of which was evident in the adoption of British Standards 600 in 1935 on E.S. Pearson's statistical work. Later US Z-1 standards were adopted in there entirely as British standard were used in England during the time of war.

These standards stimulated technological advances in terms of production quantity, quality and cost. It would be an exaggeration to say that World War II was won by sing statistical quality control methods. Some of the statistical methods researched and utilised by the allied power were so effective that they were regarded as military secrets until the surrender of Nazi Germany.

The Japanese knew about the British standards 600 in the pre-war year and translated them into Japanese during the War Japanese effort in modern statistics was expressed in mathematical language which was difficult to understand.

Quality was controlled by inspection and not every product was sufficiently inspected. Hence, Japan had to compete with price and not with quality. It was literally the ago of "Cheap and poor" products from Japan.

**Learning from US**

Having faced defeat in World War II Japan lost all that it had and could not even feed people with food, clothing and shelter. They realised that if you make poor quality products then the troops so, it was a matter of life and death. It was at his critical juncture that the US occupational (USOF) having landed in Japan ; ordered the Japanese telecommunication industry to learn the use of modern quality control and took steps to educate the industry.

This was the beginning of SQC in Japan in 1946. Quality control methods taught by the USOF were not modified for the Japanese. Though this created some problems the methods reached beyond the telecom industry. Japanese could really make tremendous progress by making heavy investments and by inviting great scholars like Dr. Deming in 1950 and Dr. Juran in 1954 the huge investment made in education and training at the crucial time paid very rich dividends in Japan.

These Japanese thus learn one bitter lesson that "if you make bad quality during war time, your country is overrun by enemy troops, so it is a matter of life and death. They eventually taught the world a lesson that if you make bad quality, during peace time, your country during peace time, your country is overrun by foreign products, which is a matter of life and sickness, commercial death, bankruptcy, impoverishment and what not.

The development of quality activities has spanned the entire of $20^{th}$ century curiously significant changes in the approach to quality activities have occurred every 20 years. Quality activities have traversed a long path from operator Inspection (1900s) to verification of quality by supervisors (1920s) to establishment if quality control departments and 100% inspection (1940s) to statistical quality control (1960s) to TQC with statistical control (1980's) TQM and statistical problem solving (1990s).

**DEFINITION**

Total Quality Management (TQM) is an enhancement to the traditional way of doing business. It is a proven technique to guarantee survival in world-class competition. Only by changing the actions of management will the culture and actions of an entire organization be transformed. TQM is for the most part common sense. Analyzing these words.

Total — made up of the whole

Quality — Degree of Excellence a Product or Service provides

Management — Act, art or manner of handling, controlling, directing etc.

**TQM CONCEPTS**

The above principles are bandied freely around in the above discussion. Its worth dwelling with each for a moment.

Be customer–focused means everything you do will be done by placing the customer in the centre. The company should regularly check customer's attitudes. This will include the external and internal customer concept.

Do it right first time so that there is no rework. This essentially means cutting down on the amount of defective work.

Constantly improve, this allows the company gradually to get better. One of the axioms use by TQM people is "A 5% improvement in 100% of the areas is easier than a 100% improvement in 5% of the areas.

Quality is an attitude The attidue is what differentiates between excellence and mediocracy. Therefore it's very important to change the attitude of the entire workforce *i.e.*, basically the way the company works company's work culture.

Telling the staff what is going on means keeping the entire workforce informed about he general direction the company is headed in typically this includes them briefings, one of the main elements to TQM.

Training and education of the workforce is a vital ingredient, as untrained staff tend to commit mistakes. Enlarging the skill base of the staff essentially makes them do a wider range of jobs and do them better. In the new system of working under TQM educating the staff is one of the principles.

Measurement of work allows the company to make decisions based on facts, it also helps them to maintain standards and keep processes with in the agreed tolerance levels.

The involvement of senior management is essential. The lack of which will cause the TQM program to fail.

Getting employees to make decision on the spot so that the customer does not face any inconvenience in empowering the employees.

Mailing it a good place to work. In many an organisation there exists a lot of fear in the staff. The fear of the boss, fear of mistakes of being sacked. TQM program is any company filled with fear cannot work, therefore fear has to be driven out of the company before starting of TQM program.

Introduce team working, its boosts employee morale. It also reduces conflict among the staff. It reduces the role of authority and responsibility, and it provides better more balanced solutions. In a lot of companies teamwork is discouraged, so TQM programs must encourage it.

Organise by process, not by function. This concentrates on getting the product to the customer by reducing the barriers between the different departments.

**Present Trend**

From maintenance of quality to continuous improvement of quality some important concepts.

1. *Product control Vs Process Control* : Process produces products. Product quality depends an process quality controlling the process is easier and cheaper than controlling the product controlling process needs knowledge tools and techniques.
2. *Upstream Vs Downstream* : Upstream is the starting area of a business process. Downstream is the tail end of the business process. Corrective action at upstream is very must cost effective. Cost goes up as and when the stream goes down. It will be in geometric proportions. Better planning is required to reduce downstream costs. It's worth spending more time for planning to reduce expensive execution time. This needs cross-functional organisation and knowledge of quality function deployment.

**BASIC APPROACH**

TQM requires six basic concepts :

1. A committed and involved management to provide long-term top-to-bottom organizational support.
2. An unwavering focus on the customer, both internally and externally.
3. Effective involvement and utilization of the entire work force.
4. Continuous improvement of the business and production process.
5. Treating suppliers as partners.
6. Establish performance measures for the processes.

There concepts outline an Excellent way to run an organization. A brief paragraph on each of them is given here.

1. Management must participate in the quality program. A quality council must be established to develop a clear vision, set long-term goals and direct the program. Managers participate on quality improvement teams and also as coaches to other teams. TQM is a continual activity that must be entrenched in the culture it is not just a one-shot program. TQM must be communicated to all people.

2. The key to an effective TQM program is its focus on the customer. An excellent place to start is by satisfying internal customers. We must listen to the "Voice of the customer" and emphasize design quality and defect prevention.

3. TQM is an organization–wide challenge that is everyone's responsibility. All personnel must be trained in TQM, statistical process control (SPC) and other appropriate quality improvement skills so they can effectively participate on project teams.

People must come to work not only to do their jobs, but also to think about how to improve their jobs, people must be empowered at the lowest possible level to perform processes in an optimum manner.

4. There must be a continue striving to improve all business and production processes. Quality improvement projects, such as on-time delivery, order-entry efficiency, billing error rate, customer satisfaction, cycle time, scrap reduction and supplier management are good places to begin.

5. On the average 40% of the sales is purchased product or service, therefore, the supplier quality must be outstanding. The focus should be on quality and life cycle costs rather than price. Suppliers should be few in number so that true partnering can occur.

6. Performance measures such as uptime, percent non-conforming, absentecism and customer satisfaction should be determined for each functional area.

Quantitative data are necessary to measure the continuous quality improvement activity.

## THE DEMING'S PHILOSOPHY

Dr. W. Edward-Deming was a protege of Dr. Walter Shewhart, who pioneered statistical process control (SPC) at Bell Laboratories. He Spent one year studying under Sir Ronald Fisher, who pioneered design of experiments.

Dr. Deming is credited with providing the foundations of the Japanese quality miracle and resurgence as an economic power. He developed the following 14 points as a theory for management for improvement of quality, productivity and competitive position.

**1. Create and publish the Aims and Purposes of the Organization**

Management must demonstrate constantly their commitments to this statement. It must include investors, customers, suppliers, employees, the community and a quality philosophy. Organization must develop a long-term view of at least 10 years and plan to stay in business by setting long-range goals. Resources must be allocated for research, training and continuing education to achieve the goals. A family organizational philosophy is developed to send the message that every one is part of the organization.

**2. Learn the New Philosophy**

Top management and every one must learn the new philosophy. Organizations must seek never ending improvement and refuse to accept non conformance customer satisfaction is the number one priority, because dissatisfied customers will not continue to purchase non confirming products and service. Every one in the organization, including the union, must be involved in the quality journey and change his or her attitude about quality.

**3. Understand the Purpose of Inspection**

Management must understand that the purpose of inspection is to improve the process and reduce it's cost. Statistical evidence is required of self and supplier every effort should be made to reduce and then eliminate acceptance sampling.

**4. Stop Awarding Business based on Price Alone**

The organization must stop awarding business based on the low bid, because price has no-meaning without quality. The goal is to have single suppliers for each item to develop a long-term relationship of loyalty and trust thereby providing improved products and service.

**5. Improve Constantly and forever the System**

Management must take more responsibility for problems by actively finding and correcting. Problems so that quality and productivity are continually and permanently improved and costs are reduced. The focus is on preventing problems before they happen variation is expected but these must be a continual striving for its reduction using control charts.

**6. Institute Training**

Each employee must be oriented to the organization philosophy of commitment to never-ending improvements management must allocate resources to train employees to perform their jobs in the best manner possible.

**7. Teach and Institute Leadership**

Improving supervision is management's responsibility. They must provide supervision with training in statistical methods and these 14 points so the new philosophy can be implemented. Instead of focusing on a negative. Fault-finding atmosphere, supervisors should create a positive, supportive one where pride in workmanship can flourish.

**8. Drive out fear, Create Trust, and Creat a Climate for Innovation**

Management must encourage open, effective communication and teamwork. Fear is caused by a general feeling of being powerless to control important aspects of one's life. It is caused by a lack of Job security, possible physical harm, performance appraisals, ignorance of organization goals, poor supervision and not knowing the job. Driving fear out of the work place involves managing for success. When people are treated with density, fear can be eliminated and people will work for the general good of the organization. In this climate, they will provide ideas for improvement.

**9. Optimize the Efforts of Teams, Groups and Staff Areas**

Management must optmize the efforts of teams, work groups and staff areas to achieve the aims and pusposes of the organization. Barriers exist internally among levels of management, among departments, within departments and among shifts. To break down the barriers, management will need a long-term perspective. All the different areas must work together. Attitudes need to be changed ; communication channels opened project teams organized and training in team work implemented.

**10. Eliminate Exhortations for the Work Forces**

Exhortations that ask for increased productivity without providing specific improvement methods can handicap an organizations. They do nothing but express managements desires. They do not produce a better product or service, because the workers are limited by the system.

**11.** (*a*) **Eliminate Numerical Quotas for the Work Force**

Instead of quotas, management must learn and Institute methods for improvement. Quotas and work standards focus on quantity rather than quality. They encourage poor workman ship in order to meet their Quotas. Quotas should be replaced with statistical methods of process control.

**11.** (*b*) **Eliminate Management by Objective**

Instead of management by objective, management must learn the capabilities of the process and how to improve them. Internal goals set by management, without a method are a burlesque.

**12. Remove Barriers that Rob People of Pride of Workmanship**

Loss of pride in workmanship exists throughout organizations because

(1) Workers do not know how to relate to the organizations mission.

(2) They are being blamed for system problems.

(3) Poor designing leads to the production of "Junk".

(4) Inadequate training is provided.

(5) Punitive supervision exists.

(6) Inadequate or ineffective equipment is provided for performing the required work.

Restoring pride will require a long term commitment by management. When workers are proud of their work, they will grow to the fullest of their job. By restoring pride, everyone in the organization will be working for the common good. A barrier for people on salary is the annual rating of performance.

**13. Encourage Education and Self-himprovement for Everyone**

What an organization needs is people who are improving with education. A long-term commitment to continuously train and educate people must be made by management Deming's 14 points and the organization's mission should be the foundation of the education program.

**14. Take Action to Accomplish the Transformation**

Management has to accept the primary responsibility for the never-ending improvement of the process, it has to create a corporate structure to implement the philosophy. A cultural change is required from the previous "business as usual" attitude. Management must be committed, involved and accessible if the organization is to succeed in implementing the new philosophy.

## GURUS OF TOTAL QUALITY MANAGEMENT

With the advent of today's reality in the Business environment through Liberation, Privatisation and Globalisation, coupled with growing concern for ecology, emergence of new economic powers, political realignments, growing literacy, concern for human rights and impact of information and telecommunication. The survival of Business has become more challenging. With global players the differentiating factor between one organisation and other is becoming narrower. There is a very little difference in the pricing or the product features. Thus, Business survival now depends more on the Quality. For the context of excellence the quality refers to that of technology and that of Management. In other words it is the Total Quality and its Management.

**Total Quality Management** necessitates realisation of Quality ideals of being Top Management led, with an integrated approach focused towards customer satisfaction through continues improvement. Often these ideals are not realised primarily because the Quality costs are high in both manufacturing and service sector, often it is un-quantified.

TQM has become the key success factor for building Business Excellence for several organisations all around the world each one of them adopt different models and different routes but invariably all of them with one ultimate goal of becoming the best.

Although the philosophies of various gurus on Quality Management appear to be varying considerably, it can clearly be seen that there exists a common thread passing through all these philosophies. The American gurus focused more on the social systems including employee involvement concepts, the Japanese gurus successfully translated these into the shop floor management systems through Just-in-time and the Western Gurus concentrated largely on the Total Qulaity Control elements of the Technical system. The **Total Quality Management Philosophy** is the integration of these three systems viz., the social system, the management system and the technical system. Hence understanding of the preaching of all these Gurus make the understanding of TQM more complete.

There have been several persons who have done extensive and long research on the subject of TQM and have propagated their own theory after such research. These persons obviously are refered to as Gurus of Quality Management and the most prominent among of them are :

| | | |
|---|---|---|
| The Early Americans | – 1950's | W. Edward Deming |
| | | Joseph M. Juran |
| | | Armand V. Feigenbaum |
| The Japanese | – 1950-70's | Dr. Kaoru Ishikawa |
| | | Dr. Genichi Taguchi |
| | | Shigeo Shingo |
| The Western | – 1980's | Philip Crosby |
| | | Tom Peters |
| | | Claus Moller |

**W. EDWARD DEMING**

Dr. Deming was born in 1890 and he did his PhD in Mathematical Physics in 1928. Thereafter he began his career with US Govt. services in Statistics Dept. working on Statistical Sampling Techniques. In 1946, he visited Japan as an Adviser to Japanese Census Program. During this point of time he was invited by JUSE-Japanese Union for Scientists and Engineers for a lecture and there after he was involved with JUSE till 1950. He won the prestigious Walter Shewart Medal from US Govt. in 1956. He was honoured with the top award of the Japanese Govt. by conferring on him the Second Order by the sacred Treasure of emperor during 1960. His work brought great impact on West in 1970's. He wrote his famous book "Out of the Crisis" in 1986. He formed the British Deming Association in 1987. US Govt. instituted a top a ward for the Quality Management in the name of Deming called DEMING'S PRIZE for Quality.

His philosophy was oriented towards Management Process. He stressed on the importance of variability reduction in process and said there are Special and Common cause of variation. While the special causes can be eliminated by the local operational people, the elimination of common causes requires Management Commitment. He invented and propagated the famous PDCA Cycle well known as Deming's improvement cycle.

In his works he has stressed the importance of Waste Elimination. He is the recent years Propagated another model for the Quality management called JOINERS TRIANGLE with Employee Involvement, Scientific problem solving and Customer delight as their apex. All his preaching has been consolidated into 14 important points called as Deming's 14 points. Besides this he has also brought out what is known as Deadly Diseases of American industry after a diagnostic study of the American industry. These are :

Lack of Constancy Purpose

Emphasis on Short term profits

Performance Evaluation and Merit rating

Management by visible figures with no consideration for hidden data.

His recent works in 1990's are on captioned topics such as :

System of profound Knowledge

Appreciation for a system

Knowledge of statistical theory

Theory of knowledge

Knowledge of psychology

**JOSEPH M. JURAN**

Dr. Juran was born in 1904, and started his career as an Engineer in 1924. His pioneering work in Quality was to bring out of a Hand Book in 1951. Even in those days he had put much emphasis in the cost of Quality that the very First chapter in his Hand book was of Quality titled as "There is Gold in the Mine".

He was invited to Japan in 1954 by the JUSE. He wrote several books on Quality Management and 12 of such books were eventually translated into 13 languages. He shot to fame very fast and evidently received more than 30 Medals from about 12 countries all over the world. He also received the prestigious Second order of the Sacred Treasure from the emperor of Japan.

His Philosophy was on Improvement orientation. His preaching popularly known as the Quality Trilogy consisting of Quality planning, Quality Control and Quality improvement had contributed very significantly to the quality management in the early days. Juran's Quality Spiral traces interactions between Quality and various activities in a product life cycle. He has recommended a Road Map for Quality improvement beginning with the Customer identification ending with the productionisation.

Dr. Juran is sceptical about the Quality circles and Zero Defect movements because he strongly believes that 80% of the problems are management controllable. The solution according to him lies in training the top management on Quality to start with.

Dr. Juran has started an institution for training and promotion of quality in US called Juran's Institute for Quality Management.

**ARMAND V. FEIGENBAUM**

Born in US secured his PhD from MIT. He authored the first book titled 'Total Quality Control'. He headed the Quality Department of General Electric – USA, where he later became the World wide Director. He became the President of the General Systems Co. Ltd. USA. He was the founder Chairman of International Academy for Quality and also the President of the American Society for Quality Control. He is winner of several awards including the famous Edward's Medal and the Lancaster Medal at USA. He is also a Member on the Board of Malcolm Balridge National Quality Award Program.

His philosophy was focused on comprehensive QC program with a specific focus on the Business Quality. He propose a 4 step approach for the Quality Control viz., Set Standards, appraise conformance to standard, Act on deviation and constantly improve Standards. He stressed the need for use of statistics for overall administration of Quality. He also broke the quality cost into the constituent elements like the appraisal, prevention and the failure cost. His crucial contribution to Quality management is to create a process of Benchmarking for the Total Quality Management.

**Dr. KAORU ISHIKAWA**

Ishikawa was born in Japan in 1915, and in 1939 got his Graduate degree in Engineering from Tokyo University. He became the Associate professor in Tokyo University in 1947 and did his PhD in Engineering to be elevated as Professor in 1960. Among the citations received by him PhD in Engineering to be elevated as professor in 1960. Among the citations received by him the important ones are, Winner of Deming's Prize, Nihon Keizai Prize and the Industrial Standardisation Prize. He was the Grant Awarded from ASQC in 1971. He is the author of several books like Guide to Quality Control, What is TQC and the Japanese way.

He pioneered Quality Control Circles QCC movement in Japan in 1960's. He simplified application of Statistical tools through the well known Q 7 tools for shop floor operators. He is inventor of the cause and effect diagram also known as Ishikawa diagrams. He was closely associated with the Company Wide Quality Control movement in Japan between 1955 to 1960.

**Dr. GENICHI TAGUCHI**

Dr. Taguchi is born in 1924. He started his career from Naval Institute of Japan between 1942-45 and then with Ministry of Public Health and Welfare. Later he joined Ministry of Education subsequently moved to Nippon Telephone at Japan.

Taguchi is the inventor of the famous Orthogonal array OA techniques for the design of Experimentation. He published his first book on OA in 1951. Taguchi also visited Indian Statistical Institute between 1954–55. He wrote a book on Design of Experiments.

Dr. Taguchi did his PhD from Kyushu University, Japan and become the visiting professor, at the Princeton University USA in 1952. Later Professor at Aoyama University in 1964. He has the privilege of becoming the national Professor Japan. He also worked for the Bell Laboratories USA.

Dr. Taguchi's philosophy is Robust Engineering Design. He blended statistics with Engineering Applications and Pioneered work in Industrial Experimentation. He is also the innovator of the Quality Loss Function concept and promoted Robust design. Related to this he propagated Signal-to-Noise Ratio Phenomenon in SPC. He developed a three stage off line QC Methods Viz., System design, Parameter design and Tolerance design.

**SHIGEO SHINGO**

Shingo was born in 1909 and graduated in Mechanical Engineering in the year 1930. He then joined Taipei Railway Factory at Taiwan in 1940 subsequently become a consultant for the Japan Management Association in 1945. He became the Head of Education Department with Japanese Govt. in 1951. He joined Toyota Motor Company., as Head of Industrial Engineering including 100 supplier companies in the year 1955. He set a world record in cycle time reduction from 4 months to 2 months in hull assembly of 65000 tons supertanker at Mitsubishi Heavy Industries. He also become the President of the Institute of Management improvement in 1962.

Shigeo Shingo has done pioneering work on the Shop floor Quality control techniques. He introduced the famous POKA-YOKE in manufacturing systems by preventive measures. He promoted ZERO DEFECT concept. He is the originator of Single Minute Exchange of Dies SMED techniques. His works in the manufacturing area short him to fame and soon he became the Successful Trainer for Dalmer Benz West Germany, Buhler Switzerland, Federal Mogul, Livernos Automation USA. He was also the Consultant for the Citeron, Peugeot (France) Daihatsu, Yamaha, Mazda, Sharp Corpn, Fuji Inds, Nippon, Hitachi, Sony, Olympus in Japan. He is the author of about 14 books. He applied ZD Concept at the Matsushita Washing Machine by introducing the Poka-Yoke system.

**PHILIP B. CROSBY**

Crosby born in USA was graduated from Western Reserve University. He joined the US naval service during Korean War. He then became the Quality Manager, of Pershing Missile Program. Later he joined ITT as Director, Quality. He wrote several books most popular among them are Quality is Free in 1979. He is Founder President of the Philip Crosby Associates since 1979 and also established Quality College at Florida.

Philip Crosby's philosophy is based on Preventive Management. He pioneered 'Do It Right First Time' and 'Zero defect' concepts. He defined Quality in terms conformance to requirements that the customer has set. He analysed and found that on an average 20%–30% of the revenue in any organisation is spent on Reworks. He said that his Zero Defect means that the company does not start out expecting defects or mistakes. All his work can be comprehenced in what is known as 4 Absolutes of Quality Management. These are be comprehenced in what is known as 4 Absolutes of Quality Management. These are :

Quality is conformance to requirements

Prevention causes Quality not appraisal

Performance standard must be Zero Defect

Measure of Quality is cost of poor quality and nothing else.

He has identified 5 characters of eternally successful organisations and has recommended 14 steps for Quality improvement.

**TOM PETERS**

Tom Peters is educated in Engineering and Business. He began his career with US Navy service and then became the Principal of McKensy and Co. Later he formed his own Tom Peter's Group Co., in USA. Based on the study of 43 excellent large US companies, he wrote his first Book "In Search of Excellence" in 1982 followed by "A Passion for Excellence", and then "Thriving on Chaos" 1988.

His philosophy is based widely on Systems and Leadership. He created Quality Management model with Leadership at the hub and People. Innovation and Customer respectively on three sides of a triangle. He suggests Leader to primarily be engaged in Listening and Facilitating. He has listed 12 important traits of successful US companies like Obsessions with Quality, passionate systems, Measurement of Quality, Quality based rewards, Training in quality for all, Multifunction teams, Factory in factory, Creation of new goals/themes, Parallel organisation for Quality Improvement, Involvement, Cost reduction and continuous improvement.

**CLAUS MOLLER**

Claus Moller is a Danish economist born in 1950's. He founded his own company called "TIME MANAGER INTERNATIONAL". He designed a very popular Management program titled "PUTTING PEOPLE FIRST " and trained several thousand people through that. Prominent among them include Japan Airlines, Russia, European Economic Community where more than 16000 people were trained from about 48 companies. He subsequently designed another successful program with focus on Job Satisfaction and Team work called "MANAGEMENT FOR EVERYONE". He worked on inter personal relations, reduction in bureaucracy for commission workers at Luxemberg and Brussels. He authored the very famous book titled "Personal Quality" in 1988.

His philisophy has deep emphasis on concept of personal quality which means setting of ideal performance level and actual performance level standards for personal quality. He says ideal performance is the goal and lists 12 golden rules to improve actual performance of individuals. These rules are :

Set personal Goals, Establish Personal Quality account, Check satisfaction of your customer, Treat NOAC, Avoid errors, Perform Effectively, Utilise resources, commit, self discipline, control stress, be ethical and demand quality. Besides he also has listed 17 hall marks of Quality.

## THE FIVE PRINCIPLES OF TQM

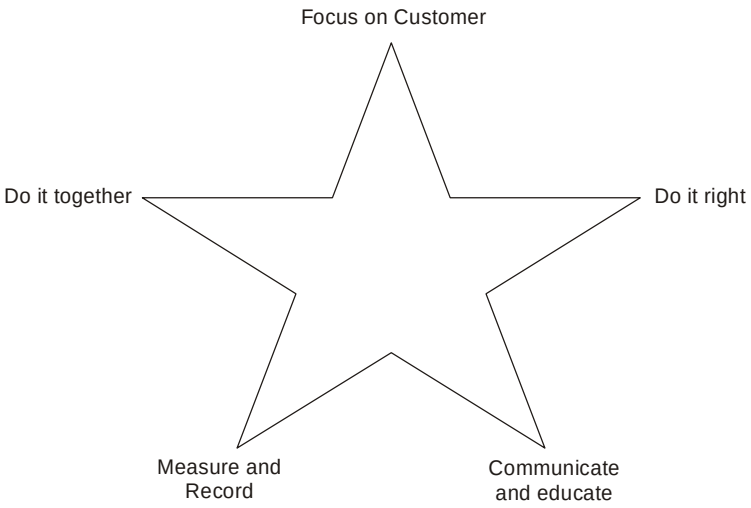



**Fig. 3.1.** Principles of Total Quality Management.

1. Concentrate on the customer
   - Be customer focused
2. Do it right
   - Do it right first time
   - Constantly improve
   - Quality is an attitude not a inspection process
3. Communication and educate
   - Tell staff what is going on
   - Educate and train
4. Measure and record
   - Measure work
5. Do it together
   - Top management must be invovled
   - Empower the staff
   - Make the business a good place to work
   - Organise by process not function

## HISTORY OF QUALITY CONTROL IN INDIA

The history of quality control or statistical quality control in India can be traced thus.

1928 – Prof. P.C. Mahalonobis initiated statistical studies and resources in their statistical laboratory, setup in the presidency college. Calcutta.

1931 – The Indian Statistical Institute was found by Prof. P.C. Mahalonobis.

1932 – The Indian Statistical Institute got recognition in the form of a learned society under the societies. Registration Act.

1948 – Dr. W.A. Shewart visited the industrial centers in India.

1952 – A team of UN experts visited India to train personel in the routine use of quality control.

1959 – The parliament of India enacted the India Statistical Institute Act declaring the institute to be of national important and empowering it to award Degrees and Diplomas.

Dr. W.E. Deming, Dr. J.M. Juran, Prof. Genichi Taguchi and Dr. Kaonu Ishikawa visited India. It's of interest to note that seeds of statistical studies and researchers were sown to India in 1928 and compared to January 1949 in Japan. Dr. Deming and Dr. Juran visited India and Japan almost during the same decade. But the massive investment made by Japan in education and training the implementation of these techniques with a sense of mission and commitment has made all the difference.

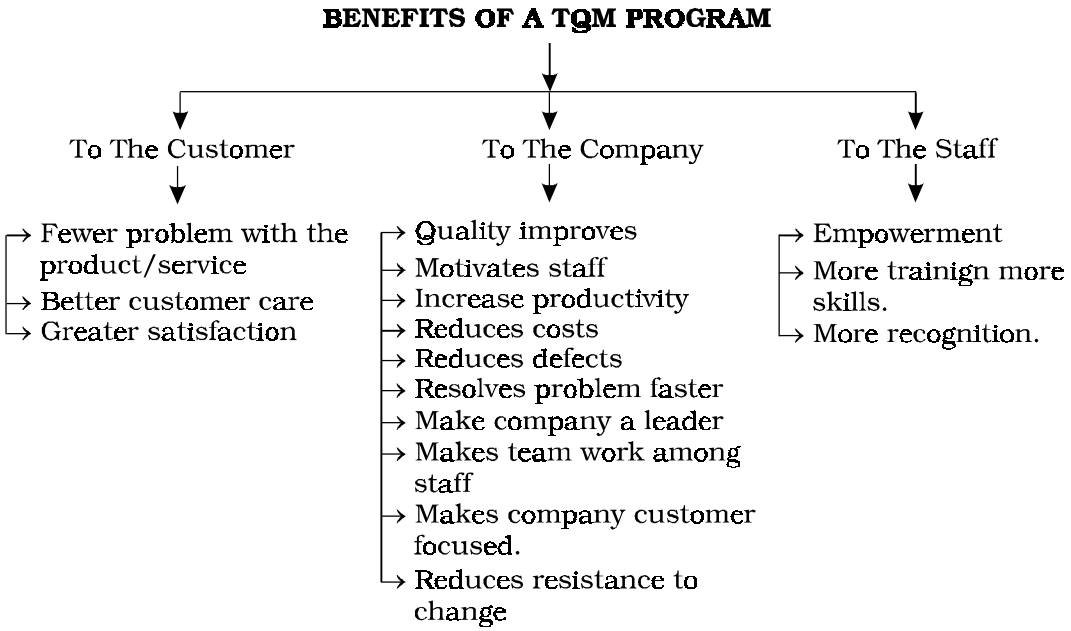



**Fig. 3.2.** Benefits of Total Quality Management.

**Involvement of Management in TQM**

The involvement of management is very much necessary to make TQM a success in any organisation. Certain steps can be taken by senior managers to facilitate TQM. They are :

➜ Decide the purpose of introducing TQM management must know its goal.

☆ What do you want to achieve out of this TQM program must be determined, otherwise the program will lack direction.

☆ Empowerment of staff, reduction of defects, improving customer loyalty etc.,

➜ Devote necessary time for the program. TQM programs take away substantial part of the manager's time and if they are not ready to spare it the program will never take off.

- ➜ Employee participation in every level should be encouraged. The involvement of the people in the organisation will ensure the success of TQM program.
- ➜ Train people. Making sure the managers are well trained in TQM, by personal checking and even conducting key quality training courses.
- ➜ The organisation must give some time for it to work : Managers might spend weeks in training, but results might not be visible immediately. It takes a long time for organisation to change their culture and attitudes, this essentially means that results will take several years to show up their face. This time lag involved in between the amount of efforts put in and the results to surface, puts many managers to loose interest in TQM.
- ➜ Maintain close contact with customer. And if needed, represent the customer interest in solving problems. Recognise efforts, Get personally involved in quality awards.
- ➜ Be prepared for resistance. People fear change, because it could lead to redundancy, which means that staff will be apprehensive and sceptical about the TQM Program. Many a time the staff simply wants to maintain their status quo, because they are familiar with it.
- ➜ Introduce new employees to quality values. Conduct orientation programs to show them the organisation level of commitment.
- ➜ Visiting other organisation who have implemented TQM will give a view of the benefits of TQM and you can learn how it should be applied. Often a group of your employees can make a visit to a non-competing company where the TQM manager will be pleased to show you around.
- ➜ Communication of quality to all the levels of the organisation. This is done by a monthly video film on quality, which will be viewed by all employees.
- ➜ The organisation has to commit a lot of money towards TQM program is the form of salary to a full-time TQM co-ordinator in case of an in-house program, payments for training courses. In addition, a lot of senior managers of visiting other companies. The organisation may also spend on give-away like printed is using a consultancy for this purpose, it will have to pay its fees.
- ➜ Emphasize continuous improvement encourage everyone to do better.
- ➜ Investments must be managed. TQM efforts will bring in solution and if these solution are not implemented the staff be disillusioned.
- ➜ Break down barriers to cross-functional co-operation.
- ➜ Nominate a TQM facilitator, a TQM director, and set up a quality council. The quality facilitator will have day to day responsibility for TQM, he will remind advice and encourage staff about TQM.

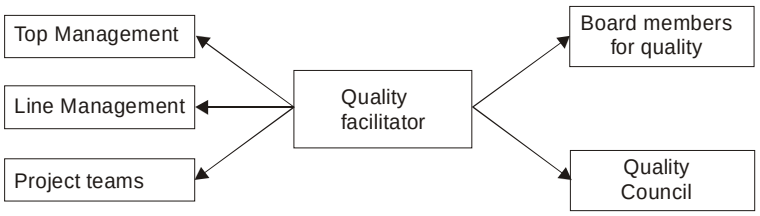



**Fig. 3.3.** Structure for TQM.

If the organisation is large, the TQM faciliator will report to a board member, otherwise to the chief executive. The quality council or the TQM steering committee is constituted of top managers and it's the deciding authority on TQM matters. The quality council should comprise the quality manager, TQM facilitator senior line management and atleast one member of the board.

➜ Get rid of unnecessary distinctions. Some companies have abolished some management titles and eliminated perquisites.

➜ Set mission and values and make sure they are communicated to every employee.

**Preparing Managers for TQM**

While preparing managers for TQM the following three factors have to be considered.

1. **Managers must be empowered.** They are expected to further delegate this and if they don't have power and resources at their disposal, how will they delegate it.
2. **Manager must evaluate their style.** They will have to consider the way they are doing their jobs currently and how they will have to change as TQM is introduced.
3. **Management training at TQM has to be given careful attention.** This will aquaint managers with TQM, dispose them favourably towards TQM, show them ways of gradually introducing TQM ideas, and convince them of TQM's benefits and gains.

**TQM ORGANIZATION**

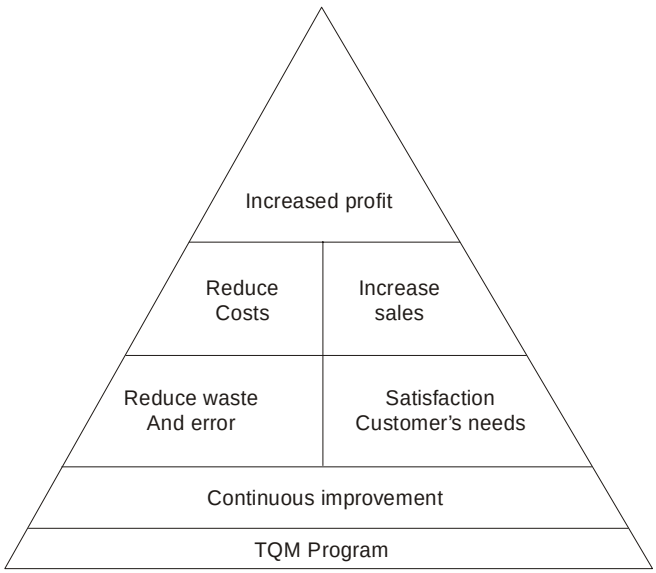



**Fig. 3.4.** Total Quality Management Organization.

TQM Program creates continuous improvement. This leads to reduction in errors and waste, which in turn leads to customer satisfaction. The benefit of this to the company is in the form of reduced costs and increased sales, which basically means increased profits.

# 4 Leadership

## INTRODUCTION

Leadership requires a keen understanding of human nature — the basic needs, wants and abilities of people. To be effective : a leader needs to know and understand the following :

1. People, paradoxically, need security and Independence at the same time.
2. People are sensitive to external rewards and punishments and yet are also strongly self-motivated.
3. People like to hear a kind word of praise.
4. People can process only a few facts at a time.
5. People trust their gut reaction more than statistical data.
6. People distrust a leader's rhetoric if the words are inconstituent with the leader's actions.

## CHARACTERISTICS OF QUALITY LEADERS

There are 12 behaviours or characteristics the successful leaders demonstrate.

1. They give priority attention to External and Internal customers and their needs.
2. They empower, rather than control, sub-ordinates, leaders have trust and confidence in the performance of their subordinates. They provide the resources, training and work environment to help subordinates do their jobs.
3, They emphasize improvement rather than maintenance. There is always room for improvement, even if the improvement is small.
4. They emphasize prevention. "An once of prevention is worth a pound of cure is certainly true. It is also true that perfection can be the enemy of creativity. There must be a balance between preventing problems and developing better but not perfect, processes.
5. They encourage collaboration rather than competition when functions areas department or work groups are in competition, they may find subtle ways of working against each other or withholding information.
6. They train and coach rather than direct and supervise. As coach they help their subordinates learn to do a better job.
7. They learn from problems. When a problem exists. What caused it ? And how can we prevent it in the future are the questions asked by leaders.
8. They continually try to improve communications, they make it evident that TQM is not just a solgan. Communication is two way-ideas will be generated by people when leaders encourage them and act upon them. Communication is the glue that holds a TQM organization together.

9. They continually demonstrate their commitment to quality. They let the quality statement be their decision making guide.
10. They choose suppliers on the basis of quality not price. Suppliers are encouraged to participate on project teams and become involved. Leaders know that quality begins with quality materials and true measures is life-cycle cost.
11. They establish organizational systems to support the quality effort. At the senior management level a quality council is provided and at the first-line supervisor level, work groups and project teams are organized to improve the process.
12. They encourage and recognize team effort. They encourage, provide recognition and reward individuals and teams. This action is one of the leader's most powerful tools.

## ROLE OF TQM LEADERSHIPS

### Customer's Satisfaction

**Introduction.** The most important asset of any organization is it's customers. An organizations success depends on how many customers, it has, how much they buy, and how often they buy satisfied customers will increase buy more and buy more frequently. The following Fig. 4.1 shows organizational diagram and how important customer to an organization.

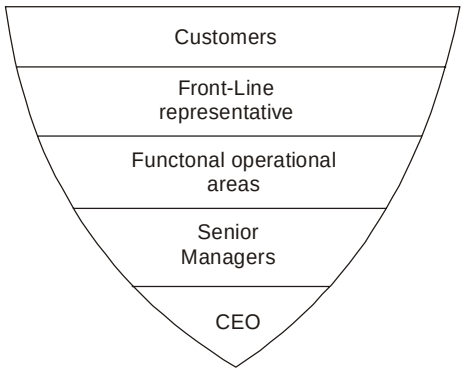



**Fig. 4.1.** Customer Satisfaction Organization.

Increasingly, manufacturing and service organization are using customer satisfaction as a measure of quality. The importance of customer satisfaction is not only due to national competition but to world wide competition.

Understanding the customer's needs and expectation is essential to winning new business and keeping existing business. An organization must give its customer a quality product or service that meets their needs at a reasonable price. Which includes on-time delivery and outstanding service.

The most successful TQM programs begin by defining quality from the customer perespective quality means meeting or exceeding the customer's expectations it is most important consideration, because satisfied customers will lead to increased profits.

**Customer Satisfaction**

Customer satisfaction is illustrated by the Teboul model in the Fig. 4.2. The customer's needs are represented by the circle, and the square depicts the product or service offered by the organization. Total satisfaction is achieved when the offer maintain the need or the circle is superimposed on the square. The goal is to cover the expected performance level better than the competiters.

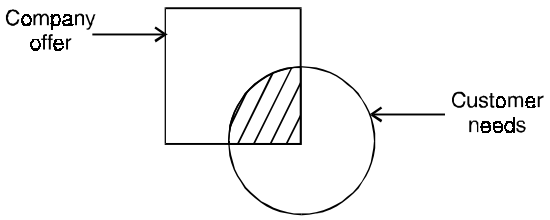



**Fig. 4.2.** Customer Satisfaction Model.

That part of the square that lies within the circle is perceived by the customer as satisfying and the part of the square outside the circle is perceived as unnecessary. It is important that the organization listen to the "voice of the customer" and ensure that its marketing, design, production and distribution process truely meet the expectations of the customers.

Customer satisfaction is not an objective statistic but more of a feeling or attitude. It is best to remember that people's opinions and attitudes are subjective by nature. Because customer satisfaction is subjective, it is hard to measure. There are so many facts to a customer's experience with a product or service that need to be measured individually to get an accurate total picture of customer satisfaction. Whether a customer is satisfied cannot be boiled down to a yes or no answer. The Teboul model, describes customer satisfaction as the degree to which the customer's satisfaction level would be the same if the experience were mediocre in the context of low expectations or if the experience were superior in the context of high expectations. Customer satisfaction's focus is creating superior experiences not mediocre experiences.

**Customer Perception of Quality**

One of the basic concepts of the TQM philosophy is continuous process improvement. This concept implies that there is no acceptable quality level because the customer's needs, values and expectations are constantly changing and becoming more demanding.

An American society for quality (ASQ) survey on end user perceptions of important factors that influenced purchages showed the following ranking.

| | | |
|---|---|---|
| 1. Performance | 2. Features | 3. Service |
| 3. Warranty | 5. Price | 6. Reputation. |

**Performance.** Performance involves "Fitness for use". Other consideration are (*a*) availability which is the probability that a product will operate when needed. (*b*) Reliability, which is freedom from failure over time. (*c*) Maintainability which is the case of keeping the product operatable.

**Features.** Identifiable features of a product or service are pschological, time oriented, contractual, ethical and technological feature are secondary characteristics of the product or service.

**Service.** An emphasis on customer service is emerging as a method for organizations to give the customer added value. Customer service is an intangible — it is made up of many things, all geared to changing the customer's perceptions. Intangible characteristics are those traits that are not quantifiable and contribute greatly to customers satisfaction. Organizations that emphasize service never stop looking for finding ways to serve their customers better, even if their customers are not complaining.

**Warranty.** The product warranty represents an organization's public promise of a quality product backed up by a guarantee of customer satisfaction. It also represents a public commitment to guarantee a level of service sufficient to satisfy the customer.

A warranty forces the organization to focus on the customer's definition of product and service quality. A warranty generates feed back by providing information on the product and service quality.

Finally, warranty builds marketing music. The warranty encourages customers to buy a service by reducing the risk of the purchase decision and it generates more sales from existing customers by enhancing loyalty.

**Price.** Today's customer is willing to pay a high price to obtain value. Customers are constantly evaluating one organization's products and services against those of it competitors to determine who provides the greatest value.

**Reputation.** Most of us find ourselves rating organizations by our overall experience with them. Total customer satisfaction is based on the entire experience with the organizations not jsut the product.

Customer are willing to pay a premium for a known or trusted brand name and often become customers for life. It costs five times as much to win a new customer as it does to keep an existing one, therefore, customer retention is an important economic strategy for any organization. Investment in customer retention can be a more effective bottom-line approach than concentrating on lowering operational costs.

**Using Customer Complaints**

Although complaints are reactive, they are one of the most important information gathering tool. A dissatisfied customer can easily become a lost customer. Many organizations are using customer dissatisfaction as the primary measure to assess their process improvement efforts.

Small organizations have a tremendous advantage in that the top-ranking official often in personal contact with key customers. Information on customer dissatisfication received into the organization at the highest level, there by providing a fast response.

Results of a study indicated that more than half of dissatisfied customers will buy again if they believe their complaint has been heard and resolved. Only one-fifth will buy again if their complaint is heard but not resolved.

By taking a positive approach, complaints can be seen as an opportunity to obtain information and provide a positive service to the customer. In reality the customer is giving the organization a second chance. Some activities are as follows.

- ☆ Develop procedures for complaint resolution that include empowering front-line personnel.
- ☆ Investigate customer's experiences by actively soliciting feed back both positive and negative and then acting on it promptly.
- ☆ Analyze complaints.
- ☆ Work to identify process and material variations and then eliminate the root cause.
- ☆ When a survey response is received a senior management should contact the customer and strive to resolve the concern.
- ☆ Establish customer satisfaction measures and constantly monitor them.

- ☆ Provide a monthly complaint report to the quality council for their evaluation 90% of all customer contact comes through an organization's front-line employees. An organization can save both customers and money by training front-line employees to solve problems directly with customers. Customers want problem solve quickly and efficienctly ; therefore, employees should know how to handle a wide range of situations that arise in the customer relationship. Recognition and reward should be linked to service quantity performance and the ability to satisfy customers front-line employees should have responsibility and authority to provide the service necessary to satisfy the customer. For example : a cashier in a resturant should be empowered to discount meal price of a dissatisfied customer without seeking management's approval.

  Studies have shown that the better the service at the point of sale. The fewer the complaints and the greater the sales volume. Its just as important to focus on employee satisfaction as customer satisfaction.

**Feed Back**

Customers feed back must be continuously solicited and monitored. Customers continuaully change their minds, their expectations and their suppliers. Customer feed back is not a one-time effort, it is on ongoing and active probing of the customer's mind feed back enables the following to an organization.

- ☆ Discover customer dissatisfaction.
- ☆ Discover relative priorities of quality.
- ☆ Compare performance with the competition.
- ☆ Identify customer's needs.
- ☆ Determine opportunities for improvement.

Even in service industries such as insurance and banking, customer feed back has become so important that it drives new product development. Effective organizations take the time to listen to the voice of the customer and feed that information back to the idea stage.

Listening to the voice of the customer can be accomplished by numerous information collecting tools. The principal ones are comment cards, surveys, focus groups, toll-free telephone lines, customer visits, report-cards, the Internet, employee feed back and American customer satisfaction index.

**Comment Card.** A low cost method of obtaining feed back from customers involves a comment card which is usually attached to the warranty card and is included with the product at the time of purchase.

**Surveys.** A customer survey is a popular tool for obtaining opinions and perceptions about an organization and its products and services.

To make surveys more useful, it is best to remember eight-points.

- ☆ Clients and customers are not the same.
- ☆ Surveys raise customers expectations.
- ☆ How you ask a question will determine how the question is answered.
- ☆ The more specific the question, the better the answer.
- ☆ You have only one chance and only 30 minutes.

- ☆ The more time you spend in survey development the less time you will spend data analysis and interpretation.
- ☆ Whom you ask is as important as what you ask.
- ☆ Before the data are collected you should know how you want to analyze and use the data.

**Focus Groups.** Customer focus groups are a popular way to obtain feed back, a focus group is a research method used to find out what the customers are really thinking. A group of customers is assembled in a meeting room to answer a series of questions, meetings, are designed to focus on current, proposed and future products and services.

**Toll-Free Telephone Number.** Toll-free (800/888) service phone numbers are an effective technique for receiving complaint feed back organizations can respond faster and more cheaply to the comaplaint. Toll-free numbers are in use by atleast 50% of all organizations with sales of at least Rs. 10 millions.

**Customer Visits.** Visits to a customer's place of business provide another way to gather information. Senior managers should be involved in these visits and not delegate them to some one else. Brainstorming sessions with the customers concerning futuristic products and services should be held at least annaully.

**Report Card.** Another very effective information gathering tool is the report card Fig. 4.3 shows a typical one. It is usually sent to each customer on a quarterly basis.

**Quarterly Report Card**

To our customer

We are continually striving to improve.

The grading scale's

A–Excellent B–Very Good

C–Average D–Poor

F–Fair

1. Product Quality Grade

   Comments .....

2. ON-Time Delivery Grade

   Comments .....

3. Service Grade

   Comments .....

4. Overall Grade

   Comments .....

5. Signed Date :

   Title Organization

**Fig. 4.3.** Report Card.

**Employee Feed Back.** Employees are often an untapped source of information. Companies listening more to the external customer but still are not listening to employees. Employees can offer insight into condition that inhibit service quality in the organization.

**The American Customer Satisfaction Index.** The American customer satisfaction index (ACSI) established in 1994, quantities quality and customer satisfaction and relates them to firms financial performance.

The index is based on findings from telephone interviews from a national sample of about 50,000 households. The index measures seven sectors of the economy, which include 40 industries and more than 200 individual companies and agencies. The seven sectors of the economy are.

1. Manufacturing (non durables)
2. Manufacturing (durables)
3. Retail
4. Transportation, communication and utilities
5. Finance and insurance
6. Services
7. Public administration and government.

**Employee Involvement**

**Introduction.** Employee involvement is one approach to improving quality and productivity. Employee involvement is not a replacement for management nor is it the final word in quality improvement. It is a means to better meet the organization's goals for quality and productivity at all levels of an organization.

**Motivation.** Knowledge of motivation helps us to understand the utilization of employee involvement to achieve process improvement.

**Maslow's Hierachy of Needs.** One of the first and most popular motivational theories was developed by Abraham Maslow. It is explained in five levels. These levels are survival, security, social, esteem and self actualization. It is shown in Fig. 4.4 once a given level is satisfied, it can no longer motivate a person.

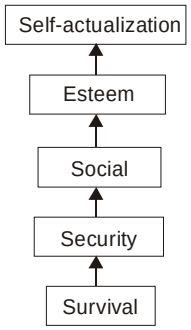



**Fig. 4.4.** Maslow's hierarchy of needs.

**Level 1 :** Means food, clothing and sheltor which is usually provided by a job.

**Level 2 :** Can mean a safe place to work and job security. Which are very important to employees.

**Level 3 :** Relates to our need to belong because we are social animals. It has been said that cutting some one out of the group is devastating to that individual. Isolation is an effective punishment.

**Level 4 :** Relates to pride and self worth. Every one regard of position or job assignment, wants to be recognized as a person of value of the organizations.

**Level 5 :** Says that individuals must be given the opportunity to go as far as their abilities will later take them. Many organizations have a policy of promoting from within. It is true that some employees do not want to move up the corporate ladder which is understandable.

**Herzberg's Two-Ractor Theory.** Frederick Herzberg extended the general work of maslow by using empirical research to develop his theory on employee motivation. He found that people were motivated by recognition, responsibility, achievement, advancement and the work itself. These factors were labeled motivators. His research showed that bad feelings were associated with salary, fringe benefits, working conditions, organization policies and technical supervision. These job-related factors were labeled dissatisfies or hygiene factors. Herzberg's hygiene factors are roughly equivalent to Maslow's lower level and the motivators are similar to the upper levels.

**Employee Wants.** While management thinks that good pay is the number one want of the employee, survey results show that this factor is usually in the middle of the ranking. Employees wants tend to follow the theories of maslow and Herzberg. If managers are to effectively motivate employees, they must align their actions closer to the motivators.

**Achieving a Motivated Work Force.** The building of a motivated work force is for the most part an indirect process. Concepts to achieve a motivated work force are as follows.

1. **Know thyself.** Managers must understand their own motivations, strengths, and weaknesses. The motivating manager knows the most valuable resource is people and his success is employees achieving their goals.

2. **Know your Employee.** Most people like to talk about themselves, the motivating manager will ask questions and really listen to the answers. As the managers learns more about the employee, he can assist the employee in directing their efforts towards satisfying their goals and well-being.

3. **Establishing a Positive Attitude.** A positive action-oriented attitude permeates the work unit. Managers are responsible for generates attitudes that lead to positive actions. Respect and sensitivity towards others is essential to the development of positive attitudes.

4. **Share the Goals.** A motivated work force needs well-defined goals that address both individual and organizational needs.

5. **Monitor Progress.** The process of goal-setting should include a road map detailing the journey with periodic milestones and individual assignments.

6. **Develop Interesting Work.** Managers should consider altering the employee assignments by means of job rotation, job enlargement and job enrichment.

7. **Communicate Effectively.** Effective communication provides employees with knowledge about their work until and the organization rather than 'grapevine' information.

8. **Celebrate Success.** Recognizing employees achievements is the most powerful tool in the manager's tool box.

These eight concepts can be used at all managerial levels of the organization.

**Suggestion System**

Suggestion system are designed to provide the individual with the opportunity to be involved by contributing to the organizations. Most of the ideas for continuous improvements will come from the team approach : They must make it easy employees to suggest improvements, review them promptly and implement them.

Stimulating and encouraging employee participation starts the creative process. There are five ground rules

1. Be progressive by regularly asking your employees for suggestion.
2. Remove fear by focusing on the process and not on the person. When employee know that puritive actions will not occur. They are more likely to respond.
3. Simplify the process so it is easy to participate. Stamp out supersfiuous paper-work, review and procedures.
4. Respond quickly to suggestions and within a specific period of time. The evaluation process must be simple and effective. The response in writing has 3 possible acceptance, rejection or reformal to a committee for further evaluation. If accepted a time frame for implementation should be given, is rejected, the reason for the rejection should be stated and if referred to a committee the evaluation time should be stated.

5. Reward the idea with published recognition so that everyone knows the value of the contribution.

The five steps approach helps to create an environment that opens communications that opens communication between employees and managers.

**Performance Appraisal**

The purpose of performance appraisals is to improve performance, let employees know now they are doing and provide a basis for promotion, salary messages, counselling and other purposes related to an employee's future. There should be a good relationship between the employees and the appraiser. The appraisal should point out strength and weaknesses as well as how performance can be improved. The most common appraisal formats are shown in Table 4.1.

**Table 4.1. Appraisal Formates**

| *Type* | *Description* |
|---|---|
| Ranking | Compare employees by ranking from highest to lowest. |
| Narrative | Given a written description of employees strength and weakness. |
| Graphic | Indicate major duties performed by the employee and rates each duty with a scale from 1 poor to 5 Exc. |
| Forced choice | Place each employee in a category with a pre determined, percentage for Example, excellent 30%, very good 25%, good 20%, fair 15% and poor 10%. |

Performance appraisals should be viewed as a positive way to get-employee involved many.

Supervisors look at appraisales as one of the unpleasant duties they most perform. Every effort should be made to avoid error in performance evaluations, culture, ethics, education level and predetermined opinions can affect evaluations.

Appraisals nourish short-term performance and destroy long-term planning. Another criticism states that individual appraisal destroys team work. If teams are to become conesive unit of "all for one and one for all" then individual ranking would undermine the entire concept. The end result would be a team that performs poorer not better. They should be based on objectivity however, "it is difficult to measure some attributes such as customer satisfaction and leadership.

A no. of practitioners have suggested improvement apprisal technique.

1. Use rating scales that have few rating categories. It is difficult to differentiate the middle range of performers.
2. Require work team or group evaluation that are at least equal in emphasis to individual focused evaluation. This action will encourage team members to help, support and co-operate with each other.
3. Require more freqwant performance reviews where such reviews will have a dominant emphasis on future performance planning. Work team and individual performance data should be collected and reviewed with an evaluation of results and lessons learned, it may be necessary to have two reviews. One immediately after completion of the task and one when the performance cycle of the task allows evaluation of results.

4. Promotion decisions should be made by an independent administrative process that draws on current job information and potential for the new job.
5. Include indexes of external customer satisfaction in the appraisal process. In order to accomplish this process, this customers and their requirements will need to be identified, performance metrics determined using a rating scale and the improvement process initiated.
6. Use peer and subordinate feed back as an index of internal customer satisfaction initiation of this activity would be similar to the previous item.
7. Include evaluation for process improvement in addition to results. Process behaviour tends to be more within the person's control. One of the basic concepts of TQM is continuous process improvement.

## CONTINUATION PROCESS IMPROVEMENT

### Introduction

We must strive to achieve perfection by continuously improving the business and production process. We continuously improve by

— Viewing all work as a process, it is associated with production or business activities,
— Making all our processes effective, efficient and adaptable,
— Maintaining constructive dissatisfaction with the present level of performance,
— Eliminating waste and rework wherever it occurs.
— Eliminating non conformities in all phases of every one's work, even if the increment of improvement is small.
— Using bench marking to improve competitive advantage
— Innovating to achieve break throughly
— Holding gains so there is no regression
— In corporating lessons learned into future activities.
— Using technical tools such as statistical process control (SPC), experimented design, bench marking, quality function deployment (QFD). etc.

Continuous process improvement is designed to utilize the resource of the organization to achieve a quality-driven culture.

### Juran's Trilogy

Process improvement involves planning. One of the best approaches is the one developed by Dr. Joseph Juran. It has three components : planning, control and improvement.

**Planning.** The planning component beging with external customers. Marketing determines the external customers and all organizational personnel determine the internal customers.

Once the customers are determined, their need are discovered. This activity requires the customers to state needs in their own words and from their own viewpoint. Ex : a stated need may be an automobile, whereas the real need is transportation or a status symbol. In addition internal customers may not wish to voice real needs out of fear of the consequence one might discover there needs by (1) being a user of the product or service (2) communicating with customers through product or service satisfaction and dissatisfaction information (3) simulaton is the laboratory.

The next step in the planning process is to develop product and or services features that respond to customer needs, meet the needs of the organizations and its suppliers, are competitive and optimize the costs of all stake holders.

The fourth step is to develop the process able to produce and or service features. This step is also performed by a multifunctional team with a liaison to the design team of particular concern will be the "scaling up" from the laboratory or prototype environment to the real process environment.

Transfering plans to operations is the final step of the planning process. When training is necessary, it should be performed by members of the process planning team. Process validtion is necessary to ensure, with a high degree of assurance that a process will consistency produce a product or service meeting requirements.

**Control.** Control is used by operating forces to help meet the product, process and service requirement. It uses the feed back loop and consists of the following steps.

1. Evaluate actual operating performance
2. Compare actual performance to goals
3. Act on the difference.

Statistical process control is the primary technique for achieving control. The basic statistical process control (SPC) tools are pareto diagrams, flow-diagrams, cause and effect diagrams, check sheet, histogram, control charts and scatter diagrams.

**Improvement.** The third part of the trilogy aims to attain levels of performance that are significantly higher than current levels. Process improvements begin with the establishment of an effective in transtructure such as the quality council. Two of the duties of the council are to identify the improvement projects and establish the project teams with a project owner. The problem solving method described in a later section may be applied to improve the process, while the quality council is the driver that ensures that improvement is continuous and never-ending. Process improvement can be incremental or break through.

### The PDSA Cycle

The basic Plan-Do-Study-Act (PDSA) cycle was first developed by Shewhart and then modified by Deming. It is an effective improvement technique it is illustrated in Fig. 4.5.

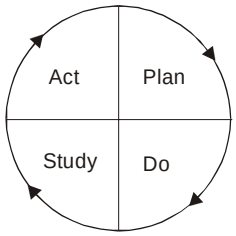



**Fig. 4.5.** The PDSA Cycle.

The four steps in the cycle are exactly as stated. First, plan carefully what is to be done. Next, carryout the plan (do it). Third, study the results. Did the plan work as intended or were the results different finally, act on the results by identifying what worked as planned and what didn't. Using the knowledge learned, develop an improved plan and

repeat the cycle. The PDSA cycle is a simple adoptation of the more eloborate problem solving method.

**Problem-Solving Method**

Process improvement achieves the greatest results when it operates within the frame work of the problem-solving method. In the initial stages of a program, quick results are frequently obtained because the solutions are obvious or an individual has a brilliant idea.

The problem-solving method is also called the scientific method has many variation depending, there are seven phases as shown in Fig. 4.6 it also shows the relationship to the PDSA cycle.

The phases are integrated in that they are all dependent upon the previous phase continuous process improvement is the objective and these phases are the frame work to achieve that objective.

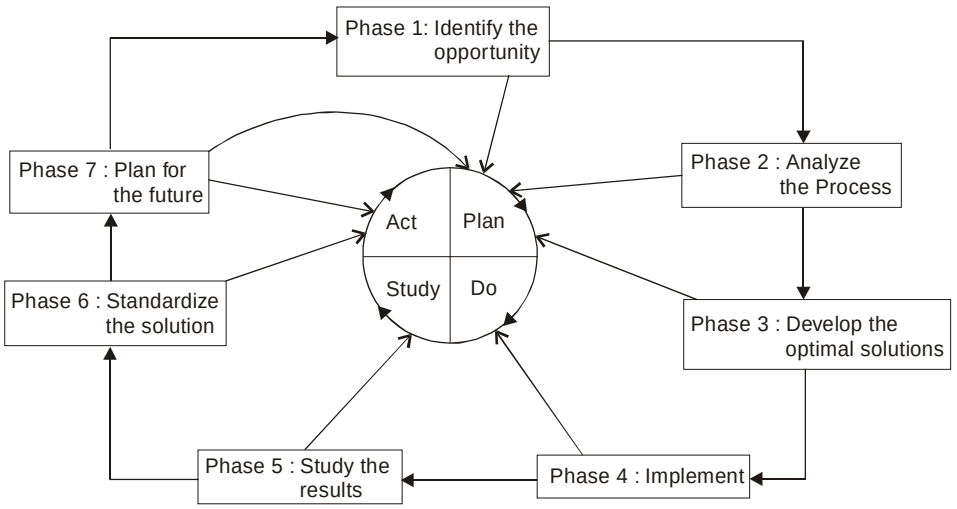



**Fig. 4.6.** Continuous Process Improvement Cycle.

**Phase 1 : Identify the Opportunity**

The objective of this phase is to identify and prioritize opportunities for improvements. It consists of three parts identify the problem, form the team and define the scope. Problems can be identified from a variety of inputs such as the following :

— Pareto analysis of repetitive internal alarm signals. For example scrap, rework, sorting and 100% test.

— Proposals from key insiders (managers, supervisors, professionals and union steward).

— Proposals from suggestion schemes.

— Data on performance of competitors (from users and from laboratory tests).

— Comments of key people outside the organization.

— Findings and comments of government regulators and independent laboratories.

— Customer surveys.

— Employee surveys.

— Brain storming by work groups.

The quality council or work group must priortize them using the following selection criteria.

1. Is the problem important and not superficial and why ?
2. Will problem solutions contribute to the attainment of goals ?
3. Can the problem be defined clearing using objective measures ?

The second part of phase 1 is to form a team. If the team is a natural work group or one where members already work together, then this part is complete.

The team leader is then selected and becomes the owner of the process improvement. Goals and milestones are established. If the improvement strategy is repair or refinement, an individual rather than a team may be assigned.

The third part of phage 1 is to define the scope. A problem well stated is half solved. Criteria for a good problem statement are as follows :

— it clearly describes the problem as it currently exists and is easily understood.

— it states the effect what is wrong, when it happens and where it is occurring, why it is wrong or who is responsible.

— it focuses on what is known, what is unknown and what needs to be done.

— it uses facts and is free of judgement

— it emphasizes the impact on the customer.

Ex. A result of a customer satisfaction, a sample of 150 billing invoices showed that 18 had errors that required one hour to correct.

In addition to the problem statement, this phases requires the following items :

1. **Authority.** Who authorized the team ?
2. **Objective and scope.** What are the expected outputs and specified areas to be improved.
3. **Composition.** Who are the team members and process and sub-process owners ?
4. **Direction and Control.** What are the guide lines for the internal operations of the team ?
5. **General.** What are the methods to be used, the resources and the specific milestones.

**Phase 2 : Analyze the Current Process**

The objective of this phase is to understand the process and how it is currently performed. Key activities are to define process boundaries, outputs and customers, inputs and suppliers and process flow : determine levels of customers satisfaction and measurements needed, gather data and identify root causes.

The first step is for the team to develop a process flow diagram. It translates complex work into an easily understood graphic description. Next, the target performance measures are defined, measurement is fundamental to meaningful needed to understand and improve the process are presently being used. If new ones are needed, they will

1. Establish performance measures with respect to customer requirements.
2. Determine data needed to manage the process.

3. Establish regular feed back with customers and suppliers.
4. Establish measures for quality/cost/time lines of inputs and outputs.

Once the target performance measures are established the team can collect all available data and information. The team should develop a plan that includes input from internal and external customers and ensurer the plan answers the following questions.

1. What problem or operations do we wish to learn about ?
2. What are the data used for ?
3. How many data are needed ?
4. What conclusions can be drawn from the collected data ?
5. What action should be taken as a result of the conclusion.

The team will idenify the cusomters and their requirement and expectations as well as their inputs, outputs and inter faces of the process and they will systematically review the procedures currently being used.

— Common items of data and informations are.
— Customer informations such as complaints and surveys.
— Design information such as specifications, drawings, function, bills of materials etc.
— Process information, such as, routines, equipment, operator raw material and component parts and supplies.
— Statistical information.
— Quality information.
— Supplies information such as process variation, on-time delivery and technical competency.

The cause and effect diagram is particularly effective in this phase. Determining all of the causes requires experience, brain storming and a through knowledge of the process.

**Phase 3 : Develop the Optimal Solution**

This phase has the objective of establishing potential and feasible solutions and recommending the best solution to improve the process. Once all the information is available the project-team being it's search for possible solutions. More than one solutions is frequently required to remedy a situation.

In this phase, creativity plays the major role and brain storming the principle technique.

There are three type of creativity :

1. Create new processes
2. Combine different process or
3. Modify the existing process.

The first type is innovation in it's highest form, combining two or more process is a synthesis activity to create a better process. This type of creativity relies heavily on bench marking. It succeeds when managers utilize the experience, education and energy of empowered work groups or project teams.

Creativity is the unique quality that separates. Mankind from the rest of the animal kingdom. Most of the problems that cause in efficiency and ineffectiveness in organization are simple problems.

Once a solutions have been determined, evaluation or testing of the doing comes next, more than one solution must be provided to determine the greater potential of the solution and their advantage and disadvantage of the solutions.

Criteria for testing solutions are cost, feasibility, effect, resistance to change, consequences and training. Solutions also may be categorized as short-range and long range.

One of the features of control charts is the ability to evaluate possible solutions whether the idea is good, poor or has no effect is evident for the chart.

**Phase 4 : Implement Changes**

Once the best solution is succeed, it can be implemented. This phase has the objective of preparing the implementation plan, obtaining approval and implementing the process improvements.

The project team usually has authority to institute remedial action, more other than not the approval of the quality council or other appropriate authority is required.

The contents of the implementation plan report must fully describe.

Why will it be done ?

How will it be done ?

When will it be done ?

Who will do it ?

What will it be done ?

Answers to these questions will designate required actions assign responsibility and establish implementation milestones. After approval by the quality council, it is desirable to obtain the advice and consent of departments, functional areas, teams and individuals that may be affected by the change.

The final element of the implementations plan is the monitoring activity that answer the following :

— What informations will be monitored or observed and what resoures are required ?

— Who will be responsible for taking the measurements ?

— Where will the measurements be taken ?

— How will the measurements be taken ?

— When will the measurements be taken ?

Measurements tools such as run charts, control charts, pareto diagrams, histogram, check sheet and questionares are used to monitor and evaluate the process change.

**Phase 5 : Study the Results**

This phase has the objective of monitoring and evaluating the change by tracking and studying the effectiveness of the improvement effort through data collection and review of progress. It is vital to institutionalize meaningful change and ensure ongoing measurement and evaluation efforts to achieve continuous improvement.

The team should meet periodically during this phase to evaluate the results to set that the problem has been solved or if fine-tuning is required.

If the team is not satisfied then some of the phases will need to be repeated.

**Phase 6 : Standardize the Solution**

Once the team is satisfied with the change, it must be institutionalized by positive control of the process, process certifications and operators certification it specifies the

what, who, how, where and when of the process and is an up-dating of the monitoring activity. Standardizing the solution prevents 'back sliding'. The quality peripherals. The system, environment and supervision must be certified. The partial check list in provides the means to intially evaluate the peripherals and periodically audit them to ensure the process will meet or exceed customer requirements for the products or service.

Finally, operators must be certified to know what to do and how to do it for a particular process. Total product knowledge is also desirable. Operator certification is an ongoing process that must periodically occur.

**Phase 7 : Plan for the Future**

This phase has the objective of achieving improved levels of process performance. Regardless of how successful initial improvement efforts are, the improvement process must continue. It is important to remember that TQM addresses the quality of management as well as the management of quality. Everyone in the organization is involved in a systematic long-term to constantly improve quality by developing processes that are customer oriented, flexible and responsive.

Continuous improvement means not being satisfied with doing a good job or process, but striving to improve that job or process. It is accomplished by incorporating process measurement and team problem solving in all work activities. TQM tools and technique are used to improve quality, delivery and cost.

Although the problem-solving method is no guarantee of success, experience has indicated that an orderly approach with yield the highest probability of success. Problem solving concentration on improvements rather than control.

**Kaizen**

Kaizen is a Japanese word for the philosophy that defines management's role in continuously encouraging and implementing small improvements involving everyone. It is the process of continuous improvement in small increments that make the process more efficient, effective, under control and adaptable. It focuses on simplication by breaking down complex processes into their sub processes and then improving them.

The Kaizen improvement focuses on the use of :

1. Value-added and non-value added work activities.
2. Muda, which refers to the seven classes of waste.
3. Principles of motion study.
4. Principles of materials handling.
5. Documentation of standard operating procedures.
6. The five S's for work place organization which are five Japanese words that mean proper arrangement (seiko)

   | | |
   |---|---|
   | Order liners | (seiton) |
   | Personal cleanliness | (seiketso) |
   | Clean up | (seiso) |
   | Discipline | (shit-suke) |

7. Visual management by means of visual displays.
8. Just-in-time principles to produce only the units in the right quantities, at the right time and with the right resources.

9. Poka-Yoke to prevent or delect errors.
10. Team dynamics. Which includes problem solving, communication skills and conflict resolution.

Kaizen realies heaving on a culture that encourage suggestions by operators who continually try to incrementally improve their job or process. This change results in a small improvement in weld quality and a subtantial improvement in oprator satisfaction. The PDSA cycle described earlier may be used to help implement Kaizen concepts.

**Reengineering**

According to Hammer and Champy reengineering is the fundamental rethinking and radical redesign of business processes to achieve dramatic improvements in critical measures of performances. Many practitioners believe that TQM is associated with only incremental in provements. The Japanese have not only relied on Kaizen but have developed policy management (host in kanri) and policy deployment (hostin tenkai) in large part to product the kind of large-scale break through that Hammer and Champy promote. Clearlly there is nothing new in the reengineering concept. It has always been part of the TQM umbrella.

**SIX SIGMA (6σ) QUALITY**

**An Overview**

'Sigma' is used to designate the distribution or spread about the mean (average) of any process. Sigma (σ) is another word for standard deviation. For a business or manufacturing process, the sigma value is a metric that indicates how well that process is performing. The higher the sigma value, (2σ, 3σ, 4σ etc.) the better the process. Sigma measures the capability of the process to perform defect-free-work. A defect is anything that results in customer dissatisfaction. With 6σ, the common measurment index is 'defects-per-unit', where unit can be virtually anything–a component, a piece of a material, a line of code, an administrative form, a time frame, a distance, etc. **The sigma value indicates how often defects are likely to occur. The higher the sigma value, the less likely a process will produce defects. As sigma value increases, costs go down, cycle time goes down, and customer satisfaction goes up. A 6σ process simply means that between the target specification and the tolerance limit six standard deviations can be fitted-in,** (Fig. 4.7 explains clearly the difference between the 3σ and the 6σ process). Further, a 6σ process capability means 3.4 ppm defects or 99.99966% good.

Our process is the reality. When we draw the histogram of our process output we come to know how we are ; we can, then, calculate the sigma (σ) value of our process. When we place the tolerance limits, as decided by the competition, on our curve (normally distributed) we come to know where we are. We may be at 2σ or 3σ, etc. We now start our journey towards 6σ. **In other words we have to shrink the variability of our process to such an extent, that the value of sigma of the process reduces to a new low, which can be fitted ± 6 times within the same tolerance limits.** This is Quality Improvement. Such an improved process hardly produces any defect.

**What is 6σ ?**

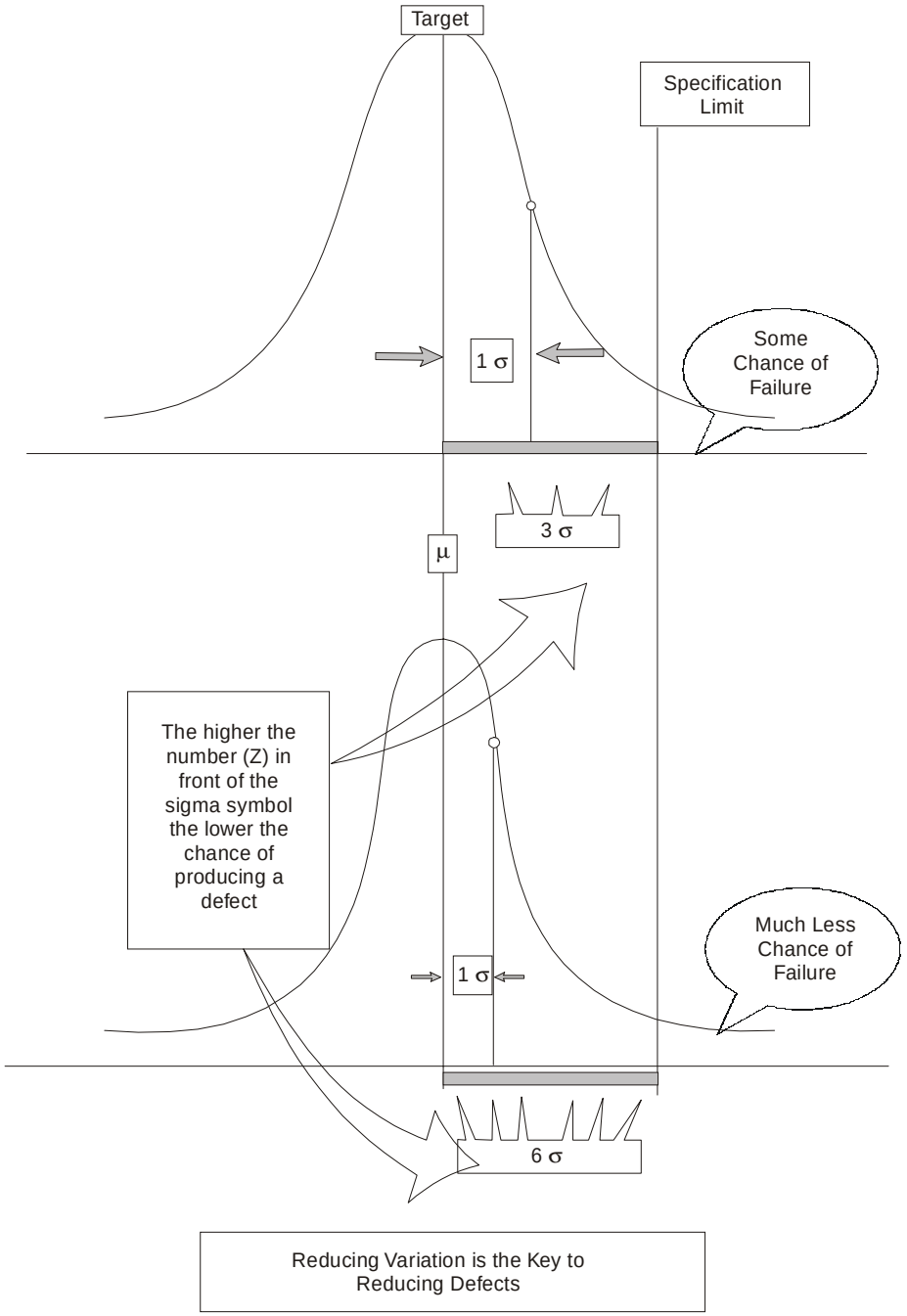



**Fig. 4.7.** Difference between 3σ and 6σ process.

**The Practical Meaning of**

| *99% Good (3.8σ)* | *99.99966% Good (6σ)* |
|---|---|
| • 20,000 lost articles of mail/hour | • Seven articles lost/hour |
| • Unsafe drinking water 15 min./day | • Unsafe drinking water 1 min./7 months |
| • 5,000 incorrect surgical procedures/week | • 1.7 incorrect surgical procedures/week |
| • 2 short or long landings at airports/day | • One short or long landing/5 years |
| • 200,000 wrong drug prescriptions/year | • 68 wrong prescription/year |
| • No electricity for almost 7 hours/month | • One hour without electricity/34 years |

**Six σ As a Goal**
**(Distribution shifted ± 1.5σ)**

| *Sigma level* | *Defects in PPM* | *Yield in %* |
|---|---|---|
| 2σ | 308,538 | 69,1462 |
| 3σ | 66,807 | 93,3193 |
| 4σ | 6,210 | 99,3790 |
| 5σ | 233 | 99,9767 |
| 6σ | 34 | 99,99966 |

**6σ According to Dr. Mikel J. Harry, CEO of Six Sigma Academy, Phoenix, USA**

- ☆ **First, it is a statistical measurement.** It tells us how good our products, services and processes really are. It allowes us to draw comparisons with other similar or dissimilar products, services and processes. We can see where we need to go and what we must do to get there. In other words, 6σ helps us establish our course and gauge our pace in the race for total customer satisfaction.
- ☆ When we say a process is 6σ, we are saying it is best in class. Such a level of capability will only yield 3.4 instances of nonconformance out of every million opportunities for nonconformance. On the other hand, when we say that some other process is 4σ, we are saying it is average. This translates to 6,210 non-conformities per million opportunities for nonconformance. In this sense, the sigma scale or measure provides us with a "goodness micrometer" for gauging the adequacy of our products, services and processes.
- ☆ **Second, it is a business strategy.** It can greatly help us gain a competitive edge. The reason for this is very simple — as you improve the sigma rating of a process, the product quality improves and costs go down. Naturally, the customer becomes more satisfied as a result.
- ☆ **Third, It is a philosophy.** It is an outlook, a way that we perceive and work within the business world around us. Essentially, the philosophy is one of working smarter, not harder. This translates to making fewer and fewer mistakes in everything we do — from the way we manufacture products to the way we fill out a purchase order. As we discover and neutralize harmful sources of variation, our sigma rating

goes up. Again, this means that our process capability improves and the defects (mistakes) go away.

**6σ According to Mr. Jack Welch, The CEO of The General Electric Co., USA :**

- ☆ 6σ is a disciplined Quality Improvement methodology that focuses on moving every process that touches the customers — every product and service — towards near perfect Quality. It is a measure of the Company's Quality.
- ☆ 'MOTOROLA' pioneered it and 'ALLIED SIGNAL' successfully embraced it. GE took the experiences of these two companies and started implementing it.
- ☆ 6σ is a top-down system.
- ☆ If you have to successfully implement 6σ, the Company should be
  - Open to change
  - Hungry to learn and
  - Anxious to move quickly on a good idea
- ☆ We became convinced that 6σ qualities could play a central role in GE's future, but we also know that it would take years of consistent communication, relentless emphasis and impassioned leadership to move GE on this bold new course.
- ☆ Today, '6σ' has spread like wildfire across the company and it is transforming everything we do. It has saved around 1200 million dollars to the company during 1998.
- ☆ '6σ' is quickly becoming a part of the genetic code of our future leadership. '6σ' training is now an ironclad pre-requisite for promotion to any professional or managerial position in the company — and a requirement for any award of stock options. Senior Executive compensation is now heavily weighted toward 6σ commitment and success-success now increasingly defined as "eatable" financial returns, for our customers and for us.
- ☆ We believed that there was an ocean of creativity, passion and energy in GE people that had no bottom and no shores. We also believed that there was an "Infinite capacity to improve everything". We believed these then, but there was no methodology or discipline attached to that belief. There is Now. It is '6σ' quality, along with a culture of learning, sharing and unending excitement.

**6σ Breakthrough Methodology of Quality Improvement**

The methodology is not very unique. It is a slight variant of the methodologies given by many quality gurus. There is nothing breakthrough in the methodology as such. But the solutions/results, we obtain by following this systematic methodology are really breakthrough. The improvement is not just in percentages but in manifold (say 100 times, 1000 times etc.)

The methodology consists of five steps namely Define (D), Measure (M), Analyse (A), Improve (I) and Control (C). Brief explanations for the same are as follows.

**Define.** The problem which requires breakthrough solution, has to be defined clearly in measurable terms. The problem selected should be vital to the customer and should have relevance to the company's business. In other words it should ensure great customer satisfaction as well as rupee savings to the company. If the company has developed its Business Strategies, the problem should fall under any one of them. Generally any customer

expects defect free products/services and timely deliveries. Majority of the problems will fall under these two categories. Defining the problem in manufacturing area is easier when compared to service areas.

**Measure.** The second most important step is measurement. We have to measure in terms of numbers to know where we are, and to decide where we go. To quote Dr. Mikel J. Harry — "If you can't express your process in the form of numbers you don't really know much about it. And if you don't know much about it, you can't control it. And, if you can't control it, you are at the mercy of chance. And, if you are at the mercy of chance, why bother with it ? Hence we must learn the language of numbers".

Data is as good as the system that measures it. Hence, before collecting the data a measurement system analysis has to be done and if it is not to the satisfactory level, corrective action has to be taken before measuring the data. Data is of two kinds — Discrete and Continuous. Continuous data is more amenable for Statistical analysis and hence as far as possible attempts should be made to convert the discrete data into continuous data. After collecting the data (discrete) on defects and counting the opportunities to make the defects, we can calculate the defects per opportunity (dpo), which is nothing but the probability of making the defects. From the statistical tables we can find out the corresponding 'Standard normal deviate', *i.e.*, the Z value or the sigma value. If it is a continuous data we can find out the sigma value by calculating the mean and the standard deviation of the process and knowing the specification limits. With this we can statistically define the problem.

After defining the problem a cause and effect diagram has to be constructed through brainstorming and segregate the causes into experimental and non-experimental causes. Solutions have to be found and implemented through Standard Operating Procedures (SOP) for the non-experimental causes at this stage itself, which brings down the variability of the process to a great extent. The experimental factors can be carried forward to the next phase-analysis.

**Analyse.** Statistical analysis has to be carried out at this stage to identify the vital experimental causes. Tests have to be conducted to find out whether the causes (factors) really make statistically significant difference in the effects (responses) when the levels of these factors are changed. The tools used are T-test, F-test, ANOVA, Chi-Square, correlation and Regression. A graphical analysis called multi-vari analysis is also done to segregate the variation of the response into with-in piece, between pieces and over time variations. After identifying the vital few experimental factors they have to be carried forward to the next pahse-Improve.

**Improve.** In this phase we will be optimizing the response. In other words we will be hitting the target value by experimenting with the level settings of the vital few factors. This is called Design of Experiment. There are various stages like Screening design/fractional factorial design, full factorial, full factorial with replication, Central composite design, Method of Steepest ascent, Evolutionary process (EVOP), Taguchi's method etc. Finally we will be tolerancing the factors at the required levels. In order to conduct the DOE, thorough planning is necessary, because the DOE is time consuming and sometimes costly.

**Control.** The last phase is to hold the gains that have been obtained from the improve phase. Unless we have good control we are likely to go back to the original state. 'Statistical Process Control' (SPC) has to be employed to control the gains. There are various kinds of control charts like I and MR, $\overline{X}$ and R, $\overline{X}$ and S and EWMA for continuous data and *p*, *np*, *c* and *u* charts for discrete data, to choose from. Also POKA-YOKE (mistake-proof) devices

can be set up to obviate the inadvertent errors. The idea of POKA-YOKE is to respect the intelligence of workers by taking over repetitive tasks that depend on vigilance or memory.

**CONCLUSION**

This article has only tried to explain the meaning of 'six sigma', without going into the depth of statistics. In effect the 'six sigma' Quality improvement methodology is a strategic bridge between the Quality Philosophy and the statistical tools available, which heavily depends on the Management drive and rigorous practice for its sustenance. To be fairly conversant with this methodology, one has to undergo a minimum of 10 days classroom training and then execute at least one real improvement project. Once the methodology is familiar. I am sure, one will not stop improving. 'Six sigma' is not just a fad ; it has yielded rich dividends in companies like MOTOROLA, GENERAL ELECTRIC, ALLIED SIGNAL, etc. It is now catching up in India and I can mention at least two major companies, *viz*., M/S BHARAT ELECTRONICS LTD and M/S WIPRO, which are rigorously practicing.

# 5 Tool and Techniques of TQM

## INTRODUCTION

One of the best technical tools for improving product and service quality is statistical process control. There are seven basic techniques. This technical tool not only controls the process and it has capability to improve it as well.

The international organization for standardization (ISO) was founded in 1946 in Geneva, Switzerland, its Mandate is to promote the development of international standards to facilitate the exchange of goods and service worldwide.

Benchmarking is not a new concept. It shares many elements with a multitude of activities, ranging from competitive analysis and Total Quality Management to ancient warfare. Since its inception as a formalized process at Xerox in 1979, it has evolved into a technique used by the majority of Fortune 500 companies. Not all companies that try it are pleased with success, but many are. Some companies have embraced the practice and want to share their experiences by offering training to others. Notable amongst this group are AT and T, IBM and Motorola. What began as a process to improve the design and manufacture of copiers is now used in variety of industries and functions. Today this diversity of applications has gone well beyond manufacturing industries and now includes studies in Government, Education, Agriculture, Health Care, and Financial Institution. In India, this concept is slowly picking up, and there are many organisations who have initiated this concept very successfully.

The purpose of this note is to answer some of the question about benchmarking and to help people understand why such attention is being focussed on the process. More importantly, it provides guidelines to help teams to get started (8-Steps Benchmarking Process) and describes pitfalls to avoid for teams that began the journey. After going through this note the leader of Benchmarking team can prepare an activity chart identifying the milestones, schedules and persons.

## BENCHMARKING AND COMPETITIVE ANALYSIS

Benchmarking can be confused with competitive analysis, which is a related but different exercise. Benchmarking shares many elements with competitive analysis. It often seeks information in the public domain and looks for creative ways to obtain and analyze data. There are significant differences, however. Two key distinguishing characteristics of Benchmarking are that the organization being studied (Benchmarked) is cooperating as a partner in the study and the focus of studying is on process and practices, not just performance measures. The table given below summarizes the major differences for organizations looking at external environment.

Characteristics of *Competitive Analysis Versus Benchmarking*

| **Characteristic** | **Competitive Analysis** | **Benchmarking** |
|---|---|---|
| Approach | Independent | Partner |
| Performed by | Individual | Team |
| Target | Competitor | "Best" Practice |
| Focus | Performance Measures | Processes and Practices |
| Objective | Competitive Intelligence | Process improvements |

**Bench Marking — Categories**

Today we have wide array of "Benchmarking" categories. Many of these distinction are quite orbitary and have nothing to do with the process itself, but deal more with the partners chosen and area of business being analysed. The Benchmarking process does not vary significantly with the three categories of partners that can be selected : Internal organisations, direct competitors, or non-competitors. There are two forms of Benchmarking : strategic and functional or operational. Operational Benchmarking focuses on the operational processes and practices and service offered by an organisation. Strategic benchmarking focuses on strategic marketing, financial, organisational and technological issues facing an organisation.

**Benefits of Benchmarking**

- Benchmarking is particularly helpful in validating proposals for change.
- Benchmarking of ten results in creative imitation and the adoption of new practices that overcome previous industry barriers.
- This search for diversity and for innovative breakthroughs applied elsewhere is at the core of benchmarking benefits.
- By sharing information, all parties benefits, because it is difficult to excel in all activites.
- Sharing information and data is often the first hurdle to be overcome in the Benchmarking process.
- Do not, however, attempt benchmarking in areas in which trade secrets or sensitive information determines the outcome of the process.
- Benchmarking, used in conjunction with other quality techniques or used alone, can influence how an organisation operates.
- If the search for "Best", or just "Better" practices is performed correctly, then the likelihood of successful outcomes is quite high.
- Success however, assumes that pitfalls are avoided and prerequisites have been met before Benchmarking is initiated.

**Some of the Prerequisites for Success**

- Management commitment and support can overcome many of the barriers to successful benchmarking.
- The key requisite for success is the organisation's readiness to accept change, given than it comprehends the needs for change.

- Rather than resting on their laurels and previous success, organisations must become receptive to new ideas.
- Sufficient resources need to be allocated, and both awareness and skills training need to be available.
- Benchmarking shares success factors with other quality management processes.
- Teamwork, analysis of data, decisions based on facts, focus on processes and continuous improvements (KAIZEN) are shared characteristics.
- The need for leaderships, a customer focus, and empowered employees are equally important.
- As in other Quality Management activities, there needs to be some one to manage the introduction and application of benchmarking in the organisation.
- Benchmarking teams need a clear understanding of both internal and external customer needs and expectations. Without this they will have difficulty selecting important subjects.
- An understanding of competitive strengths and weakness provides additional background that aids the selection processes.
- Strategic and competitive assessments not only tell the team what is important to focus on, but also give them an idea of how effectively they are currently achieving important goals.

**Some Common Misconception about Benchmarking**

- Benchmarking is not casual. It is a planned, systematic operation designed to achieve certain specific goals.
- Benchmarking is not a quick fix. It is an on-going process of setting increasingly higher goals for performance.
- Benchmarking is not copying others. It involves recognizing practices, which will fit into your operation, and adapting them.
- Benchmarking does not stand-alone. It is a facet of a broad Total Quality Program. The process must be integrated with rest of the TQM approach. (Ex. Malcolm Baldring Award, JRD QU, CII-Excellence award, BPR, and Process Improvement etc.)

**Forming the Team and Project Road Map**

- A team needs to be formed and a project road map developed.
- These are integral activities in the subject selection step.
- The team should include four to eight members who are subject matter experts and represent the various functions affected by the project.
- Teams often include several people in an advisory capacity. The project sponsor or "customer" and a benchmarking "coach" (an internal or external consultant).
- The road map serves several purposes : It keeps the team on track and it documents activities and decisions.
- This helps to educate others and to document the team's process for later use.
- Another advantage is that clearly defined projects help teams to ensure that the project scope is achievable.

- Basic elements of the road map include.
  - Project scope and objectives.
  - List of activities.
  - List of deliverables.
  - Individuals roles and responsibilities.
  - Meeting and review schedules.
  - Issues.

**The Eight Most Common Benchmarking Errors**

1. *Lack of Self-Knowledge.* Unless you've thoroughly analysed your own operations, your benchmarking efforts will not pay off. You have to know how things work in your company, how effective your current processes are, and what factor are critical. That's why internal benchmarking is an important first step.
2. *Benchmarking everything.* Be selective. Benchmarking another company's employee food service will usually not be worth the time, energy, and cost. Your TQM effort as a whole will point out the areas where benchmarking is most likely to pay off.
3. *Benchmarking projects are broad instead of focused.* The more specific the project, the easier it is and the more likely it will generate useful ideas. Benchmark a successful company's hiring procedures, not their entire human resources operations. Focus on accounts receivable handling, not the accounting department as a whole.
4. *Benchmarking produces reports, not action.* Studies have indicated that 50% of benchmarking projects result in no specific changes. The process is not an academic exercise. It should be geared toward generating and implementing actual changes.
5. *Benchmarking is not continuous.* Benchmarking is a process. Even before you reach the benchmark you've set, you should take another look at your partner's performance, or at other companies. New goals should be established and new techniques adopted. The process never ends.
6. *Looking at the numbers, not the issues.* While the measures are important, they are not the heart of the process. At some companies, benchmarking is used to set goals, but not to generate the important changes needed to meet them.
7. *Participants are not motivated.* Make sure benchmarking team members have the time to do the job. Even if the project is simply added on their regular jobs, make sure each has a stake in the success of the project. Don't consider benchmarking as "busy work" to be assigned to a group of low-level employees.
8. *Too much data.* Action are what's important, not information for its own sake. Don't measure benchmarking success by quantity of information. Always focus on key issues.

**Common Pitfalls in Benchmarking**

- Lack of management commitment and involvement.
- Not applied to critical areas first.
- Inadequate resources.
- No line organisation involvement.

- Too many subjects ; scope not well defined.
- Too many performance measures.
- Critical success factors and performance drivers not understood or identified.
- Potential partners ignored : Internal organisations, Industry leaders, or friendly competitors.
- Poorly designed Questionnaires,
- Inappropriate data : Inconsistent data.
- Analysis paralysis, excess precision.
- Communication of findings without recommendations for projects to close gaps.
- Management resistance to change.
- No repeat Benchmarking.
- No Benchmarking report/documentation.

**Eight Steps Benchmarking Process**

The Benchmarking process consists of three general activities : Planning, Analysis, and Integration/action.

Overall, the process follows the Plan-Do-Sudy-Act Cycle of all quality processes.

It is recommended to use eight steps benchmarking 7 process as mentioned below :

**Eight Steps Benchmarking Process**

| | Activity | Schedule | By Whom |
|---|---|---|---|
| Planning : | 1. Select Benchmarking subject and appropriate team | | |
| | 2. Identify performance indicators and Drivers | | |
| | 3. Select Benchmark partners | | |
| | 4. Determine data collection method and collect data | | |
| Analysis : | 5. Analyse performance gaps. | | |
| Integration/ Action : | 6. Communicate Findings and identify projects to close gaps | | |
| | 7. Implement plans and monitor results | | |
| | 8. Recalibrate benchmarks. | | |

**Benchmarking And TQM**

Benchmarking is a tool that can be expectionally useful in a TQM program. It is closely related to customer orientation. The companies that you select as benchmarking partners are those, which excel at satisfying customer in a given area.

Benchmarking is an excellent source of ideas for quality improvement projects. It provides both ideas about ways of accomplishing tasks, and specific goals in terms of levels of performance.

**Some Quotes — A Mandate for Best Practices**

"In turbulent times growth comes from understanding enterprise and core processes".

"These processes need to incorporate the best practices of exemplars to be found".

"Not reinvented here".

Two ideas "opm and opb" ("opm - other people's money and

Opb — other people's brains")

"The old wisdom "if it as n't broke, then don't fix it" the new wisdom "if it isn't perfect, then improve it".

"The old school of thought, which held that "if it ain't invented here, it can't be any good".

The curse in today's high velocity markets.

"Don't reinvent what others have learned to do better".

"Today's ralling cries — "borrow shamelessly"

"Adopt, Adapt, Advance !" "Imitate Creatively !" And "Adapt Innovatively !"

Are the anthems of business pragmatism"

"Keep on the lookout of novel and interesting ideas that other have used successfully. Your idea has to be original only in its adaptation to the problem you're currently working on". — Thomas Edison

Fool you are ..... To say you learn by your

Experience ..... I prefer to profit by others'

Mistakes, and avoid the price of my own — Prince Otto Von Bismarck

Old truth : thou shalt not steal.

New truth : thou shalt steal non-proprietary ideas shamelessly.

**Benchmarking**

**Purpose.** To establish a superior performance in the organisation by filling the gaps in performance by putting in place the best available practice.

**When to Use.** Benchmarking is a tool used while the organisation wants to implement and ensure the prevalence of TQM. This is done when each process is looked at independently and the performance is compared to that of others.

**How to Use.** To progressively stimulate an improvement a company can use the foll types of benchmarking.

**Internal Benchmarking.** The main aim of IB is to optimize performance through removal of errors the comparison in case of IB is generally between functions and similar organisation.

**Competitive Benchmarking.** This type of benchmarking can be carried out on the basis of product, functions department or on a co. wide basis this is a cross comparison with in one industrial sector aimed at establishing best practice through the identification of gaps between your own and your competitors performance.

**Comparative Benchmarking.** This type is one where comparison takes place across all business sectors aimed at establishing the practice in all areas.

**The Rocks of Benchmarking**

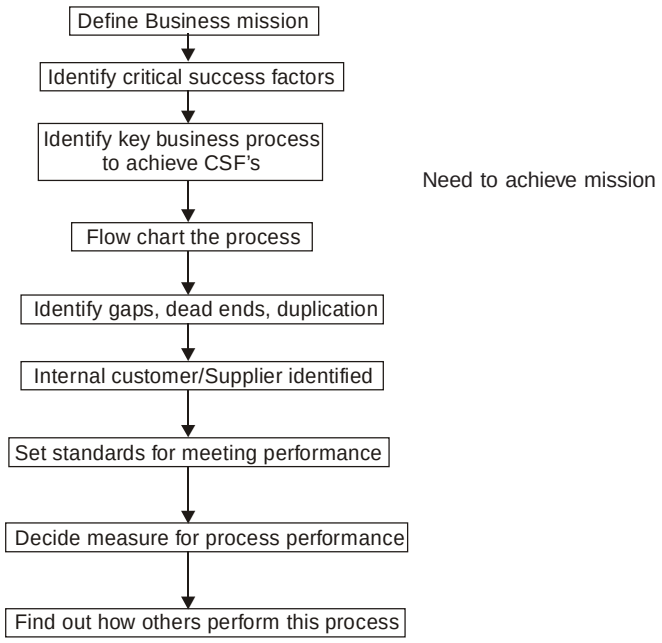



**Fig. 5.1. Steps of Benchmarking**

**Benefits**

It gives the introduction to measurement. It brings about focus in the mission and helps to identify measures for key business processes. Benchmarking help organisation bring about an external focus and being closer to the markets.

**Deming Wheel (PDCA)**

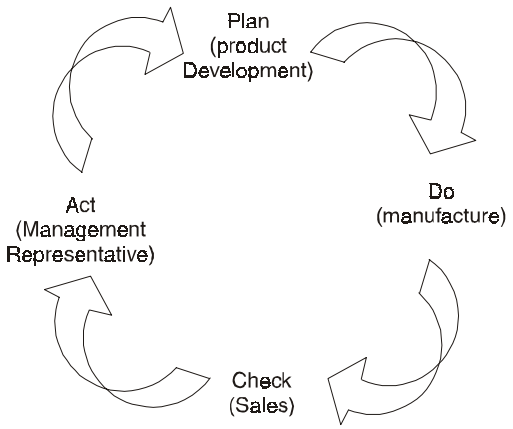



**Fig. 5.2. Deming Wheel.**

**Introduction.** With the HQ at Geneva. These standards tell suppliers and manufacturers what is required of a quality oriented management system.

**Quality Management Systems.** Quality is not a chance occurrence. It has to be built up consciously and stage by stage through suitable processes, procedures, resources responsibilities and an appropriate organisation structure that knits all these factors together. The amalgamation of all these factors which are aimed at achieving desired quality levels consistently in what is termed as a Quality Management System (QMS).

**The Need for Quality Management System.** Quality standards were hitherto largely specified in terms of tolerances, acceptance levels and other "specs". With the gradual transition of the basic concepts of Quality Control from inspection, to the present stage of managing for quality in all activities, it was but natural that quality standards too changed accordingly. These days the customers are equally concerned about the manner in which quality is managed in the supplier's organisation, as they are with the actual quality of the purchase product.

All organisations do have a QMS, although most often it is informal or semi-formal. The drawback with such systems is that in certain areas there is a total absence of established in certain areas there is a total absence of established policies, procedures, responsibility and authority. These are termed as 'System Deficiencies'. Even if procedures and policies exist, there is ambiguity with regard to their practice. Therefore such systems fails randomly due to the prevailing confusion regarding the system or due to the absence of a few key people, who inadvertently emerge as the only ones who area aware of how the system works.

A formal quality system helps remove uncertainty by formalizing the Who, Why, Where, What and How of Quality so that processes for achieving quality are established, whereby customer satisfaction can be achieved consistently irrespective of the personnel involved. A quality system standard helps to check or implement a system against standard checkpoints, which ensure that all aspects of managing quality have been looked into. It also indicates to the customers, the quality status of the supplier's organisation, which these days is becoming as important as the quality status of the product.

**The Evolution of ISO 9000.** The Defence Services have long been associated with quality and have played a pioneering role especially in the area of quality standards, particularly during the world wars. Quality system standards were first developed and widely used as a means of supplier assessment by American Defence orgnisations such as the NATO, etc. Such standards were later developed by other nations as well. During the post-war period, the need for such standards were felt in general industry too. Therefore Britain refined its defence standards and formulated the BS : 5750. In 1987, the ISO formulated the ISO 9000 series of standards, after considering similar standards of various countries, but designing it predominantly based on the BS 5750.

**Uses of The ISO 9000 Standards.** The standard can be used for these purposes :

*Internal Audit.* An organisation can audit its existing systems with reference to the standard to provide feedback to management regarding the deficiencies of the system.

*Vendor Assessment.* To assess and ensure a vendor's capability to consistently meet the quality standards for products standards desired by the organisation.

*Supplier Capability*. To provide confidence to its customers regarding its capability to consistently meet the quality standards desired by the customer.

**Contents of ISO 9000.** The ISO 9000 is a series of five standards. A brief description of each is given on next page to highlight the essential difference between them.

*ISO 9000*. This contains guidelines on the selection and use of this series of standards, along with definitions of key terms and explanation of basic quality concepts.

*ISO 9001*. This is the most comprehensive standard in the series. It specifies system requirements in purchase, Design and Development, Production, Installation and Servicing. This standard is not necessary for all organisations having an inhouse design/development department. It is necessary only for those organisations who design or develop at least a few of their product specifically tailored to a customers' requirements. Examples of this would include a jobbing foundry, an architectural organisation, etc. For example, this standard applies to an organisation such as Tata Steel due to the nature of R and D work where special grades of steel are sometimes developed to specifically cater to stated requirements. Exports of machinery and equipment from the Adityapur Complex specifically designed to customers' requirements also makes this particular standard applicable to Tata Steel.

*ISO 9002*. 90% of the organisation registered to this series in Britain conform to 9002. This standard specifies system requirements only for Purchasing, Production, Inspection, Installation and servicing. It does not concern Design, otherwise identical to the 9001. It is exactly identical to the 9001 excluding the sections on Design/Development.

*ISO 9003*. This is the least used standard in the series. It specifies system requirements only for Final Inspection and Testing. It is applicable only to warehouses, stockiest, distributors and the like.

*ISO 9004*. This is not a standard in the real sense. It contains excellent guidelines on implementation and highlights important aspects to be considered while designing a system.

**Essence of the Standard.** To put it in the simplest of terms the standard (9001-3) recommends that an organisation should :

Decide its Quality Policy

Determine how the policy is to be implemented and design a system accordingly

Implement the system

Review the system regularly to gauge its effectiveness

Review the system regularly to gauge its effectiveness

(The 9004 provide guidelines to the organisation on all the above activities)

Quality system have to be tailor-made to suit the organisation's requirements, so as to be effective. Therefore, the standard only provides the basic framework for the system and offers unlimited freedom within the basic framework.

Let's take the example of the Quality Policy. The standard does not specify the contents of the policy, which could be, say, anywhere between the two extremes-to produce zero-defects or to produce a certain level of defects. However, what the standard does specify is that whatever be the policy, the management should ensure that it is understood and implemented at all levels in the organisation.

Similarly, for activities such as design, production and the like, the standard only specifies the presence of quality control elements such as planning, review, correction action etc. There is complete freedom in the modalities an organisation may use for satisfying these elements.

The important aspects to be noted are :

The standard specifies that the quality system be documented (through Quality manuals, work instructions, organisation charts, process charts, and the like).

In any customer-supplier relationship (internal/external the onus is on the supplier to fully clarify and comprehend all customer requirements, stated as well as implied. For example, the Purchasing Department, should clarify and review all requirements stated in the purchase requisition before placing the order. This applies similarly to the Design Department, with regard to Design inputs, to the Marketing Department, while accepting an order and to any other department in a similar manner.

The standard does not specify requirements for Cost of Quality, continuous improvement and does no dwell upon employee participation, morale, attitudinal change etc., though all these are mentioned in the form of guidelines in the 9004 ; all of which are important ingredients of the TQM philosophy. Therefore, an ISO 9004 has only taken one more step towards being the TQM company and is not necessarily a TQM company. However, a TQM company will always conform to a Quality System standard, which may either be the ISO 9000 or an equivalent.

**Certification.** Certification is not mandatory for all organisations that conform to the standard. For example, if an organisation decides that vendors for a particular product should conform to the ISO 9000. It can send its own team of qualified auditors to assess the vendor's organisation. However, an ISO 9000 certificate is a proof of conformance, which therefore obviates such individual assessments.

Accreditation's to the standard is achieved only after the QMS of an organisation is assessed and approved by a recognized third party certifying body. After certification the organization is assessed at regular intervals by the same certifying body to ensure continuous conformance. The body has to be accredited to the 'National Accreditation Council for Certification Bodies'', (NACCS), UK, or the Dutch Government's "Radd Voorde Certificate." Presently there are no approved certifying bodies in India. More likely, organisations such as the Bureau of Indian Standards, the Confederation of Engineering Industry (CEI) may get themselves credited for the same. Until then, Indian as the British Standards Institute, Lloyd's Register Quality Assurance Limited (LRCA), Bureau Veritas Quality International (BVQI) etc., or similar organisations is other countries.

**International Implications.** Though the standard has been adopted as a EURONORM, and also by 90 countries wolrdwide, some of the advanced countries such as the USA, Japan, France, Germany etc. have not adopted it. This is probably because their existing standards are either superior or on par with the 9000. In fact the entire concept of Euronorms itself, for various products, is facing rough weather due to the conciliatory nature of the norms.

However, the fact remains that Quality System Standards are here to stay and conformance to such standards should be an imperative step in an organisation's march towards Total Quality.

**Glossary of Terms.** The following definitions are important since the usage of these words in the standard are in a slightly different form than their normal usage.

1. Supplier : Your organisation (as a supplier)
2. Purchaser : Your organisation's customer
3. Vendor/Sub-contractor : The source of goods purchased by your organisation.

## QUALITY BY DESIGN

**Introduction.** Quality by design principles are changing by the managers think and conduct business. Lossely designs quality by design is the practice of using a multidisciplinary team to conduct conceptual thinking, product design and production

planning all at one time, it is also known as concurrent engineering, simultaneous engineering or parallel engineering. Quality by design has recently encouraged changes in management structures.

The major functions within an organization would complete their task by "throwing it over the wall" to the next department in the sequence and would not be concerned with any internal customer problems that right arise, quality by design or concurrent engineering requires the major functions to be performed at the same time. This system provides for immediate feedback, which prevents problems with quality and productivity from occurring. Fig. 5.3. shows the flow diagram for both sequential or traditional engineering on the left and quality by design or concurrent engineering on the right. When each of the specialists early. Input to the product definition and specifications, cost is minimized and performance is maximized. Thus, better-quality products are manufactured for less cost with shorter time to market.

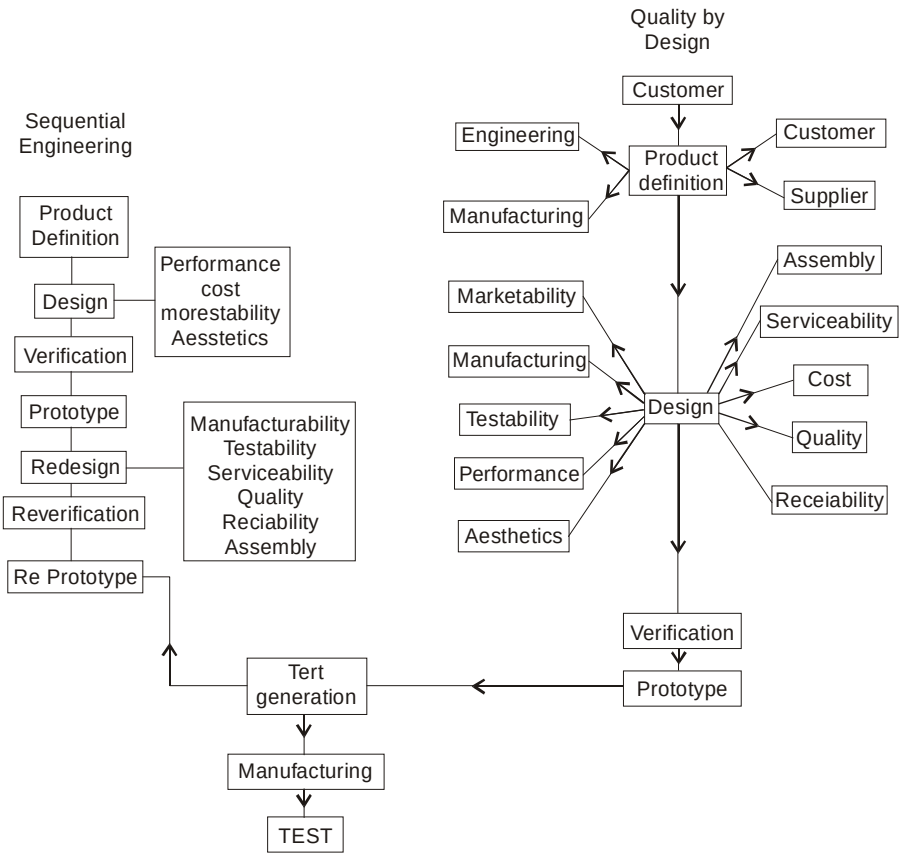



**Fig. 5.3. Product development flow diagram.**

The quality by design or concurrent engineering method combines all these steps into one. The product is designed to be successful at each stage of its life cycle. It is designed correctly the first time, considering all attributes and facets of its life, such as marketability, assembly and service ability, before release to testing and small production.

**Rationale for Implementation of Quality by Design.** Project budgets for all industries are becoming most crucial to any product's marketability. Accounting methods and budgets were not as critical as they are today. In the past, consumers had only a few brands to choose from the price was dictated the demand for quality product at reasonable profit. Imported products helped balance the demand for quality products at reasonable prices and allowed consumers to set the market price. Quality by design helps control this by shifting all the design to the beginning of the project rather than throughout its whole life-cycle as shown in Fig. 5.4.

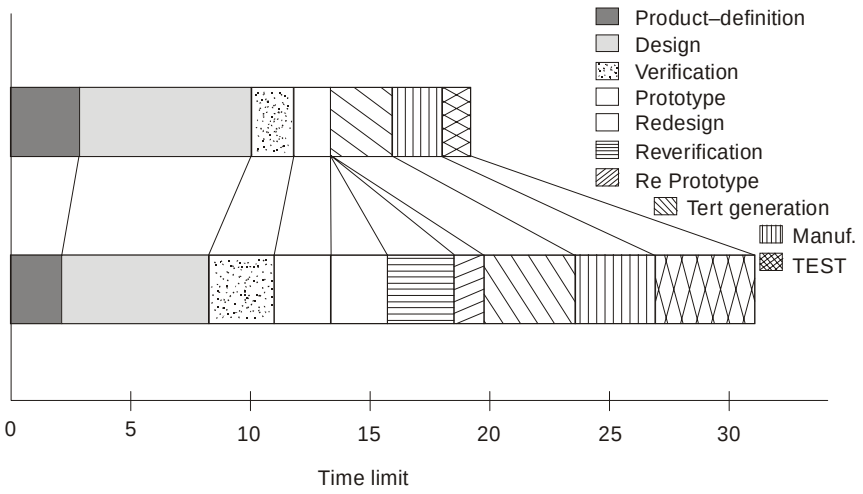



**Fig. 5.4.** Hypothetical Product Development Time Life.

The amount of time required in the quality by design model for product-definition and specifications can be significantly greater than that required in the sequential engineering model.

By using quality by design, the product is designed within production capabilities in order for statistical process control to be effective. Producing products well within process capabilities will cause a chain reaction of customer satisfaction. Customers returns will decreases and rework costs will also decrease. Thus organizations are taking two steps for ward and one gaint step back every day they continuously inspect products that could have been designed within process capabilities rather than at or below process capabilities.

Failure mode and effect analysis was first developed over 60 years ago.

The Aerospace industry adopted FMEA during the 1960's.

It is now in wide spread use in the Automotive industry.

Many manufacturers of automotive vehicles are now insisting that suppliers use FMEA on new products and during product changes.

Its use in other industries is a comparatively recent phenomenon.

**What is FMEA ?**

FMEA is a design and pre-product planning technique which structures the engineers thought processes to consider every conceivable way in which a component or product may fail. It is an analytical technique which :

Identifies potential failures mode

Assesses potential effects to a customer

Identifies potential causes of failure

Identifies need for changes

Minimises potential cause of failure

Facilitates inter-departmental dialogue

Facilitates identification of critical characteristics in a process

An FMEA enables engineers to look at all aspects of design and process in a formalized and structured way.

The potential problems are recorded, numerically assessed and ranked in order, for action to be taken.

**When to Use F.M.E.A**

An FMEA study can be carried out at any stage during the development of a product. However the ideal times to use this technique are :

At the original concept stage when specification are being established.

As soon as the design is finalised but before any manufacturing of tooling etc., is commenced.

When major changes are to be carried out, either to the design or process alternation.

After an FMEA study has been conducted and corrective actions taken ; *i.e.*, reassessment.

**Benefits of FMEA ?**

FMEA is a upstream, off-line activity and represents a quality prevention cost. It is concerned with building quality into the product prior to manufacture. Properly conducted, it should therefore lead to :

A reduction of scrap, and rework activities.

Reduction in inspection and process control activities.

Reduction of failure conditions.

Reduction in warranty costs.

Overall reduction in manufacturing costs.

Improved customer satisfaction.

**Design and Process FMEA**

Design and process FMEAs represent two different stages of analysis and are conventionally recorded separately.

DESIGN FMEA should be carried out as early as possible in the life of a product. Its purpose is to ensure that the product will function properly when manufactured. Each future is analysed for potential failure.

PROCESS FMEA study should be carried out as soon as the drawings are completed. It is used to identify potential modes of failure of each process, component and assembly.

**Preparation of an FMEA Document**

All the following steps are essential in the production of an FMEA document.

1. FMEA NUMBER

   Enter the FMEA document number, which may be used for tracking.

2. ITEM

   Enter the name and number of the system, subsystem or component, for which the process is being analysed.

3. PROCESS RESPONSIBILITY

   Enter the QEM, department and group. Also indicate the supplier name if known.

4. PREPARED BY

   Enter the name, telephone number and engineer of the company responsible for preparing the FMEA.

5. MODEL YEAR(S)

   Enter the intended model year(s) that will be utilised and/or be affected by the design/process being analysed (if known).

6. KEY DATE

   Enter the initial FMEA due date, which should not exceed the schedule start of product date.

7. FMEA DATE

   Enter the date the original FMEA was compiled, and the latest revision date.

8. CORE TEAM

   List the name of the responsible individuals, and departments which have the authority to identify and/or perform tasks. (It is recommended that all teams members, names, departments, telephone numbers, addresses, etc., be included on distribution lists).

9. PROCESS FUNCTION/REQUIREMENTS

   Enter a simple description of the process of operation being analysed (eg. Turning, drilling, tapping welding, assembling). Indicate as concisely as possible the purpose of the process or operation being analysed. Where the process involves numerous operation (e.g. assembling) with different potential modes of failure, it may be desirable to list the operations as separate processes.

10. POTENTIAL FAILURE MODE

    Here we identify and describe potential failure modes. It is important to consider all possible failure modes. Work on the principle that if it can go wrong it probably will e.g. Bent, Binding, Dirty, Open, Short, Grounded Tool worn.

11. POTENTIAL FAILURE EFFECTS

    Here we identify and assess the potential effects of the failure and describe the effect of the failure in terms of the fitness for purpose of the product. E.g. Noise, Erratic operation, Cannot fasten, Cannot bore/tap etc.

12. SEVERITY(S)

    Estimating the severity of the effect of the failure again on a scale 1-10

Typically :

| | |
|---|---|
| Minor Severity | 1 |
| Low Severity | 2-3 |
| Medium Severity | 4-6 |
| High Severity | 7-8 |
| Very High Severity | 9-10 |

13. POTENTIAL FAILURE CAUSES

Here we address ourselves to the potential causes of failures, and list all the assignable causes of possible failure. Use cause and effect diagrams. E.g. : Improper torque, over, under ; inadequate venting/gating etc.,

14. OCCURRENCE(O) (CONTINUED)

*Suggested Evaluation Criteria :*

(The team should agree on an evaluation criteria and ranking system, which is consistent, even if modified for individual process analysis.)

| *Probability of failture* | | *Ranking* | *Possible* | *Failure* | *Rates Cpk* |
|---|---|---|---|---|---|
| Very High : | Failure is almost inevitable | 10<br>9 | > 1 in<br>1 in | 2<br>3 | ≤ 0.33<br>≥ 0.33 |
| High : | Generally associated with processes similar to previous process that have often failed. | 8<br>7 | 1 in<br>1 in | 8<br>20 | ≥ 0.51<br>≥ 0.67 |
| Moderate : | Generally associated with processes which have experienced occasional failure, but not in major proportions. | 6<br>5<br>4 | 1 in<br>1 in<br>1 in | 80<br>400<br>2,000 | ≥ 0.83<br>≥ 1.00<br>≥ 1.17 |
| Low : | Isolated failures associated with similar process | 3 | 1 in | 15,000 | ≥ 1.33 |
| Very low : | Only isolated failure associated with almost identical process | 2 | 1 in | 1,50,000 | ≥ 1.50 |
| Remote : | Failure is unlikely. No failure ever associated with almost identical process. | 1 | 1 in | 15,00,000 | ≥ 1.67 |

15. CURRENT PROCESS CONTROLS

Current process controls are descriptions of the controls that either prevent to the extent possible the failure mode from occurring or detect the failure mode should it occur. These controls can be process control such as fixture **error-proofing** or statistical process control (SPC) or can be post-process evaluation.

The evaluation may occur at the subject operation or at subsequent operations that may detect the subject failure mode.

16. DETECTION(D)

*Estimate the probability of Non detection of failure before it reaches the customer :*

*Suggest Evaluation criteria.* (The team should agree on an evaluation criteria and ranking system, which is consistent, even if modified for individual process analysis.)

Likelihood the Existence of a Defect will be detected by
Controls before Next or Sunsequent Process, or Before Part or Component
Leaves the Manufacturing or Assembly Location.

| | | Ranking |
|---|---|---|
| Absolute Certainty of Non-detection | Controls will not or can not detect the existence of a detect | 10 |
| Ver low : | Controls probably will not detect the existence of a detect | 9 |
| Low : | Control have a poor chance of detecting the existence | 8<br>7 |
| Moderate : | Controls may detect the existence of a detect | 6<br>5 |
| High : | Controls have a good chance of detecting the existence of a detect. (Process automatically detects failure) | 4<br>3 |
| Very High : | Controls will almost certainly detect the existence of a defect (Process automatically prevents further processing) | 2<br>1 |

17. RISK PRIORITY NUMBER

The risk number (RPN) is simply calculated as the product of occurrence, severity and detection.

RPN = O × S × D

18. RECOMMENDED ACTION

This is based on

(*a*) The value of the calculated RPN.

High RPN's require urgent action.

It is useful to rank RPN's (high to low) in order of priority similar to Pareto Analysis.

(*b*) Significantly high individual values of occurrence, severity or detection.

19. RESPONSIBILITY

Enter the organisation and individual responsible.

20. ACTION TAKEN

Determine actions to be taken to eliminate or reduce potential concerns. Issue instructions for action to be taken and record actions taken. Then go to step 21 and revise O, S, D and RPN. Repeat until satisfied.

REMEMBER AN FMEA IS A LIVE DOCUMENT WHICH RECORDS ALL DESIGN AND PROCESS CHANGES

**Typical Failure Mode Area**

| | | | | |
|---|---|---|---|---|
| Bent | Corroded | Grounded | Omitted | Shorted |
| Blistered | Cracked | Leaking | Open Circuited | Tight |
| Bound | Damaged | Loose | Out of tolerance | Warped |
| Brittle | Deformed | Melted | Porous | |
| Broken | Discoloured | Misaligned | Rough | |
| Burrs | Eccentricity | Misassembled | Sharp edges | |

**Implemention ISO-9000**

There are a number of steps that are necessary to implement a quality management system.

1. **Senior Management Commitment.** The most important step in implementing a quality system that will meet or exceed an ISO 9000 standard is to acquire the full support of upper management. The Chief Excessive Officer (CEO) must be willing to commit the resources necessary to achieve certification.

2. **Appoint the Management Representative.** Once the commitment has been made, the process can proceed by adopting a project team approach and treating the same as other business undertaking and then management representative. This person is responsible for co-ordinating the implementation and maintenance of the quality system and is the contact person for all parties involved in the process, both internal and external. The implementation of the quality system should involve everyone in the organization. The standard requires the management representative be a person who is able to ensure that the quality system is effectively implemented and maintained irrespective of other responsibilities.

3. **Awareness.** This step requires an awareness program. Because the process is going to affect every member of the organization as well as require their input, and everyone should understand the quality system. They should know how it will affect day-to-day operation and the potential benefits.

4. **Appoint an Implementation Team.** After everyone has been informed of the organizations intentions to develop the quality system, an implementation team should be assembled. This team should be drawn from all levels and areas of the organizations.

5. **Training.** The implementation team, supervisors and internal audit team should be trained. This activity can be accomplished by sending team leaders for training and by bringing the training in house for all team members through a one or two day seminar.

6. **Time Schedule.** This activity develops a time schedule, for implementation and resistration of the system. The time frame will very demanding on the size and type of organizations and the extent of its existing quality system.

7. **Select Element Owners.** The implementation team selects owners for each of the system elements. Many of there owners will be members of the implementation team. Owners may has be assigned more than one elements. Each owner has the option of selecting a team to assist in the process.

8. **Review the Present System.** Review all the present-quality system. Copies of quality manuals, procedures, work instructions and forms presently in use are obtained. These documents are sorted into system elements to determine what is available and what is needed to complete the system.

9. **Write the Documents.** Before written quality and procedure manuals — they can be combined into one document. Write appropriate work instructions to maintain the quality of specific functions. This process should involve every employee, because the best person to write a work instructions is the one who performs the job on a regular basis.

10. **Install the New System.** Integrate the policies, procedures and work instructions into the day-to-day workings of the organizational and document what is being done. It is not necessary for all elements to be implemented at the same time.

11. **Internal Audit.** Conduct an internal audit of the quality system. This step is necessary to ensure that the system is working efficiency and to provide management with information for the comprehensive management review.

12. **Management Review.** Conduct a management review, the management review is used to determine the effectiveness of the system in achieving the stated quality goals.

13. **Pre-assessment.** If a good job has been done on the previous steps, preassessment is not necessary.

14. **Registration.** This step requires three parts, choosing a registrar, submitting an application and conducting the registrar system audit. Choosing a registrar include cost, lead time, your customer's acceptance of the registrar, the registrar's accreditation and similarity with your industry. The application for registration should also include supplying the registrar with the policy and procedure manuals for their review. The time involved in the resistrar's system audit and procedure manuals for their review. The time involved in the register's system audit will vary depending on the size and complexity of the organization and the number of auditors involved. It is usually one to three days and will consist of an opening meeting to describe the process the auditors will follow, the audit itself and a closing meeting to discuss the finding of the auditors will follow, the audit itself and a closing meeting to discuss the finding of the audit.

### Documentation of ISO 9000

A quality system is the method used to ensure that the quality level of a product or service is maintained. The system documentation can be viewed as a hierarchy containing four tiers as shown in Fig. 5.5.

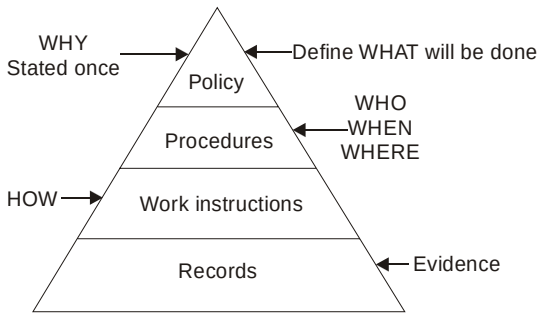



**Fig. 5.5.** The Documentation Pyramid.

If the system is properly structured, changes at one level will seldom affect the levels above it but may effect those below.

**Policy.** The first tier of documentation is the policy manual. It defines what will be done and why. A quality policy manual should be written, it should be clear, precise practice and easy to understand.

The remainder of the policy manual addresses. What will be done to comply with the standard being used. The way of looking at the policy manual is to think of it as the commandments of the system. Each element of the standard is addressed individually and usually requires one page or less.

**Procedure.** The second tier of documentation is the quality procedure. There procedures describe the methods that will be used to implement and perform the stated policies. The procedures define who should perform specific tasks when the task should be done and where documentation will be made showing that the task was performed. They dictate the strategies that will be used to ensure the quality of the system. It should be noted that procedure are not required for all elements. Many organizations combine the policy and procedures into one document.

**Work Instructions.** Work instructions are usually department, machine, task, or product oriented and spell out how a job will be done. A work instructions may be in the form of a declared drawing, recipe, routing sheet, specific job function. The writing of a work instructions is best carried out by the employee who performs the task. This person knows the process and the problems encountered in that process. A documentation specialist may be needed to do the actual writing. It creates a pride of ownership in the document, making it more likely to be carried out, employee participation helps to ensure that future improvement with be suggested.

**Records.** Records are a way of documenting that the policies, procedures and work. Instructions have been followed. They may be forms that are filled out, stamp of approval on a product or a signature and date on some type of document such as a routing sheet. Records are used to provide traceability of actions taken on a specific product or batch of products.

### Quality

**Introduction.** In today's marketplace, companies must be driven by the customer. Customers have certain needs and expectations. It is the company's responsibility to deliver a product or service that satisfies the customer. In the ideal situation, the features, advantages and benefits of the company's product fit the customer's expectations. When this occurs, the company has no problem successfully marketing its product, resulting in high levels of satisfaction for the customer and higher profits for the company.

The actual situation is often very different. When a company's product doesn't match the customer's expectations, the product does not sell. This forces the company to use marketing technique to move the product. Some options available are :

Adjust Price (offer rebates)

Increase Sales Commissions

Carry Larger Inventory

Advertise the Product

Develop a Public Relations Campaign

All of these techniques are expensive ; some are very expenisve. The result of using them is a loss of profits. The lesson here is that by manufacturing products that fit customer expectations, profits will increase. QFD (Quality Function Deployment) was developed to do just that.

It is infinitely more difficult to wrestle market share away from a viable competitor than it is for the first producer into a market to attract customers to buy and use a new product whose time has come. The company with the expectations, at a cost that represents value, have a competitive advantage. But these companies must have the shortest product development cycle to maintain the advantage.

Quality Function Development can be an aid to achieving our goals of :

- Quality
- Cost
- Timeliness
- Value

**Why QFD ?**

A lack of a well-defined product development process in R and D/Design Department can be the root cause for the ruination of a company. QFD (Quality Function Deployment) offers many features, which are helpful in defining this process, something that for many reasons is often very difficult.

Although many companies do not have a well-defined new product development process, they manage to market blockbuster products or services which fill the needs of customers or users. At other times, their new products or services may fail because customer does not purchase them. In other words, success comes when customer needs are well understood and met by the products or services. QFD provides a formal process to understand user needs and to weave these needs across the entire product development cycle, including manufacturing.

**Definition of QFD**

"QFD is a very systematic and organised approach of taking customer needs and demands into consideration when designing new product and services or when improving existing products and services." Another name for this approach is "customer-driven engineering" because the voice of the customer is diffused throughout the product (or service) development life cycle.

QFD is a planning tool that defines a process for developing products or services. The aptitude to plan is rare in the human race. Managers are evaluated on short-term results, which further inhabits this aptitude for planning. It is difficult to use Deming's PDCA (Plan, Do, Check, Act) cycle to improve the product development process if the P of PDCA Cycle is weak. QFD is applying TQM philosophy to product development by focusing on P. Using QFD counteracts the inherent weakness embedded in human nature — that of avoiding planning.

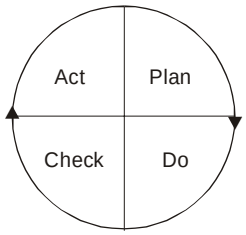



**Fig. 5.6.** Deming's Cycle of Continuous Improvement (PDCA Cycle)

**Where Did QFD Come From ?**

Yoji Akao of Tamagawa University is the key contributor to QFD Development in Japan. There are 30 matrices in his approach. The QFD team can pick and choose the matrix which would be of most use for a particular phase of product development.

Another expert in the area of QFD is Fukahara. He is associated with the Central Japan Quality Control Association. He mainly focuses on the house of quality, namely the product definition aspect. There is also the four phase (or four matrices) approach promoted by the American Supplier Institute (ASI). There are, of course, many other approaches of QFD.

**What is the Best Approach ?**

Each approach has its pros and cons. What is the best approach ? The answer is not very straightforward. Each company has a unique set of employees, products, customers, systems, and so on, giving birth to unique circumstances and culture. The best solution is to develop a custom-fit QFD model to meet the particular needs of your company. Instead of cookbook approach to apply a tool like QFD, you may get bogged down.

The collective brainpower of the employees is the most valuable resources of any company. One of the major goals of using TQM philosophy is to enable employees to make meaningful contributions as a result of using their collective brain power. QFD allows this freedom.

**House of Quality**

The House of quality consists of 11 parts, each of which is briefly explained below :

1. This part of the house contains customer demands, described in customers' own words. These demands are subdivided in order to obtain the best possible description of customer preferences ; *i.e.,* the description, which best allows translation into technical terms. Information is obtained from traditional market survey and from team discussion.
2. This part of the house indicates the relative importance of customer demands. Information is obtained from market surveys based on statistical methods. It is the customer's voice which counts in QFD, and when it concerns the relative importance of customer demands. A suitable 5 point scale should be used to get relative importance.

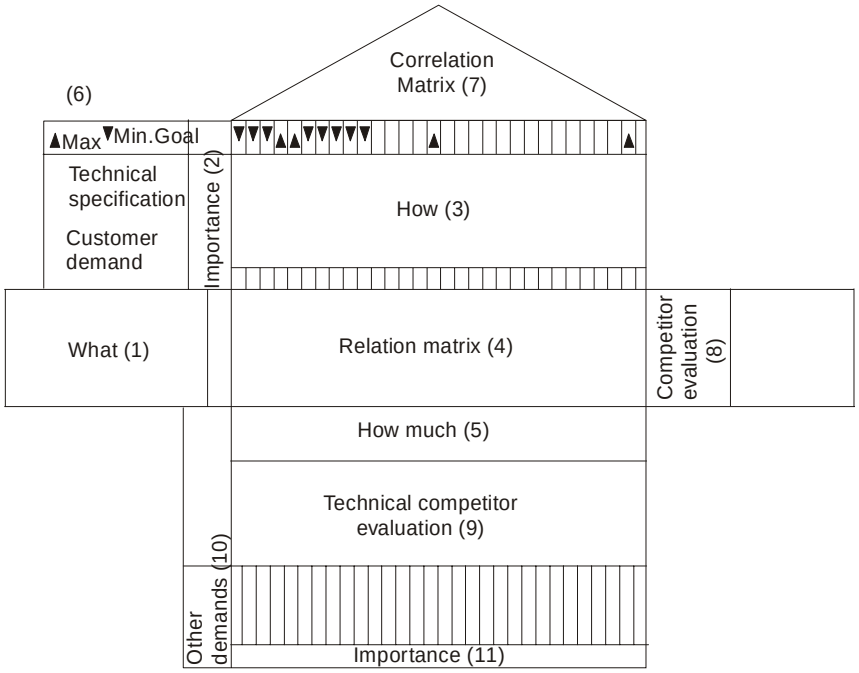



**Fig. 5.7.** House of quality

3. Here, customer demands are converted, or translated, into corresponding technical properties. To obtain as precise a description as possible, they are divided into primary, secondary and tertiary properties, and it is important that these properties are quantifiable.
4. Next, the degree of correlation between customer demands and the proposed technical solutions is established. A symbol at the point of intersection between a customer demand and a technical property shows the strength of this correlation. This symbol could, for example, be :

   ◎ or + + + = strong

   ○ or + + = medium

   △ or + = weak

   The absence of any symbol means that the particular property does not contribute anything to meeting the customer's demand. The symbols may vary, of course but the important thing in to achieve a consensus about the correlation between demand and properties.
5. The technical 'target' values and the measurement scales used for individual properties are to be shown.
6. Arrows, or other symbols (*e.g.* + or –), indicate the direction in which the engineer, for technical reasons, wishes to change the property concerned. A + or ↑ means that the engineer wants to increase the value of the property in question.

7. The roof, also called the correlation matrix, is one of the most important parts of the house of quality. This matrix shows the correlation between individual properties in the sense that, in some cases, increasing one property inevitably results in another having to be reduced. This is especially important in connection with product changes. Symbols are used to indicate the degree of correlation between individual properties, *e.g.*, :

   ◎ or SP = strong positive correlation

   ○ or P = positive correlation

   ⨯ or N = negative correlation

   # or SN = strong negative correlation

8. Competitor analysis also plays an important role in QFD. Under this part a customer-based profile of an existing product is compared with an evaluation of its strongest competitors. The scale used for this approximates to the other scales used in customer surveys as closely as possible. The fewer changes of scale used in a market survey the better.
9. Apart from customer evaluation of existing products and their strongest competitors, products are also evaluated technically. This takes place in where engineers evaluate products solely from a technical point of view, thus revealing any discrepancies between technical and marketing claims. Ideally, what customer regards as a better product. If not, there is either an error somewhere in the evaluation, or a conflict which can only be resolved at management level.
10. Apart from customer demands, ther house is also supplied with information about official requirements, standards, and so on. This happens in this section (10).
11. This is the key point, where the actual transformation of customer demand to technical specification takes place ; for each column (each technical property), customer rankings from (2) are multiplied by the numerical values attached to the correlation symbols. Three pluses (+ + +/◎) normally means multiplying by 9, 2/0 pulses multiplying by 3, and a single plus/multiplying by 1. The figures are then added together, giving a score for each technical property. This score is the direct equivalent of the evaluation of customer demand, allowing a direct evaluation of each property's relative importance.

    It can be seen from this that the house of quality involves several different aspects of product development. It does not provide instant solutions at the push of a button. Rather, it is an ingenious way of summarising all the relevant points involved in the development of new products.

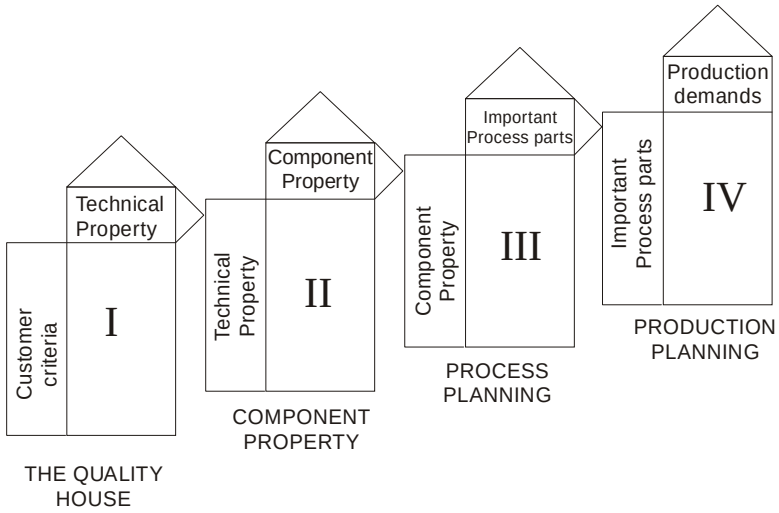



**Fig. 5.8.** The four most important house of QFD

The principle applied in the house of quality is the basis for the construction of other matrices, or houses, developed during the later stages of QFD. QFD in its most sophisticated form, consists of more than 30 matrices, involving everything from cost deployment to technology deployment. These basic houses are illustrated in above figure, which shows the deployment of customer's wishes all the way down to individual production demands.

The QFD methods is rapidly being adopted all over the world. Japanese firms are in the vanguard of this advance course, but many Western companies, *e.g.*, Philips, Electrolux, Digital, Hewlett-Packard, Volvo, Ford and G.M. are hard on their heels. The method is more or less standard in most big Japanese firms, some, such as Toyota, going so far as to require their subcontractors to employ it too.

The House of Quality can also be explained through a simple diagram as given below :

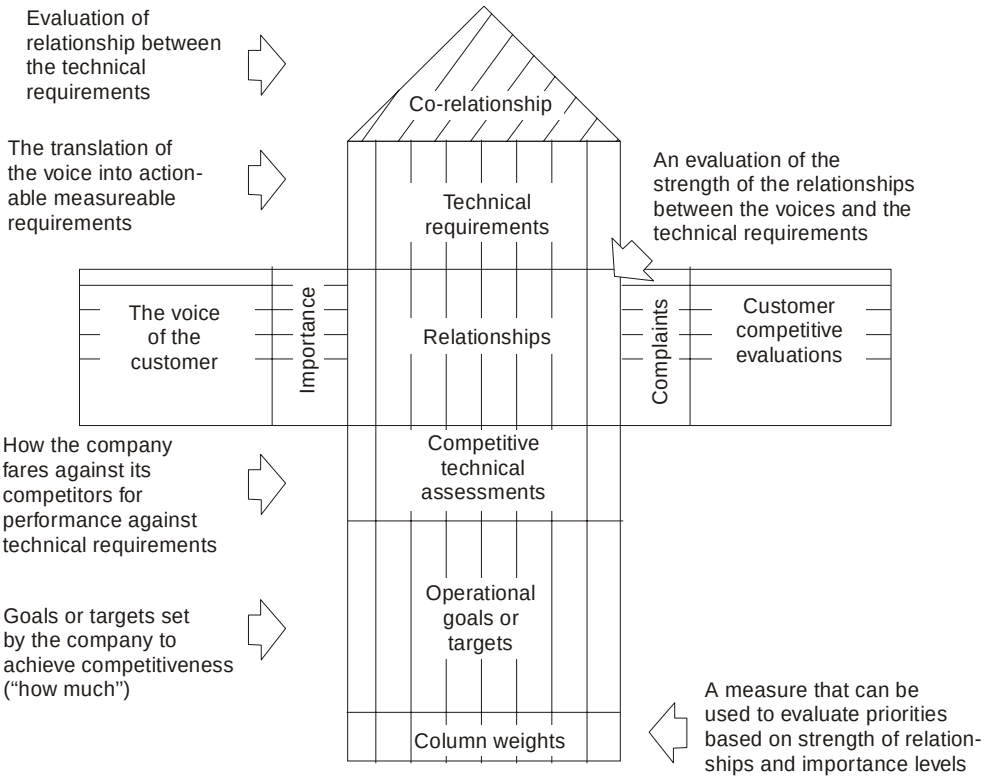



**Fig. 5.9.** The Basic ingredients of the QFD matrix : adding the technical information portion of the matrix.

**What are the Benefits of Using QFD ?**

The ultimate benefit of QFD for any company is its contribution to meeting and exceeding customer needs. Thus obtaining higher market share and profits. There are, however many other tangible benefits that participants of QFD team can experience. Some of them are listed here :

1. Products definition is firmer and takes place earlier in the new product develop-ment life cycle. This minimizes engineering changes and results in better quality.
2. QFD addresses major issues and compaints expressed by customers during the early stages of product definition. Hence, the number of complaints about and dissatisfaction with new products decreases with time. This benefit is seen after several product cycles.
3. Cross-functional wall break down with QFD since the team must address isues that affect all departments. Sub-optimization of resources in a company is mini-mized and communication between departments improves.

4. Team members develop a deeper understanding of customer needs and have the customer's voice as basis for making tradeoffs, resulting in superior decisions for the organization.
5. The analytic vigor of QFD causes streamlining or elimination of many internal processes that do not add value to the new product development process.
6. Customer needs are evaluated with respect to competitive products and services. This allows identification of the internal processes that need improvement to stay competitive.

   Documentation is an essential ingredient of QFD. Hence, one of its greatest benefits is that we build product intelligence. This documentation provides the following advantages :

   • It helps new engineers come aboard faster

   • Easily accessible documentation reduces chances of repeating mastakes of the past.
7. The accumulation of knowledge decreases the need of having someome with seniority lead the project. A senior leader contributes significantly to the success of the project. Also, under a TQM environment the team will feel empowered and possess the authority generally associated with having a senior leader.
8. QFD be beneficial in understanding and identifying a market niche where customer needs are not being met. This provides opportunities for introducing niche products.
9. QFD provides an excellent framework for cross-functional deployment of quality, cost and delivery.
10. QFD allows for quick changes, which is very important for the new product development process. It is possible to revise previous decisions when new information becomes available during product development, for example, if the competition introduces a new product or a state of the art technology becomes available. Detailed documentation keeps all information visible to the QFD team at all times.

All the above benefits result in a robust new product development cycle and minimize difficulties and problems. QFD is one of the best ways to introduce TQM to the marketing, design, and manufacturing environments.

The method has been around for so long in Japan that the first documented results are starting to come in. Yoji Akao, a Japanese professor, carried out a survey of the use of QFD in Japan in 1986. On the basis of this, he concluded that the five most important results of the use of QFD are as follows :

• It is much easier to define what design quality actually is.

• Quality problems are reduced at an earlier stage in the process.

• Product planning is much easier to carry out.

• Information about competitors is considerably improved.

• The barriers between departments are broken down.

These are considerable achievements, and there are good reasons for thinking that firms have got a lot more out of these methods that they let on. Many firms regard advances in the QFD area as company secrets, surrounded by the same sort of security measures as actual technological innovations.

**QFD Success Factors :**

- Accurate customer voice.
- Strong Management Commitment.
- A good consultant.
- A realistic time line.
- Regular project reviews.
- Milestone celebration to keep interest high and to develop a sense of closure.
- Sharing with other teams to facilitate deeper learning.

**Drawback of QFD**

Yoji Akao identified the following problems areas :

1. The quality house has a tendency to grow too big, thus becoming unwidely.
2. It is often difficult to get the right kind of information from customers. Customer demands are not something you just collect through simple market surveys. These surveys often require painstaking preliminary studies.
3. Many of the answer that customers give are difficult to categorize as demands.
4. It can be difficult to determine the connection between customer demands and technical properties.

Despite teething problems, there is no doubt that the QFD method is here to stay. It is imperative that more is done to spread its use in India. Both business schools and technical colleges must introduce relevant courses, and, above all, a wide range of relevant supplementary education in the field must be made available.

As far as Indian companies are concerned, companies who are practicing QFD are TVS Suzuki, Godrej, ABB, KEC, Thermax-Pune, Indalco-Nanjangudu, Essae Teroka and many more.

# 6 Statistical Process Control

**Introduction**

One important tool in statistical quality control is the shewhart control chart. The simplicity of the control chart, many engineers, production personnel and inspectors. Used for the entirely new point of view. Measured quality of "Manufactured Product is always subject to a certain amount of variation as a result of chance. Some stable "system, of chance causes" is inherent in any particular scheme of production and inspection. Variation within this stable pattern is inevitable. The reason for variation outside this stable pattern may be discovered and corrected.

The power of the shewhart technique lies in its ability to separate out these assignable causes of quality variation. Moreover, by identifying certain of the quality variation as inevitable chance variations, the control chart tells when to leave a process alone and thus prevents unnecessarily frequent adjustments that tend to increases the variability of the process rather than to decrease it. The control chart technique permits better decisions on engineering tolerances and better comparisions between alternative designs and between alternative production methods.

One of the best technical tools for improving product and service quality is statistical process control (SPC). There are seven basic techniques. The word statistical is somewhat of a misnomer the first four technique are not really statistical.

**7 QC TOOLS**

**What is QC problems — Solving**

In our daily work, there are times when these thoughts come to our mind, "Everytime I do this, job, there are many failures" or "This is tiring. Isn's there is a better way of doing it ?" In such situations, one may choose to improvise or solve it by oneself. But there are times when we cannot.

In such a situation, the people in the same workplace, facing a similar problem can gather, and the combined wisdom can result in good solutions. Instead of thinking alone, get the co-operation of others to solve it, and there will be improvements in work and increase in work efficiency. This will further result in workplace improvements and the development of the company. This also signals the employees, contribution to the company.

There are two ways of approaching problem-solving. These are the emergency measures and preventive measures. For example, when company — A lodges a claim, we will immediately apologise — emergency measures. In this instance the measures taken are of great importance. However, it is also important that we make certain that preventive measures are taken against the cause of the claim. The problems (theme) at the workplace are, for example, defective goods, occurrence of accidents, transitional production plans and sales plans, deformities and abnormalities.

At times, the main cause is hidden and we may still find that efficiency is low and there are a lot of dissatisfied customer.

Do not be content with this condition. Aim for even higher goals and keep taking up preventive measures. That is what QC Problem-Solving is all about.

**7 QC Tools.** The Samurai warrior had seven tools, such as a sword, helmet, bow guard and arrow and so on ; he would never venture any where without these tools, which he needed for protection and success. In a similar vein, the seven quality control tool are essential for to-day's workers, engineers, professionals, and managers.

1. **Pareto Diagram.** The Pareto principle was named after the Italian economist who had developed certain mathematical relationships of vital few and trivial many as applied to distribution of wealth. In studying the problems, it can be generally observed that 80% of the problems result from only 20% of the potential causes.

The primary purpose and use of pareto diagrams is to focus improvement efforts on the most important causes by identifying the vital few and trivial many causes.

The Pareto Chart Indicates the following :

1. What are the problems,
2. Which problem needs to be tackled on priority.
3. What percentage (%) of the total does each problem present.

**Areas of Application**

| | |
|---|---|
| Sales | — Customer complaints analysis, warranty costs, Market Share |
| Production | — Analysis of Non-conformance, machine and men Utilization |
| Maintenance | — Machine down time, break down, spares requirement. |
| Safety | — Injury types and causes |
| Finance | — Costs, etc. |

How to construct Pareto Diagram

1. Select the problem area (say customer complaints).

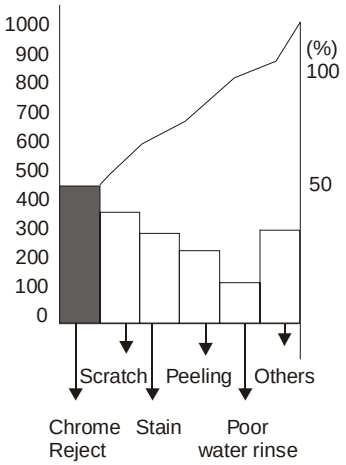
2. Decide the method and the period for data collection.

3. Arrange the data of the items in the descending order.4. Draw axis on graph with the scale of unit indicated.

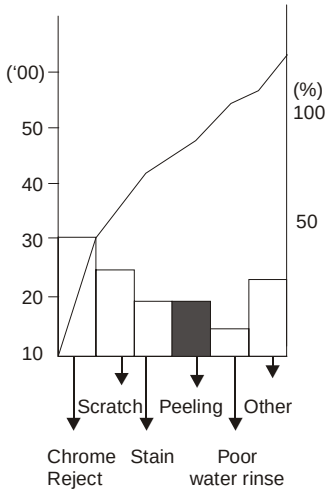
5. Draw the bar graph in the descending order.

Defects in Casting revelled After machining


Pareto Diagram (A)



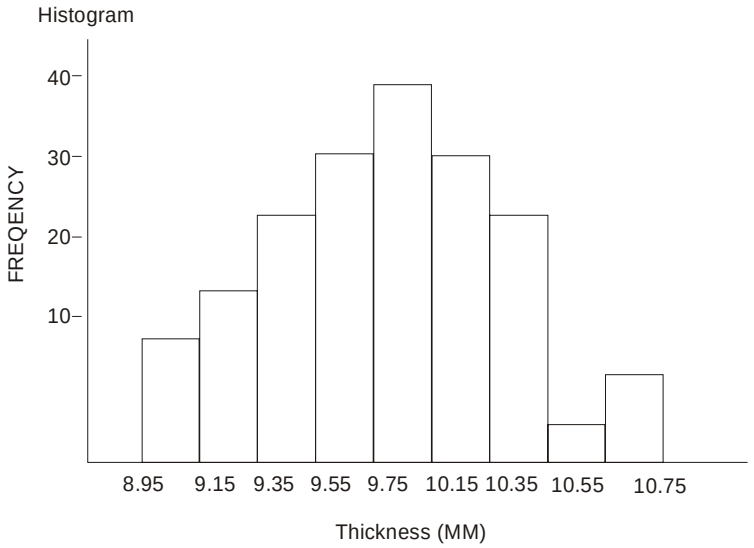

Pareto Diagram (B)




2. **Histogram.** A histogram is a bar graph which shows the frequency distribution of the data of a group about the central value. The histogram is an important diagnostic tool because it gives a "Birds's-eye-view" of the variation in a data set.

A histogram can be used for

1. Comparisons of process distribution before and after the improvement action (production, vendor performance, administration, purchase, inspection, etc.)
2. Comparison of different groups (production, vendor to vendor difference etc.)
3. Relationship with specification limits.




3. **Cause and Effect Diagram.** A cause and effect diagram (also known as Ishikawa diagram or fishbone diagram) in a pictorial representation of all possible causes which are supposed to influence an "effect" which is under consideration.

For every effect there are likely to be several causes. They can be classified under men, methods, materials, machines, policies, procedures, plant etc. These categories are only suggestions. You may use any category that emerges or helps people think creatively.

**Steps in Constructing a Causes and Effect Diagram**

Address to the member the problem or the "Effect" in question and ask the members what the possible causes could be adopt structured brain storming method (in brain storming encourage ideas, never criticise, allow to develop on other ideas, write all ideas on a flip chart or black board).

Start constructing the diagram. Write the effect or the problem on the right hand side in a rectangular box.

Cluster the causes of the effect under large heading and write against bones.

**Interpretation of C and E Diagram**

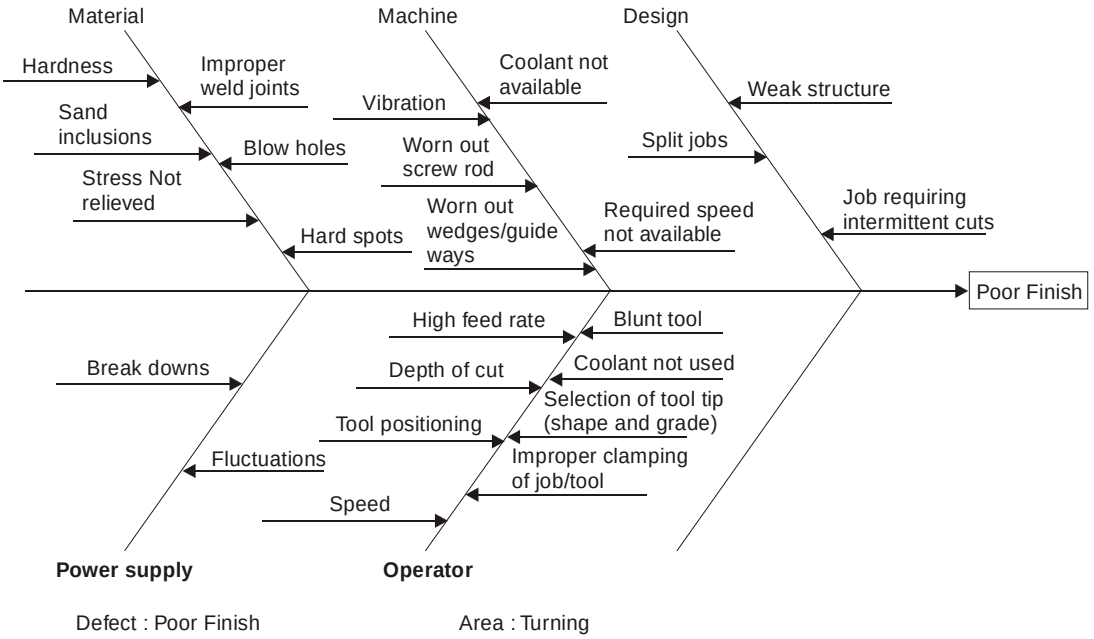
In order to find most basic cause of the problem

Look for causes that appear repeatedly.

Reach a team consensus

Gather data to determine the relative frequencies of the different causes.

**Cause and Effect Diagram**




4. **Check Sheet**

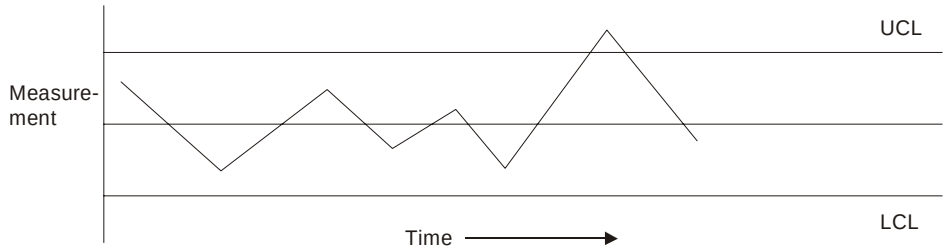
A check sheet is a data gathering format prepared in such a way that the data collection is simplified.

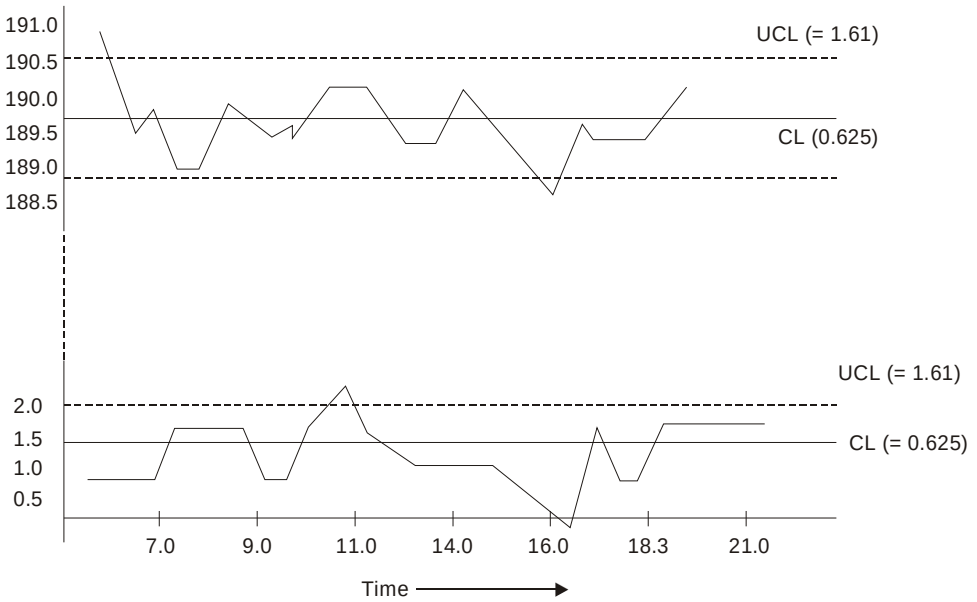
The check sheet preparation considers the representatives of the information to be recorded and simplifies the data that is to be actually recorded everytime to a mere check work. Check sheets are simply an easy to understand form used to answer the question "how often certain events are happening" ? It starts the process of translating into facts.

5. **Control Chart**

A control chart is a chart to examine whether a process is in stable condition or to ensure that process is maintained in stable condition.

The control limits are indicated by two line viz. Upper control limit and lower control limit. If the points are within the control limit lines, then the process is in stable condition. The fluctuation of the points within the control limit line results from common causes built into the process. However points outside the limits come from a special cause.






6. **Stratification**

Stratification is the technique of obtaining data in different groups based on segregated causes.

In general the poor quality is resulted due to the influence of multiple causes. To identify the principle cause of poor quality it is necessary to collect the data in different groups according to the different causes.

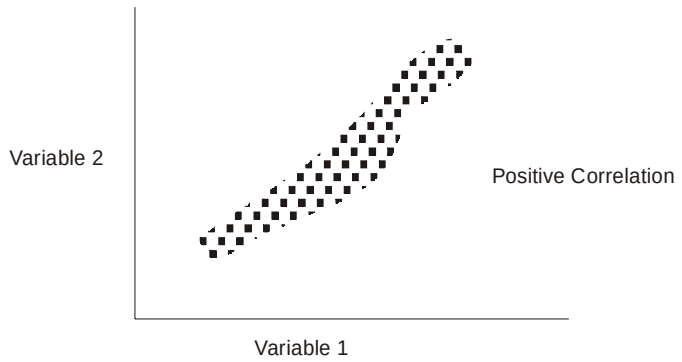
**Areas of Application**

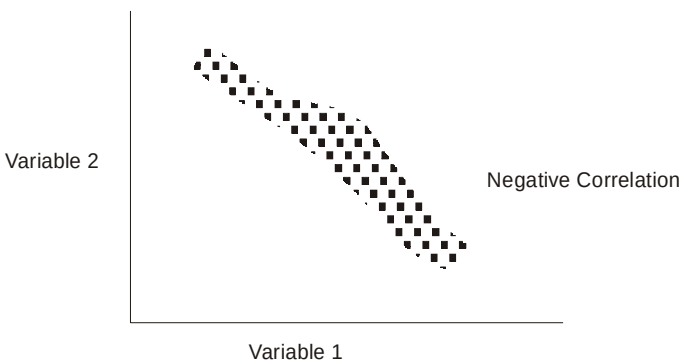
- Raw Material — Supplier wise stratification
  Batch wise stratification
- Production — Machine wise stratification
  operator wise stratification shift wise
- Finance — Stratification of income and expenditure as per different categories.
- Safety etc., — Accident type wise stratification.

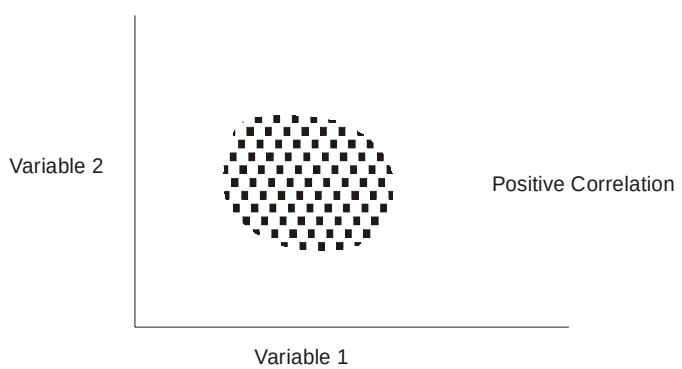
7. **Scatter Diagram**

Scatter diagram is a simple statistical tool to understand in a better way the relationship between two variables.

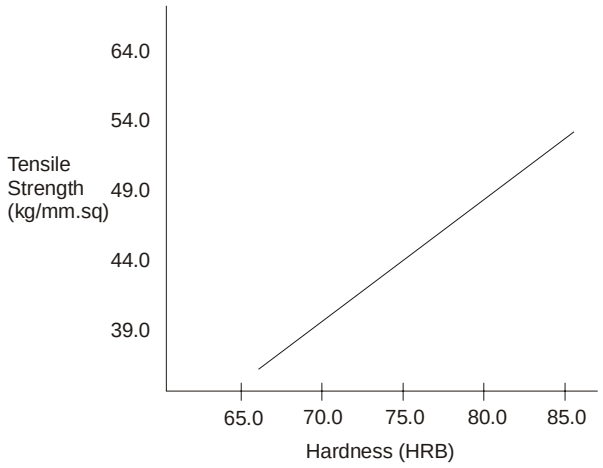
It makes clear whether a relationship exists between two variables and the strength of that relationship.


Variable 2
Positive Correlation
Variable 1


Variable 2
Negative Correlation
Variable 1


Variable 2
Positive Correlation
Variable 1

**Scatter Diagram**




**Causes of Variation in Quality**

The variation in the quality of the product in any manufacturing process in broadly classified into two classes : Chance causes and Assignable causes.

**Chance Causes**

The chance causes are those causes which are inherent in manufacturing process by virtue of operational and constructional features of the equipments involved in a manufacturing process. This is because of :

(1) Machine vibration

(2) Voltage fluctuation

(3) Temperature fluctuation

(4) Tool Chatter

(5) Composition variation of material etc.

The chance causes are very difficult to trace out, even though it is possible to trace out, it is not economical to eliminate. The chance causes results in only a minute amount of variation in process.

Variation in the chance causes is due to internal factors only. The general pattern of variation under the chance causes will follow a stable statistical distribution (Normal distribution).

**Assignable Causes**

These are the causes which creates an extraordinary variation in the product quality. Assignable causes variable can always be traced to a specific source. Assignable causes occur due to

(1) Lack of skill in operation

(2) Wrong maintenance practices

(3) New vendors

(4) Errors in setting jigs and fixtures

(5) Raw materials defect etc.

Variation due to these causes can be controlled before the defective items are produced. Any one assignable cause can result in a amount of variation in process. If the assignable causes are present, then system will not follow a stable statistical distribution.

## CONTROL CHARTS

**Definition.** A control chart is defined as a statistical tool used to detect the presence of assignable causes in any manufacturing systems and it will be influenced by the pure system of chance causes only.

Control charts are of two types : Variable control charts and Attribute control charts.

### Variable Control Charts

A variable control chart is one by which it is possible to measure the quality charactertistics of a product. The variable control charts are

(*i*) $\overline{X}$ — chart

(*ii*) R — chart

(*iii*) $\sigma$ — chart

### Attribute Control Chart

An attribute control chart is one in which it is not possible to measure the quality characteristics of a product *i.e.*, it is based on visual inspection only like good or bad, success or failure, accepted or rejected. The attribute control charts are !

(*i*) *p* — chart

(*ii*) *np* — chart

(*iii*) *c* — chart

(*iv*) *u* — chart

### Objectives of Control Charts

1. Control charts are used as one source of information to help whether an item or items should be released to the customer.
2. Control charts are used to decide when a normal pattern of variation occurs, the process should be left alone when an unstable pattern of variable occurs which indicates the presence of assignable causes it requires an action to eliminate it.
3. Control charts can be used to establish the product specification.
4. To provide a method of instructing to the operating and supervisory personnel (employees) in the technic of quality control.

Symbols or Notations

$\overline{X}$ : Mean of the sample

$\overline{X}^{\sigma}$: Standard deviation of the sample

$\overline{X}^{1}$ : Mean of the population or universe

$\sigma^{1}$ : Standard deviation of the population.

**Central Limit Theorem**

Irrespective of the shape of the distribution of the universe, the average value $\overline{X}$ of a sample size '*n*' ($\overline{X}_1$, $\overline{X}_2$. $\overline{X}_3$, ..... $\overline{X}_n$) drawn from the population will tend towards a normal distribution as *n* tends to infinity

Relationship between $\overline{R}$ and $\sigma^1$

$$\sigma^1 = \frac{\overline{R}}{d_2}$$

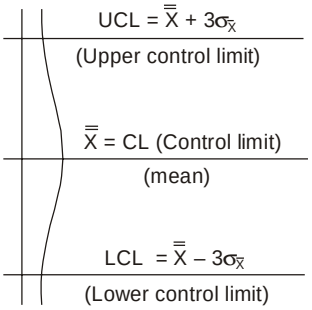
$\overline{R}$ = Mean Range

$d_2$ = Depends upon sample size from the tables.

Also $\sigma_{\overline{X}} = \frac{\sigma^1}{\sqrt{n}}$

where *n* = sample size.

Determination of control limits for $\overline{X}$ chart when the range is known :




$\overline{\overline{X}}$ = Mean of all the sample

$$UCL = \overline{\overline{X}} + 3\sigma_{\overline{X}}$$

$$= \overline{\overline{X}} + 3\frac{\sigma^1}{\sqrt{n}}$$

$$= \overline{\overline{X}} + 3\frac{\overline{R}}{d_2\sqrt{n}}$$

$$UCL = \overline{\overline{X}} + A_2\overline{R}$$

where $A_2 = \frac{3}{d_2\sqrt{n}}$

a constant which is obtained from the tables similarly

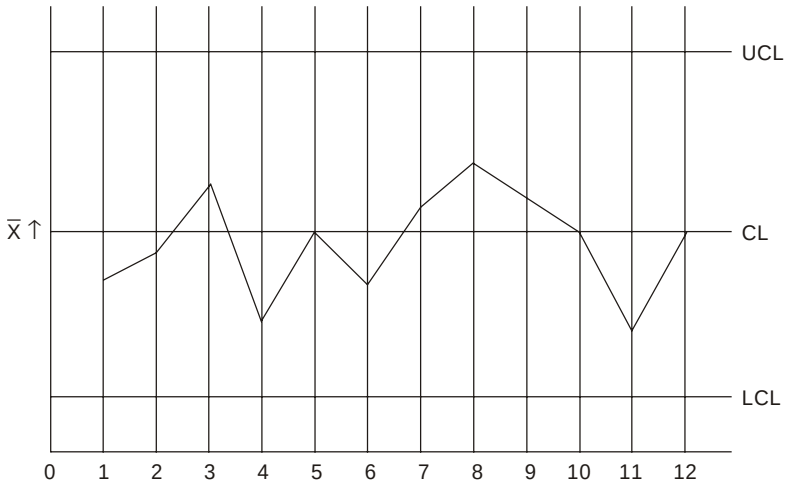
LCL = $\overline{\overline{X}} - A_2\overline{R}$.

**Interpretation of Control Charts**

After plotting the points on the $\overline{X}$ – R charts, it shows two possible states of control. They are 1. State of statistical control and 2. State of lack of control.

**State of Satistical Control**

A manufacturing process is said to be in a state of statistical control whenever it is operated upon by a pure system of chance causes. The display of points in the $\overline{X}$ chart and R chart will be distributed evenly and randomly around the central line and all the points should fall between the UCL and LCL which is shown below.




**State of Lack of Control**

A process is said to be in a state of lack of control when the state of statistical control does not hold good. In such a state we interpret the presence of assignable causes. The reasons for lack of control are

(*i*) points violating the control limits

(*ii*) run

(*iii*) Trend

(*iv*) Clustering

(*v*) Cycle pattern.

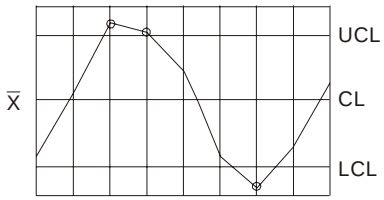
**Violation of Control Limits**

A chart is said to be a violation of control limits, whenever the points cross the UCL, or LCL or both, reasons for this are

(*a*) wrong setting

(*b*) Physical or environmental conditions

(*c*) Damage or mishandling of parts




Subgroup Number

**Run** (upward or downwards)

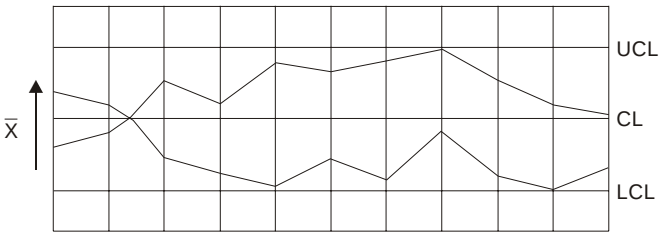
A run is said to occur whenever seven or more consecutive points occur on one side of the central line, run can be either above the central line or below the central line, which is shown below. Reasons for run are

(*a*) Different kinds of raw material used

(*b*) New worker

(*c*) Carelessness of operator

(*d*) Minor failure of machine parts




**Trend**

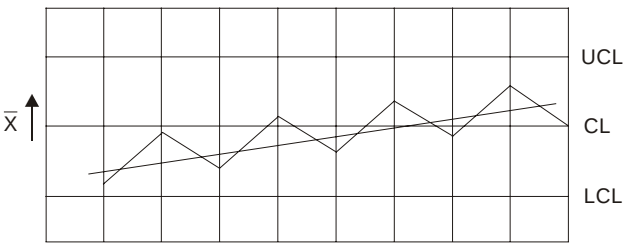
This refers to the gradual movement of points from one side of the central line to the other side of the central line. A trend is said to occur whenever a correlation exists between the points with reference to the sub-group number. Trends can be upward or downward. The reasons for this are

(*a*) Tool wear

(*b*) Worker fatigue

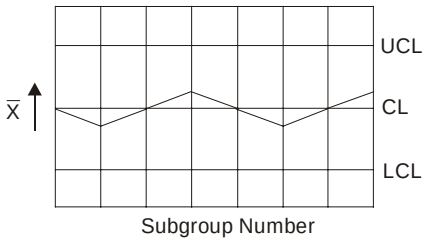
(*c*) Effect of temperature and humidity

(*d*) Accumulation of waste products.



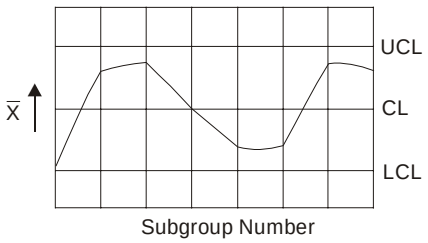
**Clustering**

This occurs whenever the points do not spread over the entire region between UCL and LCL. Instead the points exists very close to the central or on the central line, itself. Clustering indicates systematic differences within the subgroup.




**Cycle Pattern**

This pattern shows systematic differences between the sub-groups. Such things can occur whenver the assignable causes are present.




After plotting the $\overline{X}$ – R charts, the important points to be noted are :

1. Accept the process as confirming to the standards if the plotted points are randomly and evenly distributed around the central line and lie within the control limits of both the charts.
2. If any plotted points fall on or above the UCL on the range chart, it indicates an increase in the process dispersion which must be corrected by appropriate technical action.
3. If any plotted points fall on or above the UCL (*a*) LCL in the $\overline{X}$ chart, it indicates a shift in the process average. This should be corrected by taking suitable technical action.

**SOLVED PROBLEMS**

**Problem 1.** *Control charts for* $\overline{X}$ *and R are maintained on a certain dimension of a manufactured part which is specified as 2.05 ± 0.02 cms. Subgroup size is 4. The values of* $\overline{X}$ *and R are computed for each subgroup. After 20 subgroups.* $\Sigma\overline{X} = 41.283$ *and* $\Sigma R = 0.280$. *If the dimensions fall above USL, rework is required, if below LSL, the part must be scrapped. If the process is in statistical control and normally distributed.*

(*a*) *Determine the* $3\sigma$ *control limit for* $\overline{X}$ *and R chart.*

(*b*) *What is process capability*

(*c*) *What can you conclude regarding its ability to meet specifications*

(*d*) *Determine the percentage of scrap and rework*

(*e*) *What are your suggestions for improvement.*

**Solution.** $\Sigma\overline{X}$ = 41.283

$\Sigma R = 0.280$

*n* = Sample size = 4

Number of subgroup (K) = 20

The specification limits are 2.05 ± 0.02

Upper specification limit USL = 2.07 cm

Lower specification limit LSL = 2.03 cm

From the tables, for a subgroup size 4

$A_2 = 0.73$ $d_2 = 2.059$

$D_3 = 0.0$ $D_4 = 2.28$

$$\overline{\overline{X}} = \frac{\Sigma\overline{X}}{K} = \frac{41.283}{20} = 2.08415$$

$$\overline{R} = \frac{\Sigma R}{K} = \frac{0.28}{20} = 0.014$$

(*a*) Control limit for R – chart

$UCL = D_4\ \overline{R}$

$= 2.28 \times 0.014$

$= 0.0319$

$LCL = D_3\ \overline{R}$

$= 0 \times 0.014$

$= 0.0$

$CL = \overline{R} = 0.014$

Control limits for $\overline{X}$ – chart

$UCL = \overline{X} + A_2\overline{R}$

$= 2.06415 + 0.73 \times 0.014$

$= 2.07437$

$LCL = \overline{\overline{X}} - A_2\overline{R}$

$= 2.06415 - 0.73 \times 0.014$

$= 2.05393$

$CL = \overline{\overline{X}} = 2.06415$

(*b*) Process Capability

Since the process is in a state of statistical control.

$CL = \overline{\overline{X}} = \overline{X}^1 = 2.06415$

$$\sigma^1 = \frac{\bar{R}}{d_2} = \frac{0.014}{2.059}$$

$$= 0.00679$$

The process capability $= 6\sigma^1$

$$= 6 \times 0.00679$$

$$= 0.04074$$

(*c*) USL – LSL = 2.07 – 2.03 = 0.04

Since the $6\sigma^1$ is greater than USL – LSL, the process is not capable of meeting the specification limit *i.e.*, 0.0407 > 0.04.

**Note :**

1. If $6\sigma^1$ is less than (USL-LSL), the process is capable of meeting the specification. These should not be any rejection. If rejection occurs we can conclude that, the process is not centered properly.
2. If $6\sigma^1$ is equal to (USL-LSL), the process is exactly capable of meeting the specification limits. But tight tolerances are provided. We have to prefer a skilled operator for operating the machine.
3. If $6\sigma^1$ is greater than (USL-LSL), the process is not capable of meeting the specifications limits. The rejections are inevitable.

(*d*) UNTL (upper natural tolerance limit)

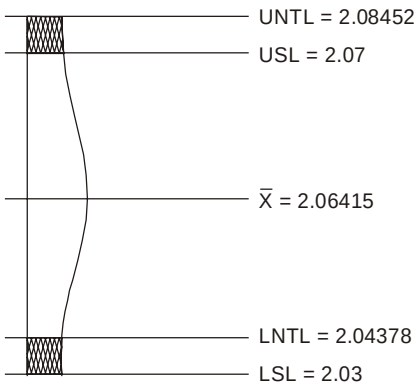
$$\text{UNTL} = \bar{X}^1 + 3\sigma^1$$

$$= 2.06415 + 3(0.00679) = 2.08452$$

LNTL (lower natural tolerance limit)

$$\text{LNTL} = \bar{X}^1 - 3\sigma^1 = 2.04378$$

USL = 2.07 LSL = 2.03




It is clear from the figure that the percentage of scrap is zero. The percentage of rework is

$$Z = \frac{\text{USL} - \bar{X}^1}{\sigma^1} = \frac{2.07 - 2.06415}{0.00679} = 0.86$$

The probability from the tables for Z = 0.86 is

0.8051 = 80.51%

Therefore the rework is 100 – 80.51 = 19.49%

(*e*) Since the percentage of rework is 19.49%, to minimise this, the possible ways are

(*i*) Change the process centre to the specification mean *i.e.*, from 2.06415 to 2.05. The calculations are shown below :

$$= \frac{2.07 - 2.05}{0.00679} = 2.94$$

Probability from Normal tabels is 0.9984

That is 1 – 0.9984 = 0.0016 = .16%

The percentage of rework is .16%

Since it is symmetric the percentage of scrap is also 0.16%

(*ii*) Widening the specification limits, for this we have to consult the design engineer, whether the product performs its function satisfactorily or not.

(*iii*) Decrease the dispersion, for this we have to prefer a skilled operator and very good raw material and a new machine, practically which is difficult.

(*iv*) Leave the process alone and do the 100% inspection.

(*v*) Calculate the cost of scrap ans rework, whichever is costly make it zero, accordingly change the process centre.

**Problem 2.** *Subgroup of 5 item each are taken from a manufacturing process at regular intervals. A certain quality characteristic is measured and* $\overline{X}$ *, R values computed for each subgroup. After 25 subgroup* $\Sigma\overline{X}$ *= 357.5,* $\Sigma R = 8.8$*. Assume that all the points are within the control limits on both the charts. The specifications are 14.4 ± 0.4.*

(*a*) *Compute the control limits for* $\overline{X}$ *and R chart.*

(*b*) *What is the process capability*

(*c*) *Determine the percentage of rejections if any.*

(*d*) *What can you conclude regarding its ability to meet the specifications.*

(*e*) *Suggest the possible scrap for improving the situation.*

**Solution.** K = 25

$n = 5$

$\Sigma\overline{X} = 357.5$

$\Sigma R = 8.8$

Specification limits = 14.4 ± .4

USL = 14.4 + .4

= 14.8

LSL = 14.4 – 0.4

= 14.0

$$\overline{\overline{X}} = \frac{\Sigma \overline{X}}{K} = \frac{357.5}{25} = 14.3$$

$$\overline{R} = \frac{\Sigma R}{K} = \frac{8.8}{25} = 0.354$$

From tables, for a subgroup size of 5.

$A_2 = 0.58$

$d_2 = 2.326$

$d_3 = 0.0$

$D_4 = 2.11$

Control limits for R-chart

$$UCL = D_4 \overline{R} = 2.11 \times 0.352$$
$$= 0.7427$$

$$LCL = D_3 \overline{R} = 0.0 \times 0.352$$
$$= 0.0$$

$$CL = \overline{R} = 0.352$$

(*a*) Control limits for $\overline{X}$-chart.

$$UCL = \overline{\overline{X}} + A_2 \overline{R}$$
$$= 14.3 + 0.58 \times 0.352$$
$$= 14.5041$$

$$LCL = \overline{\overline{X}} - A_2 \overline{R}$$
$$= 14.3 - 0.58 \times 0.352$$
$$= 14.0958$$

$$CL = \overline{\overline{X}} = 14.3$$

(*b*) Since the process is in a state of statistical control

$$\overline{\overline{X}} = \overline{X}^1 = 14.3$$

$$\sigma^1 = \frac{\overline{R}}{d_2} = \frac{0.352}{2.326}$$

$$= 0.15133$$

Process capability $= 6\sigma^1$

$$= 6 \times 0.15133$$
$$= 0.907$$

(*c*) USL – LSL = 14.8 –14.0

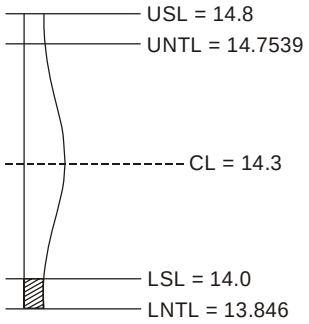
$$= 0.8$$

Since $6\sigma^1 >$ (USL – LSL), the process is not capable of meeting the specification limits. *i.e.,* 0.907 > 0.8. Rejections are inevitable

$$UNTL = \overline{X}^1 + 3\sigma^1$$
$$= 14.3 + 3 \times 0.15133$$
$$= 14.7539$$

$$LNTL = \overline{X}^1 - 3\sigma^1$$
$$= 14.3 - 3 \times 0.15133$$
$$= 13.846$$

$$CL = \overline{X}^1 = 14.3$$
$$USL = 14.8$$
$$LSL = 14.0$$




Percentage of rejection

$$Z = \frac{X - \overline{X}^1}{\sigma^1} = \frac{14.0 - 14.3}{0.15133}$$
$$= -1.98$$

Probability from tables = 0.0239

Percentage of rejection = 2.39%

(*e*) To minimise the percentage of rejection the possible ways are : change the process centre to the specification mean *i.e.*, 14.3 to 14.4. The calculations are shown below.

$$Z = \frac{14.0 - 14.4}{0.15133} = -2.64$$

The probability is 0.0041

Percentage of rejection = 0.41%

Since it is symmetric the total percentage of rejections are 0.41 × 2 = 0.82%.

**Problem 3.** *A control chart has been used to monitor a certain characteristic. The process is sampled in a subgroup size of 4 at an interval of 2 hours.* $\overline{X}$ *-chart has* $3\sigma$ *control limits of 121 and 129 with the target value of* $\overline{X}^1 = 125$.

(*a*) *If the product is sold to a user who has a specification of* $127 \pm 8$. *What percentage of the product will not meet the specification assuming normally distributed output.*

(*b*) *If the target value of the process can be shifted without effect on the process standard deviation, what target value would minimise the amount of product being outside the specifications.*

(*c*) *At this new target value what percentage of the product will not meet the specification requirements.*

**Solution.** Subgroup size '*n*' = 4

UCL = 129

LCL = 121

$$CL = \overline{\overline{X}} = \overline{X}^1 = 125.$$

Specification limits = 127 ± 8

USL = 135

LSL = 119

from tables, for a subgroup size of 4.

$A_2 = 0.73$

$d_2 = 2.059$

$D_3 = 0.0$

$D_4 = 2.28$

$$UCL = \overline{\overline{X}} + A_2 \overline{R}$$

$$\overline{R} = \frac{UCL - \overline{\overline{X}}}{A_2}$$

$$= \frac{129 - 125}{0.73}$$

$$= 5.48$$

$$\sigma^1 = \frac{\overline{R}}{d_2} = \frac{5.48}{2.059} = 2.66$$

Process capability = $6\sigma^1$

$= 6 \times 2.66$

$= 15.96$

USL – LSL = 16

Since $6\sigma^1$ < (USL – LSL), the process in capable of meeting the specification limits.

$$UNTL = \overline{X}^1 + 3\sigma^1$$

$$= 125 + 3 \times 2.66$$

$$= 132.98$$

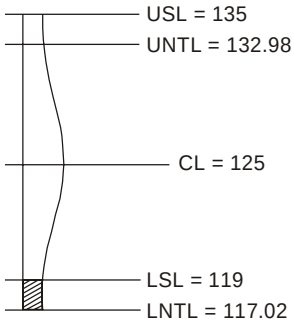
$$LNTL = \overline{X}^1 - 3\sigma^1$$

$$= 125 - 3 \times 2.66$$

$$= 117.02$$

USL = 135

LSL = 119




Percentage of rejection

$$Z = \frac{119 - 125}{2.66} = \frac{LSL - CL}{\sigma^1}$$

$$= -2.25$$

(*a*) Probability from table 0.0122 = 1.22%

(*b*) In order to minimise the percentage of rejection change the process target from 125 to 127.

The percentage of rejection

$$Z = \frac{119 - 127}{2.66}$$

$$= -3.00$$

Probability from tables

$$= 0.00135$$

Percentage of rejection = 0.135

(*c*) Since it is symmetric the total percentage of rejection

$$= 0.135 \times 2 = 0.27\%.$$

**Problem 4.** *Subgroup of 4 items each are taken from a manufacturing process at regular intervals. Å certain quality characteristic is measured and* $\overline{X}$ *, R values are computed for each subgroup. After 25 subgroup.* $\Sigma\overline{X} = 15350$, $\Sigma R = 411.4$.

(*a*) *Compute the control limits for* $\overline{X}$ *, R chart.*

(*b*) *Assume all the points are falling within the control limits on both the charts. The specification limits are 610 ± 15. If the quality characteristic is normally distributed what percentage of product would fail to meet the specifications.*

(*c*) *Any product that falls below L will be scrapped and above U must be reworked. It is suggested that the process can be centered at a level so that not more than 0.1% of the product will be scrapped. What should be the aimed value of* $\overline{X}^1$ *to make the scrap exactly 0.1%.*

(*d*) *What percentage of rework can be expected with this centering.*

**Solution.** $\Sigma \overline{X} = 15350$

SR = 411.4

K = 25

$n = 4$

$$\overline{\overline{X}} = \frac{\Sigma \overline{X}}{K} = \frac{15350}{25} = 614$$

$$\overline{R} = \frac{\Sigma \overline{R}}{K} = \frac{411.4}{25} = 16.456$$

From tables, for a subgroup size of 4

$A_2 = 0.73$

$d_2 = 2.059$

$D_3 = 0.0$

$D_4 = 2.28$

Control limits for $\overline{X}$-chart

$$USL = \overline{\overline{X}} + A_2 \overline{R}$$
$$= 614 + 0.73 \times 16.456$$
$$= 626.012$$

$$LCL = \overline{\overline{X}} - A_2 \overline{R}$$
$$= 601.987$$

$$CL = \overline{\overline{X}} = 614$$

Control limits for R-chart

$$UCL = D_4 \overline{R}$$
$$= 2.28 \times 16.456$$
$$= 37.5196$$

$$LCL = D_3 \overline{R}$$
$$= 0 \times 16.450$$
$$= 0.0$$

$$CL = \overline{R} = 16.456$$

(*b*) Specification limits are

$610 \pm 15$

USL = 625

LCL = 595

$$\overline{X}^1 = \overline{\overline{X}} = 614$$

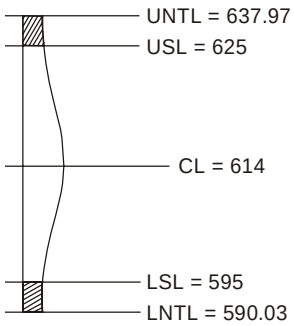
$$\sigma^1 = \frac{\overline{R}}{d_2} = \frac{16.456}{2.059} = 7.99$$

$$\text{UNTL} = \overline{X}^1 + 3\sigma^1$$
$$= 614 + 3 \times 7.99$$
$$= 637.97$$

$$\text{LNTL} = \overline{X}^1 - 3\sigma^1$$
$$= 590.03$$

$$\text{USL} - \text{LSL} = 30$$

$$\text{Process capability} = 6\sigma^1$$
$$= 6 \times 7.99$$
$$= 47.94$$




Percentage of scrap

Since $6\sigma^1 > (\text{USL} - \text{LSL})$, the process is not capable of meeting the specification limits. Rejections are inevitable.

Percentage of scrap

$$Z = \frac{X - \overline{X}^1}{\sigma^1}$$

$$= \frac{595 - 614}{7.99} = -2.37$$

Probability from tables = 0.0089 = 0.89%

Percentage of rework

$$Z = \frac{X - \overline{X}^1}{\sigma^1}$$

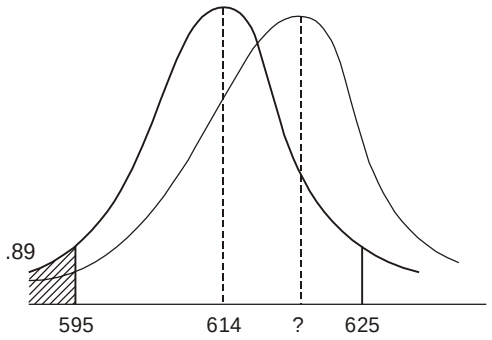
$$= \frac{625 - 614}{7.99} = 1.37$$

Probability from tables

$$= 0.947 = 91.47\%$$

$$\text{Rework} = 100 - 91.47\%$$
$$= 8.53\%$$

For area to be 0.1% towards left the value of $Z = -3$.




$$-3 = \frac{595 - \overline{X}^{1}\ \text{new}}{7.99}$$

$\overline{X}^{1}$ new = 595 + 3 × 7.99 = 618.97

The percentage of rework

$$Z = \frac{625 - 618.97}{7.99} = 0.75$$

Probability = 0.7734

= 77.34%

Percentage of rework = 100 – 77.34

= 22.66%.

**Problem 5.** *A product has a specification of 48 ± 3 on one of its quality characteristic. Assuming a normal distribution.*

(*a*) *What would be process average and std. deviation if the specifications have to fall exactly at*

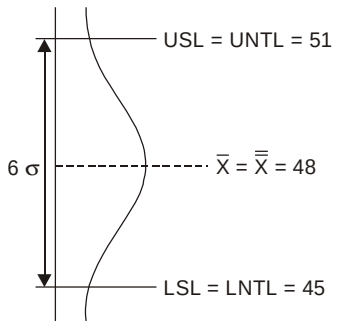
$\overline{X}^{1} \pm 3\sigma^{1}$

(*b*) *Assuming the process capabilities meet the values established in. What would be the 3σ control limits for monitoring this characteristic for* $\overline{X}$ *, R chart using a sub-group size of 4.*

**Solution.** Specification limits = 48 ± 3

USL = 51 = UNTL

LSL = 45 = LNTL



(*a*) $\overline{X}^1 \pm 3\sigma^1 = 48 \pm 3$

$\overline{X}^1 = 48$

$3\sigma^1 = 3$

$\sigma^1 = 1$

Also the process is exactly cabable of meeting the specification limit.

$USL - LSL = 6\sigma^1$, $\sigma^1 = 1$

The process average = $48 = \overline{\overline{X}}$

(*b*) $n = 4$

for a subgroup size of 4, from tables

$A_2 = 0.73$

$d_2 = 2.059$

$D_3 = 0.0$

$D_4 = 2.28$

$$\sigma^1 = \frac{\overline{R}}{d_2} \quad \Rightarrow \quad \overline{R} = \sigma^1 \times d_2$$

$= 1 \times 2.059$

$= 2.059$

Control limits for R-chart

$UCL = D_4 \overline{R} = 2.28 \times 2.059$

$= 4.6945$

$LCL = D_3 \overline{R} = 0.0$

$CL = \overline{R} = 2.059$

Control limits for $\overline{X}$-chart

$USL = \overline{\overline{X}} + A_2 \overline{R}$

$= 48 + 0.73 \times 2.059$

$= 49.50$

$LCL = \overline{\overline{X}} - A_2 \overline{R}$

$= 48 - 0.73 \times 2.059$

$= 46.49$

$CL = \overline{\overline{X}} = 48.$

**Problem 6.** *A certain product has a specification of 120 ± 5. At present the estimated process average is 120 and* $\sigma^1 = 1.5$.

(*a*) *Compute the* $3\sigma$ *limits for* $\overline{X}$ *, R chart based on a subgroup size of 4.*

(*b*) *If there is a shift in the process average by 2%, what percentage of product will fail to meet the specifications.*

(*c*) *What is the probability of detecting the shift by* $\overline{X}$ *-chart.*

**Solution.** Specification limits = 120 ± 5

USL = 125

LSL = 115

$\overline{X}^1 = 120, \sigma^1 = 1.5$

$\overline{X}^1 = \overline{\overline{X}}$

$n = 4$

From tables for a subgroup size of 4.

$A_2 = 0.73$

$d_2 = 2.059$

$D_3 = 0.0$

$D_4 = 2.28$

$$\sigma^1 = \frac{\overline{R}}{d_2}$$

$$\overline{R} = \sigma^1 \times d_2 = 1.5 \times 2.059$$
$$= 3.0885$$

Control limits for R-chart

$$UCL = D_4 \overline{R}$$
$$= 2.28 \times 3.0885$$
$$= 7.04178$$

$$LCL = D_3 \overline{R}$$
$$= 0.0$$

$$CL = \overline{R} = 3.0885$$

Control limits for $\overline{X}$-chart

$$UCL = \overline{\overline{X}} + A_2 \overline{R}$$
$$= 120 + 0.73 \times 3.0885$$
$$= 122.2546$$

$$LCL = \overline{\overline{X}} - A_2 \overline{R}$$
$$= 120 - 0.73 \times 3.0885$$
$$= 117.7454$$

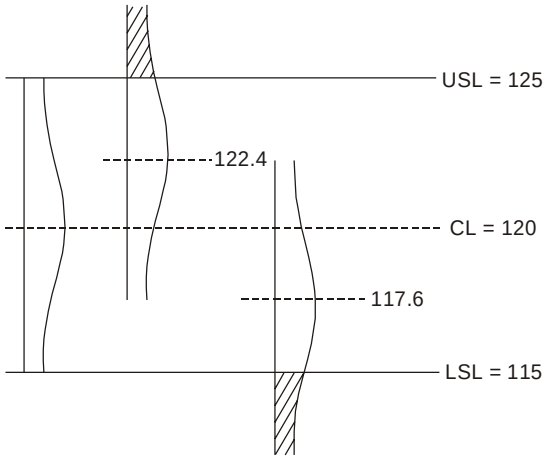
$$CL = \overline{\overline{X}} = 120$$

(*b*) Shift in the process average = ± 2%

$$\overline{X}^1 \text{ new} = 120 \times 1.02 \qquad (+ 2\%)$$
$$= 122.4$$

$\overline{X}^1$ new = 120 × 0.98 (– 2%)

= 117.6




Below $Z = \frac{115 - 117.6}{1.5}$

$= -1.73$

probability = 0.0418

= 4.18%

Above : $Z = \frac{125 - 122.4}{1.5}$

$= 1.73$

Probability = 0.9582

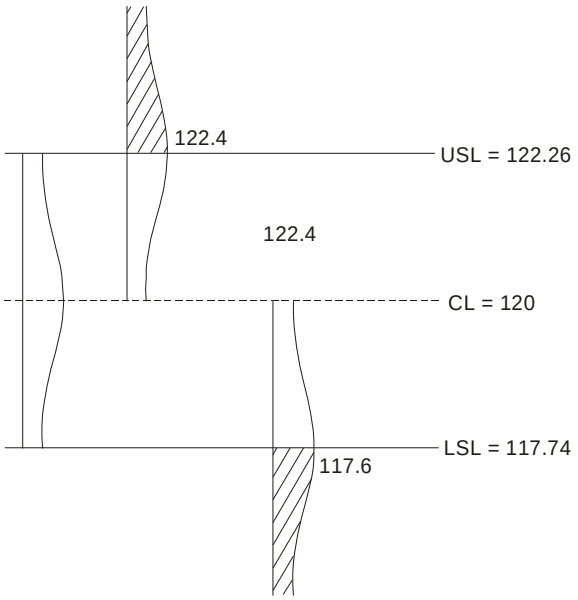
= 95.82%

*i.e.*, 100 – 95.82 = 4.18%

(*c*) With respect to $\overline{X}$-chart

$$\sigma_{\overline{X}} = \frac{\sigma^1}{\sqrt{n}} = \frac{1.5}{\sqrt{4}}$$

$\sigma_n = 0.75$




Below $Z = \dfrac{117.74 - 117.6}{0.75} = 0.1866$

Probability = 0.5714

= 57.14%

Above $Z = \dfrac{122.26 - 122.4}{0.75} = -0.1866$

$= -0.1866$

Probability = 0.4287 = 42.86%

*i.e.,* = 100 – 42.86 = 57.14%.

**Problem 7.** *For a certain characteristic of a product of sample size 2 after 25 subgroups.* $\Sigma R = 0.81$ *and* $\Sigma \overline{X} = 27.635$. *The specification limits are* $1.12 \pm 0.087$.

(*a*) *In the process harmonised to the specifications.*

(*b*) *What are the rejections percentage if any*

(*c*) *Is the process capable of meeting the specifications.*

(*d*) *Harmonise the process to the specifications and obtain the control limits for* $\overline{X}$ *-R chart after harmonising the process to specification.*

**Solution.** $n = 2$, K = 25, $\Sigma R = 0.81$, $\Sigma \overline{X} = 27.635$.

Specification limits = $1.12 \pm 0.087$

USL = 1.207

LSL = 1.033

From tables for a subgroup size of 2.

$d_2 = 1.128$

$A_2 = 1.88$

$D_3 = 0.0$

$D_4 = 3.27$

$$\overline{\overline{X}} = \frac{\Sigma \overline{X}}{K} = \frac{27.635}{25} = 1.1054 = \overline{X}^1$$

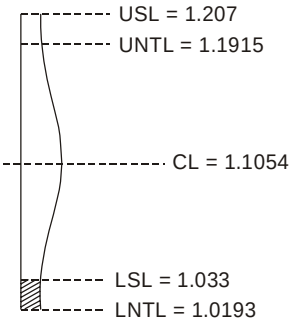
$$\overline{R} = \frac{\Sigma R}{K} = \frac{0.81}{25} = 0.0324$$

$$\sigma^1 = \frac{\overline{R}}{d_2} = \frac{0.0324}{1.128} = 0.0287$$

$$UNTL = \overline{X}^1 + 3\sigma^1$$
$$= 1.1054 + 3 \times 0.0287$$
$$= 1.1915$$

$$LNTL = \overline{X}^1 - 3\sigma^1$$
$$= 1.1054 - 3 \times 0.0287$$
$$= 1.0193$$

$$CL = \overline{X}^1 = 1.1054$$




(*a*) It is clear from the figure that the process is not harmonised with the specifications (LNTL is below LSL) (For a process to be harmonised, LNTL, UNTL must fall well within the USL and LSL or must be just equal to them.)

(*b*) The percentage of rejections

$$Z = \frac{1.033 - 1.1054}{0.0287} = -2.5$$

Probability = 0.0059

Percentage of rejection = 0.59%

(*c*) USL – LSL = 1.207 – 1.033
= 0.174

$$6\sigma^1 = 6 \times 0.0287$$
$$= 0.1722$$

$6\sigma^1$ (USL-LSL) *i.e.,* 0.1722 < 0.174 the process is capable of meeting the specification limits.

(*d*) In order to harmonise the process to the specifications change the process centre to the specifications mean

*i.e.,* $\overline{\overline{X}} = 1.12$

The control limits for $\overline{X}$-chart

$$UCL = \overline{\overline{X}} + A_2 \overline{R}$$
$$= 1.12 + 1.88 \times 0.0324$$
$$= 1.1809$$

$$LCL = \overline{\overline{X}} - A_2 \overline{R}$$
$$= 1.059$$

$$CL = \overline{\overline{X}} = 1.12$$

Control limits for R-chart

$$UCL = D_4 \overline{R} = 3.27 \times 0.0324$$
$$= 0.1059$$

$$LCL = D_3 \overline{R} = 0.0 \times 0.0324 = 0.0$$

$$CL = \overline{R} = 0.0324.$$

**Problem 8.** *Specifications on a certain product characteristics are given as 155 ± 20. The subgroup size is 5. After 50 subgroup* $\Sigma \overline{X} = 7660$ *and* $\Sigma R = 880$. *Assume the process is in control and normally distributed.*

(*a*) *Determine the control limits for* $\overline{X}$ *-R chart.*

(*b*) *What portion of non confirmed items are produced.*

(*c*) *If the process is recentered exactly at 155. What portion of non-confirmed item is being produced.*

**Solution.** Specification limits are 155 ± 20

USL = 175

LSL = 135

$n = 5$

K = 50

$$\Sigma \overline{X} = 7660, \ \overline{\overline{X}} = \frac{\Sigma \overline{X}}{K} = \frac{7660}{50} = 153.2$$

$$\Sigma R = 880, \ \overline{R} = \frac{\Sigma R}{K} = \frac{880}{50} = 17.6$$

Control limits for $\overline{X}$-chart

From tables for a subgroup size of 5.

$A_2 = 0.58$

$d_2 = 2.326$

$D_3 = 0.0$

$D_4 = 2.11$

$UCL = \overline{\overline{X}} + A_2 \overline{R}$

$= 153.2 + 0.58 \times 17.6$

$= 163.408$

$LCL = \overline{\overline{X}} - A_2 \overline{R}$

$= 153.2 - 0.58 \times 17.6$

$= 142.992$

$CL = \overline{\overline{X}} = 153.2$

Control limits for R-chart

$UCL = D_4 \overline{R}$

$= 2.11 \times 17.6$

$= 37.136$

$LCL = D_3 \overline{R} = 0$

$CL = UCL = \overline{R} = 17.6$

Since the process is in the state of statistical control.

$\overline{\overline{X}} = \overline{X}^1 = 153.2$

$$\sigma^1 = \frac{\overline{R}}{d_2} = \frac{17.6}{2.326} = 7.566$$

$UNTL = \overline{X}^1 + 3\sigma^1$

$= 153.2 + 3 \times 7.566$

$= 175.898$

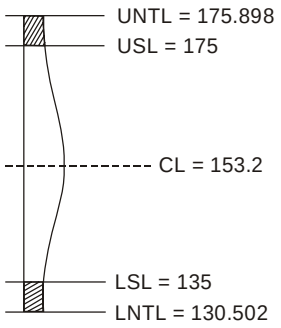
$LNTL = \overline{X}^1 - 3\sigma^1$

$= 153.2 - 3 \times 7.566$

$= 130.502$

$USL = 175$

$LSL = 135$




The percentage of product which does not confirm with the specification is shown by shaded area

$$Z = \frac{135 - 153.2}{7.566}$$

$= -2.4$

Probability $= 0.0081$

$= 0.81\%$

and $Z = \frac{175 - 153.2}{7.566}$

$= 2.88$

$= 0.9980$

$= 99.8\%$

$100 - 99.8 = 0.2\%$

Total portion of the product which does not confirm with the specifications

$= 0.81 + 0.2$

$= 1.01\%$

(*c*) If the centre is 155, then

$$Z = \frac{135 - 155}{7.566} = -2.64$$

Probability $= 0.0041$

$= 0.41\%$

$$Z = \frac{175 - 155}{7.566} = 2.64$$

Probability $= 0.9959 = 99.59\%$

Rejections $= 100 - 99.59 = 0.41\%$

The total rejections will be $= 0.41 + 0.41$

$= 0.82\%$.

**Problem 9.** *The following are the* $\overline{X}$ *– R values of 20 subgroup of 5 readings.*

| *S.G. No.* | $\overline{X}$ | *R* |
|---|---|---|
| *1* | *34* | *4* |
| *2* | *31.6* | *2* |
| *3* | *30.8* | *3* |
| *4* | *33.8* | *5* |
| *5* | *31.6* | *2* |
| *6* | *33.0* | *5* |
| *7* | *28.2* | *13* |
| *8* | *33.8* | *19* |
| *9* | *37.8* | *6* |
| *10* | *35.8* | *4* |
| *11* | *38.4* | *4* |
| *12* | *34.0* | *14* |
| *13* | *35.0* | *4* |
| *14* | *33.8* | *7* |
| *15* | *31.6* | *5* |
| *16* | *33.0* | *7* |
| *17* | *32.6* | *3* |
| *18* | *31.8* | *9* |
| *19* | *35.6* | *6* |
| *20* | *33.0* | *4* |

$\Sigma\overline{X}$ = 669.2

$\Sigma R$ = *126.0*

(*a*) *Determine the control limits for* $\overline{X}$ *and R chart.*

(*b*) *Construct the* $\overline{X}$ *and R chart and interpretate the result.*

(*c*) *What is process capability.*

(*d*) *Does it appear that the process is capable of meeting the specification limits.*

(*e*) *Determine the percentage age of rejections if any*
*The specification limits are = 33 ± 5.*

**Solution.**

$\Sigma\overline{X}$ = 669.2

$\Sigma R$ = 126.0

$$\overline{\overline{X}} = \frac{\Sigma\overline{X}}{K} = \frac{669.2}{20} = 33.46$$

$$\overline{R} = \frac{\Sigma R}{K} = \frac{126}{20} = 63$$

For a subgroup size of 5, from tables

$A_2 = 0.58$

$d_2 = 2.326$

$D_3 = 0.0$

$D_4 = 2.11$

Control limits for R-chart

$$UCL = D_4 \bar{R} = 2.11 \times 6.3$$

$$LCL = D_3 \bar{R} = 0.0$$

$$CL = \bar{R} = 6.3$$

It is seen from the data that two subgroups are crossing the UCL which indicates the presence of asignable causes. So the homogenisation is necessary.

$$\bar{R}_1 = \frac{126.0 - 14 - 19}{20 - 2}$$

$$= \frac{93}{18} = 5.17$$

Again control limits for R-chart

$$UCL = D_4 \bar{R}_1 = 2.11 \times 5.17$$

$$LCL = D_3 \bar{R}_1 = 0.0$$

$$CL = \bar{R}_1 = 5.17$$

Again one more subgroup is crossing the UCL

$$\bar{R}_2 = \frac{126 - 14 - 19 - 13}{20 - 3} = 4.7$$

Again control limits for R-chart

$$UCL = D_4 \bar{R}_2 = 2.11 \times 4.7 = 9.917$$

$$LCL = D_3 \bar{R}_2 = 0 \times 4.7 = 0.0$$

$$CL = \bar{R}_2 = 4.7$$

Now all the points are falling with in the control limits. The final values are UCL = 9.917

LCL = 0.0

CL = 4.7

Control limits for $\bar{X}$-chart

$$UCL = \bar{\bar{X}} + A_2 \bar{R}_2$$

$$= 33.46 + 0.58 \times 4.7$$

$$= 36.186$$

$$LCL = \overline{\overline{X}} - A_2\overline{R}_2$$
$$= 30.734$$

$$CL = \overline{\overline{X}} = 33.46$$

It is seen from the data that three subgroups are crossing the control limits. Which indicates the presence of assignable causes, so homogenisation is necessary.

$$\overline{\overline{X}}_1 = \frac{669.2 = 37.8 - 38.4 - 28.2}{20 - 3}$$

$$= 33.22$$

Again control limits for $\overline{X}$-chart

$$UCL = \overline{\overline{X}}_1 + A_2\overline{R}_2$$
$$= 33.22 + 0.58 \times 4.7$$
$$= 35.946$$

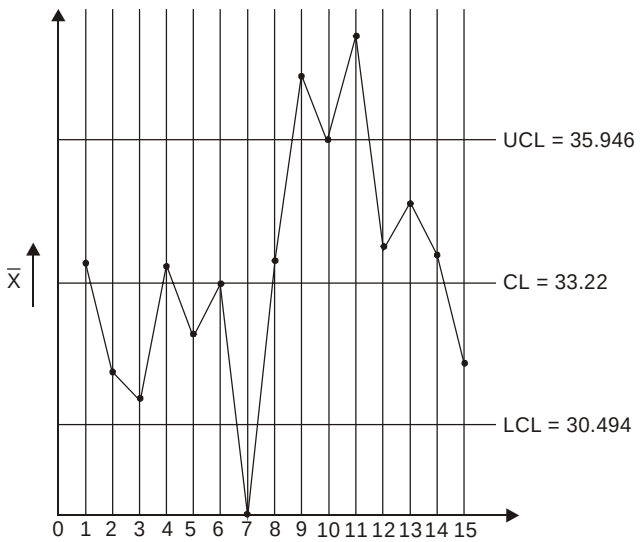
$$LCL = \overline{\overline{X}}_1 - A_2\overline{R}_2$$
$$= 33.22 - 0.58 \times 4.7$$
$$= 30.494$$

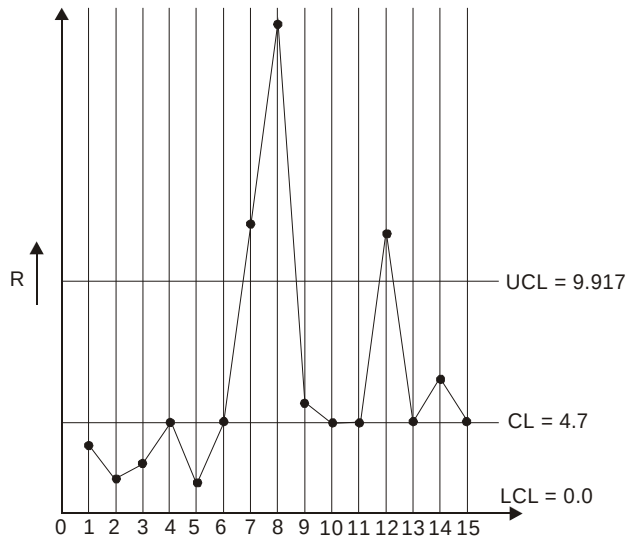
$$CL = \overline{\overline{X}}_1 = 33.22$$

Now all the points are falling within the control limits. The final values are

UCL = 35.946
LCL = 30.494
CL = 33.22

The charts are plotted for the final values.






(*b*) Interpretation R-chart is not in control, some points are crossing the UCL, $\overline{X}$-chart is not in control. Points are crossing the control limits. So the process is not in a state of statistical control.

$$\sigma^1 = \frac{\overline{R}_2}{d_2} = \frac{4.7}{2.326} = 2.02$$

The process capability = $6\sigma^1$

$= 6 \times 2.02$

$= 12.12$

(*d*) USL – LSL = 10

Since $6\sigma^1 >$ (USL – LSL), the process is not capable of meeting the specifications limits.

(*e*) UNTL = $\overline{\overline{X}}^1 + 3\sigma^1$

$= 33.22 + 3 \times 2.02$

$= 39.28$

LNTL = $\overline{\overline{X}}^1 - 3\sigma^1$

$= 33.22 - 3 \times 2.02$

$= 27.16$

CL = $\overline{\overline{X}}_1 = 33.22$

USL = 38

LSL = 28

Below Z = $\frac{28 - 33.22}{2.02} = -2.58$

Probability = 0.0052 = 0.52%

Above Z = $\frac{38 - 33.22}{2.02}$ = 2.36

Probability = 0.9909 = 99.09%

Therefore 100 – 99.09 = 0.91%

Total Rejection = 0.52 + 0.91 = 1.43%.

## EXERCISE PROBLEMS

**1.** The argon gas pressure in four feet flourescent lamp tube was to be controlled to ensure the correct wattage of the bulbs. Control charts are to be initiated to monitor argon gas. Pressure by sampling in a subgroup size of 3. The following data is collected.

| *Sub Group No.* | *Mean* | *Range* |
|---|---|---|
| 1 | 2.1 | 0.25 |
| 2 | 2.55 | 0.3 |
| 3 | 2.65 | 0.1 |
| 4 | 2.75 | 0.2 |
| 5 | 2.70 | 0.2 |
| 6 | 2.90 | 0.3 |
| 7 | 2.50 | 0.25 |
| 8 | 2.50 | 0.0 |
| 9 | 2.0 | 0.2 |
| 10 | 2.8 | 0.3 |
| 11 | 2.7 | 0.2 |
| 12 | 2.8 | 0.25 |
| 13 | 2.6 | 0.3 |
| 14 | 2.5 | 0.1 |
| 13 | 2.6 | 0.3 |
| 14 | 2.5 | 0.1 |
| 15 | 2.8 | 0.2 |
| 16 | 3.0 | 0.4 |
| 17 | 2.8 | 0.3 |
| 18 | 2.5 | 0.15 |
| 19 | 3.4 | 0.2 |
| 20 | 1.96 | 0.2 |

(*a*) Construct $\overline{X}$-R chart and plot the data.

(*b*) Interpret the state of production process.

(*c*) It is required to ensure a minimum pressure at 2.54 mm of Hg. What control limits would you recommend for $\overline{X}$-R chart.

**2.** Control charts are to be established to monitor the weight in ounces of the cotents of the container being filled on an assembly line. The container should hold atleast 10 ounces in order to guarantee this weight. The process must be set to deliver slightly more than this wt. Sample of 5 have been taken every 50 minutes. The mean and ranges are given below. (*a*) Contract the $\overline{X}$-R chart and interpret the state of control. (*b*) Do you think that the process can meet the specification of L = 10.5 ounce.

| *S.G. No.* | *Mean* | *Range* |
|---|---|---|
| 1 | 11.7 | 1.9 |
| 2 | 11.7 | 3.4 |
| 3 | 11.9 | 1.0 |
| 4 | 12.2 | 1.3 |
| 5 | 11.0 | 1.9 |
| 6 | 12.7 | 2.5 |
| 7 | 12.0 | 2.6 |
| 8 | 12.1 | 2.1 |
| 9 | 12.6 | 2.2 |
| 10 | 12.1 | 1.2 |
| 11 | 11.7 | 1.8 |
| 12 | 12.8 | 2.8 |
| 13 | 12.0 | 2.5 |
| 14 | 11.8 | 0.9 |
| 15 | 12.2 | 3.2 |
| 16 | 12.4 | 1.6 |
| 17 | 12.6 | 2.7 |
| 18 | 12.4 | 1.7 |
| 19 | 13.5 | 1.4 |
| 20 | 11.7 | 2.3 |

**3.** A machine is working to a specification of 12.58 ± 0.05. A study of 50 consecutive pieces show the following measurement put in 10 groups of 5 each.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 12.62 | 12.63 | 12.62 | 12.61 | 12.59 | 12.57 | 12.57 | 12.58 | 12.61 | 12.56 |
| 12.62 | 12.56 | 12.56 | 12.66 | 12.58 | 12.63 | 12.56 | 12.57 | 12.60 | 12.59 |
| 12.60 | 12.60 | 12.57 | 12.62 | 12.57 | 12.60 | 12.61 | 12.60 | 12.62 | 12.62 |
| 12.61 | 12.59 | 12.58 | 12.61 | 12.59 | 12.59 | 12.59 | 12.62 | 12.60 | 12.60 |
| 12.65 | 12.60 | 12.63 | 12.60 | 12.60 | 12.59 | 12.60 | 12.60 | 12.65 | 12.54 |

(*a*) Determine the control limits for average and range charts.

(*b*) Plot the data and interpretate the results.

(*c*) Determine the process capability.

(*d*) Does it appear that machine is capable of meeting the specifications requirements. If not determine the percentage of rejection.

**4.** The frequency of exitation of quartz crystal used to generate passes to the stepped motor of quartz watches is required to satisfy a specification of 32768 ± 4. To control the frequency of the activity of regulation of slicing angle of quartz is monitored through $\overline{X}$ and R charts. A subgroup size of 3 crystals are examined from the production line. The $\overline{X}$ values in axis of 32700 and the values of R are as follows.

| *S.G. No.* | $\overline{X}$ | *R* |
|---|---|---|
| 1 | 70 | 2 |
| 2 | 70 | 3 |
| 3 | 68 | 1 |
| 4 | 70 | 0 |
| 5 | 67 | 3 |
| 6 | 68 | 5 |
| 7 | 54 | 3 |
| 8 | 67 | 4 |
| 9 | 68 | 0 |
| 10 | 68 | 0 |
| 11 | 68 | 2 |
| 12 | 70 | 4 |
| 13 | 69 | 6 |
| 14 | 70 | 1 |
| 15 | 64 | 1 |
| 16 | 70 | 3 |
| 17 | 70 | 1 |
| 18 | 70 | 2 |
| 19 | 72 | 2 |
| 20 | 70 | 1 |

(*a*) Can the process meet the specifications.

(*b*) In the process correctly centered.

(*c*) With the existing process level if control is activated what percentage of the rejections can be expected.

(*d*) In order to correctly centre the process with respect to the specifications. What value of control limits would you recommend for $\overline{X}$ and R chart.

**5.** The following table given the values of mean and range of a sample of observations. The sample size in 5.

| *S.G. No.* | *Mean* | *Range* |
|---|---|---|
| *1* | *4.17* | *0.14* |
| *2* | *4.2* | *0.30* |
| *3* | *4.32* | *0.20* |
| *4* | *4.26* | *0.26* |

| | | |
|---|---|---|
| *5* | *4.22* | *0.12* |
| *6* | *4.3* | *0.24* |
| *7* | *4.4* | *0.50* |
| *8* | *4.25* | *0.17* |
| *9* | *4.49* | *0.58* |
| *10* | *4.6* | *0.23* |
| *11* | *4.4* | *0.62* |
| *12* | *4.6* | *0.23* |
| *13* | *4.31* | *0.28* |
| *14* | *4.29* | *0.38* |
| *15* | *4.46* | *0.22* |
| *16* | *4.27* | *0.09* |
| *17* | *4.33* | *0.17* |
| *18* | *4.62* | *0.2* |
| *19* | *4.49* | *0.2* |
| *20* | *4.32* | *0.4* |

(*a*) Plot the data on $\overline{X}$, R chart.

(*b*) What can you conclude about the process.

(*c*) If the specifications are $4.35 \pm 0.5$ what do you say about the process. Explain in detail.

**6.** The following data are obtained over a 10 day period to initiate $\overline{X}$ and R charts. Control charts for a quality characteristic of a certain product that had required a substantial amount of rework. All the figures applied to a product made on a single machine by a single operation. The subgroup size is 5. Two subgroups were taken per day.

| *S.G. No.* | $\overline{X}$ | *Range* |
|---|---|---|
| 1 | 177.6 | 23 |
| 2 | 176.6 | 8 |
| 3 | 178.4 | 22 |
| 4 | 176.6 | 12 |
| 5 | 177.0 | 7 |
| 6 | 179.4 | 8 |
| 7 | 178.6 | 15 |
| 8 | 178.8 | 6 |
| 9 | 178.2 | 7 |
| 10 | 179.6 | 12 |
| 11 | 179.8 | 9 |
| 12 | 176.4 | 8 |
| 13 | 178.4 | 7 |
| 14 | 178.2 | 4 |

| | | |
|---|---|---|
| 15 | 180.6 | 6 |
| 16 | 179.6 | 10 |
| 17 | 177.8 | 9 |
| 18 | 178.4 | 7 |
| 19 | 181.6 | 10 |
| 20 | 177.6 | 6 |

(*a*) Determine the trial control limits for $\overline{X}$, R charts.

(*b*) Plot the $\overline{X}$, R chart and offer your comment on the process.

(*c*) The specified requirements for the quality character are given as 171 ± 11. If it falls below 160 it will be scrapped. If above USL of 182 it may be reworked. Because scrapping an article is much more costly then rework, it is desired to hold the scrap to a low figure, without causing the excessive rework. The process average can be shifted by a relatively simple adjustment. What would you suggest as the aimed value for the process centering in the immediate feature why ?

**7.** ABC company has to add trace elements into its feed to provide the vital vitamins. It prepares a media base and mixes the trace elements at a pre-mixing stage and disperses the media to achieve the homogenity in the mix. The trace element formulation requirement to satisfy a specification of 1% ± 0.35%. The subgroup size is 3. The following 20 sets of $\overline{X}$ and R values are given below.

| *Sample No.* | $\overline{X}$ *(%)* | *R(%)* |
|---|---|---|
| 1 | 0.88 | 0.1 |
| 2 | 0.86 | 0.15 |
| 3 | 0.92 | 0.20 |
| 4 | 1.04 | 0.22 |
| 5 | 1.03 | 0.4 |
| 6 | 1.11 | 0.3 |
| 7 | 1.1 | 0.1 |
| 8 | 1.2 | 0.1 |
| 9 | 1.14 | 0.2 |
| 10 | 0.86 | 0.36 |
| 11 | 1.12 | 0.7 |
| 12 | 1.18 | 0.26 |
| 13 | 0.80 | 0.05 |
| 14 | 0.99 | 0.06 |
| 15 | 1.04 | 0.08 |
| 16 | 1.11 | 0.26 |
| 17 | 1.0 | 0.4 |
| 18 | 0.8 | 0.46 |
| 19 | 0.8 | 0.3 |
| 20 | 0.81 | 0.3 |

(*a*) Construct the $\overline{X}$, R chart and interpret the state of production process.

(*b*) Do you think that the product can meet the specifications. If not how much is not meeting the specifications.

**8.** The data which is given below are resistance measurements on different gauges of $\overline{X}$-chart with a sample size of 2. The mean and ranges are given below.

| *S.G. No.* | *Mean* | *Range* |
|---|---|---|
| 1 | 9.35 | 1.1 |
| 2 | 9.35 | 2.1 |
| 3 | 8.83 | 1.9 |
| 4 | 10.1 | 0.8 |
| 5 | 8.35 | 0.7 |
| 6 | 8.7 | 0.2 |
| 7 | 10.18 | 0.2 |
| 8 | 8.85 | 0.5 |
| 9 | 8.85 | 0.3 |
| 10 | 9.9 | 1.2 |
| 11 | 9.15 | 0.5 |
| 12 | 8.75 | 1.9 |
| 13 | 7.45 | 0.2 |
| 14 | 7.7 | 0.2 |
| 15 | 8.25 | 1.3 |
| 16 | 8.4 | 1.6 |
| 17 | 9.7 | 0.2 |
| 18 | 9.85 | 2.1 |
| 19 | 8.5 | 0.2 |
| 20 | 10.0 | 1.4 |

(*a*) Is the process in control.

(*b*) The resistance requires to satisfy a specification of $9.2 \pm 2.5s$. Do you think that the process can meet the specifications. What is the percentage of rejection if any.

(*c*) What is the corrective action if any.

**9.** $\overline{X}$ and R charts have been maintained on a certain quality characteristic. All the points are falling within the control limits on both the charts. A sudden change in the process occurs that increases $\overline{X}^1$ by $1.5\,\sigma^1$ but does not change $\sigma^1$. If the subgroup size is 3, approximately what percentage of the points would you expect to fall outside the control limits on $\overline{X}$-chart because of change in $\overline{X}^1$. Assume the normal distribution for the quality characteristic and control limits are based on the observation made before the shift in the process centre.

**10.** The mean cantilever strength of cast resin port insulators of 11 KV type A from a controlled manufacturing process from 20 subgroup of size 3 was found in control chart analysis to be 580 kg. The mean range was 8 kg. If $\overline{X}$-R charts are maintained.

(*a*) What is the probability that a fall in the mean strength to 575 kg is detected by the $\overline{X}$-chart.

(*b*) If the $\overline{X}$-R charts are maintained with an increased subgroup size of 6, what is the probability that a fall in the mean strength of 575 kg is detected by $\overline{X}$-chart.

**11.** The following measurements are obtained from a sample of 5 items taken at regular intervals from the manufacturing process.

| Sample | Measurements in cm. | | | | | |
|---|---|---|---|---|---|---|
| No. | 1 | 2 | 3 | 4 | 5 | |
| 1 | 0.997 | 1 | 0.998 | 0.996 | 1.002 | = 4.993 |
| 2 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | = 4.997 |
| 3 | 0.996 | 1.002 | 0.999 | 1 | 1 | = 4.997 |
| 4 | 0.996 | 1 | 0.999 | 1.002 | 0.999 | = 4.996 |
| 5 | 1 | 1.001 | 1 | 1 | 0.998 | = 4.999 |
| 6 | 0.999 | 0.996 | 1.002 | 1 | 0.998 | = 4.995 |
| 7 | 0.999 | 1.002 | 1 | 1.001 | 1.001 | = 5.003 |
| 8 | 0.999 | 0.999 | 1.002 | 1 | 0.996 | = 4.996 |

(*a*) Determine the trial control limits.

(*b*) Construct the $\overline{X}$-R chart and plot the data.

(*c*) What values of $\overline{\overline{X}}$ and $\overline{R}$ you recommend for the future period.

(*d*) If the specification limits are $1 \pm 0.003$ what do you conclude about the process to meet the specifications.

**PROCESS CAPABILITY**

The first priority is to bring a process into a state of statistical control. That is to say, make the process stable and therefore predictable.

But this is not enough ; there is more to be done. Once we have concluded that a process is in statistical control (by the absence of *special causes* on the control charts) there could still be two possible conditions prevailing, *i.e.*,

(*a*) the process is stable (*i.e.*, in control) and *capable*. In other words it continually produces parts which are acceptable.

(*b*) the process is stable and *incapable, i.e.*, it continually and predictably produces parts which are not acceptable.

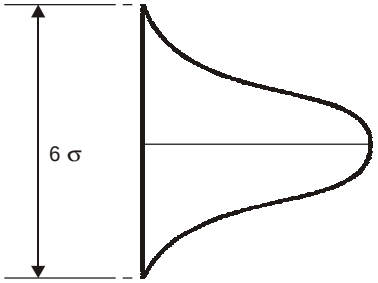
Obviously, it is the first alternative we want. So there must be some examination to test for the inherest *capability* of the process.

This is done by comparing the output which the process can be expected to produce under normal and random conditions with that required by the customer.

The *natural spread* of a process is the distribution of parts produced when the process is operating in statistical control and being affected only by random variations.

So if we ran a process and measured every individual part produced the distribution would more than often follow the normal bell-shaped curve with a standard deviation of $\sigma$.

The natural spread across the basis is 6σ




So this tells us what the process is actually giving us. We still don't know if this is good enough for what we *want*.

To determine this, we must compare the process spread with the design requirements, *i.e.*, the tolerance. The tolerance is, of course, the difference between the upper specification limit (USL) and the lower specification limit (LSL).

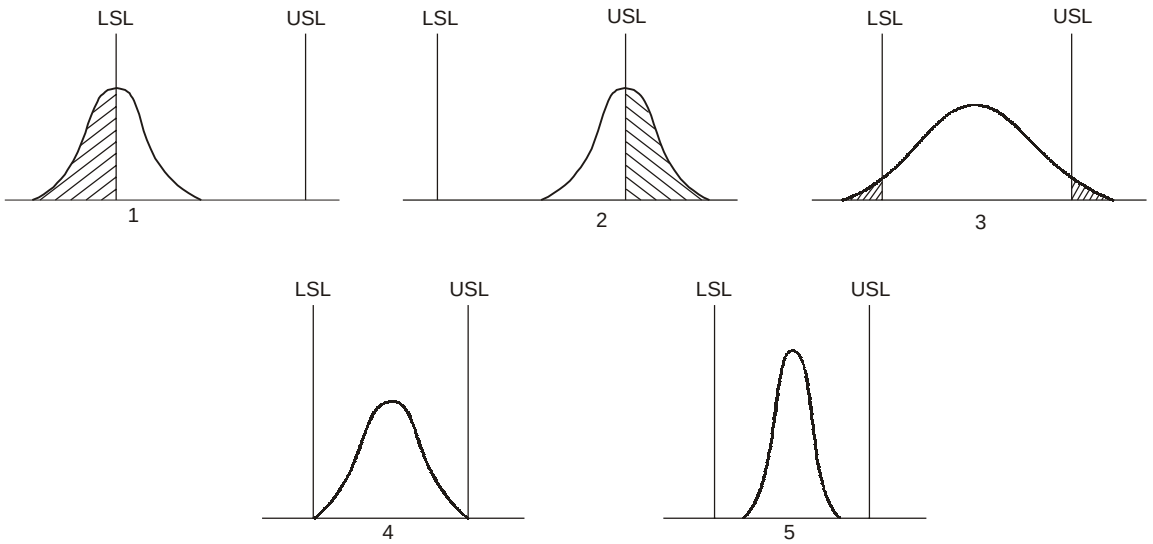
The following diagrams illustrate the output distribution from five different processes.

Processes 1 and 2 are producing components beyond the specification limits (either too low or too high). This is because the process is wrongly set. Adjustment of the setting will bring the distribution within the specification limits.

On the other hand, process 3 is producing unacceptable parts which are *both* too high and too low. This is because the process spread is too wide. No end of adjustment to the setting will cure this problem. The piece-to-piece variability must be reduced to make the process capable of producing acceptable parts.

Process 4 is just capable of producing acceptable parts.

process 5 is the best.




This is the objective : to reduce the spread about a centre line which is on target setting.

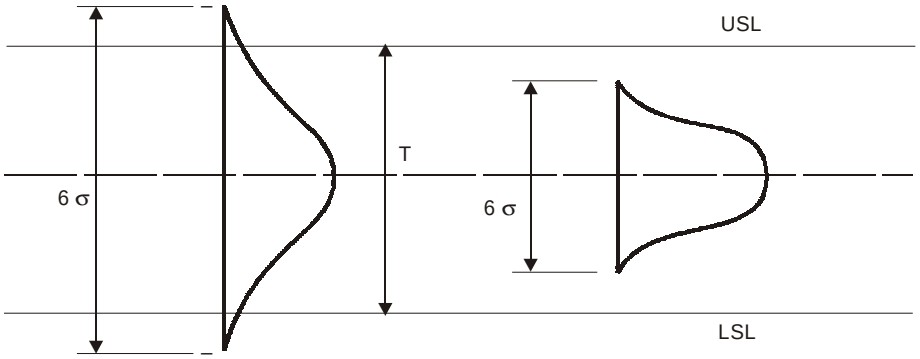
For a variable process where we draw average/range charts *the measure of capability* can be indicated by *capability indices*.

There are two indices and we shall deal with each in turn.

**The $C_p$ index**

The $C_p$ index gives a measure of capability for a process which is centred on the required design value, *i.e.*, it is centred on nominal.

The $C_p$ index is found by comparing the tolerance (T) with the natural spread (6σ).




$$C_p = \frac{T}{6\sigma}$$

*Caution* : Here there is some confusion because some organisations actually invert the formula and use

$$C_p = \frac{6\sigma}{T}$$

So make sure you are clear as to which convention is being used by you, your suplliers and your customers.

As can be seen, when the process spread has the same value as the tolerance 6σ will be equal to T and so

$$C_p = \frac{T}{6\sigma} = 1$$

This is the MINIMUM VALUE FOR $C_p$ if the process is to be capable (or the maximum, depending on the organisation's convention).

To be on the safe side, and to allow for a bit of latitude, many companies (in the automotive business for example) stipulate that for a process to be capable.

$C_p$ must be at least 1.33

**Calculating $C_p$**

$$C_P = \frac{T}{6\sigma}$$

$\sigma$ = standard deviation for individuals = $\frac{\overline{R}}{d_2}$

Therefore $6\sigma = \frac{6\overline{R}}{d_2}$

FOR SAMPLE SIZE OF $5 d_2 = 2.326$

So $6\sigma = \frac{6\overline{R}}{6.326}$

*i.e.*, $6\sigma = 2.58\,\overline{R}$

Remember this is only when we are using a sample size of 5 in drawing the variable control chart.

Therefore $C_P = \frac{T}{2.58\overline{R}}$ (for sample size $n = 5$)

*Example 1*

In a process, $\overline{R}$ was found to be 0.035 mm

The design called for USL = 4.95 mm

and LSL = 4.88 mm

Then T = 4.95 – 4.88

= 0.07

and $C_P = \frac{0.07}{2.58 \times 0.035}$

$= \frac{0.07}{0.09}$

*i.e.*, $C_P = 0.78$

Since $C_P$ is less than 1, this process is *not capable* of producing enough parts to the satisfaction of the customer.

In such a situation two options are available :

(*a*) negotiate to have the specification changed to widen the tolerance. This sometimes happens but customers are reluctant to relax specifications.

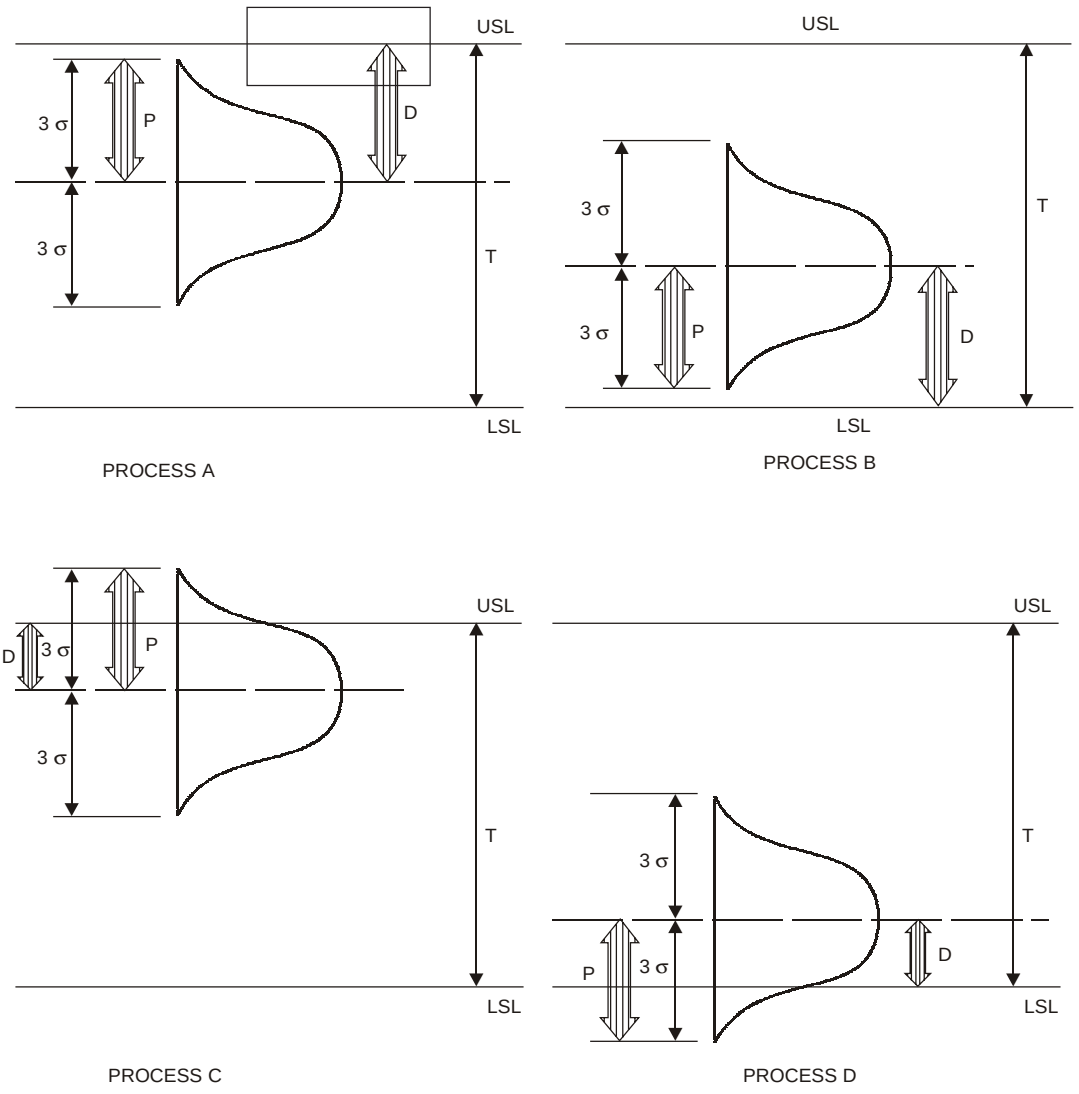
(*b*) carry out 100% inspection to sift out the bad parts. This should only be a temporary measure. The objective is to *improve the process* to make it capable. (N.B. in any case 100% inspection is *not 100% effective*.)

**The $C_{pk}$ index**

We have seen that the $C_p$ index indicates the capability of a process which is *centred on nominal.* In some situations the process is *not centred on nominal.*

In such cases a different approach is required to calculate the capability index.

For example, the situations could be as shown below :




To take into account the seeting of the process it is necessary to consider the process mean ($\overline{\overline{X}}$) in relation to the specifications limits.

In this case, the capability index $C_{px}$ is found by

$$C_{pk} = \frac{USL - \overline{\overline{X}}}{3\sigma} \quad \text{or} \quad \frac{\overline{\overline{X}} - LSL}{3\sigma}$$

whichever is the lesser value.

In other words, the specification limit to be considered is the one nearer to the procees mean.

In the above diagrams for different process settings this relationship is shown by the ratio $\frac{D}{P}$.

Again, for the process to be capable, $C_{pk}$ must be at least 1 and preferably greater than 1.33. So in the four cases above

Processes A and B are capable, and

Processes C and D are incapable.

This interesting thing is that $C_p$ for all four processes would be the same, *i.e.*, $\frac{T}{6\sigma}$ and would be greater than 1, indicating capability.

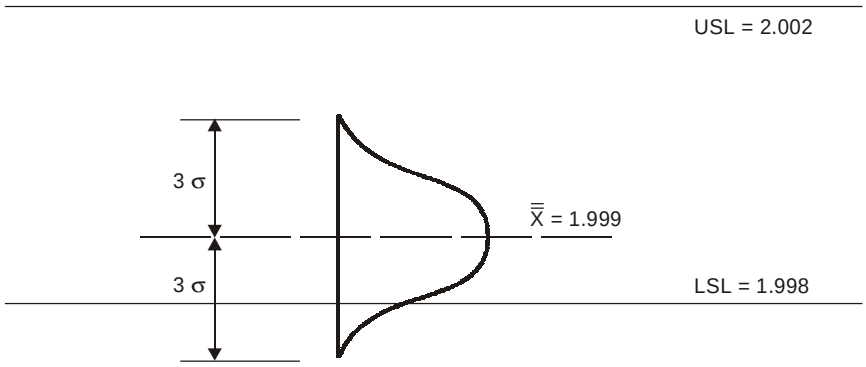
That is why it is misleading to use $C_p$ to calculate when the process is not set on nominal.

*Note :* When the process is centred on target then $C_p$ and $C_{pk}$ are the same value.

*Example* : In an operation the design called for a shaft of diameter $2.00 \pm .002$ inches.

A control chart was drawn for samples of 5 and $\overline{R}$ was found to be 0.0009 and $\overline{\overline{X}}$ was 1.999.

The process could be depicted as follows :




Therefore $C_{pk} = \frac{\overline{\overline{X}} - LSL}{3\sigma}$

$3\sigma = 1.29\,\overline{R}$ (for sample size $n = 5$)

So $C_{pk} = \frac{1.999 - 1.998}{1.29 \times 0.0009}$

$= \frac{0.001}{1.29 \times 0.0009}$

*i.e.*, $C_{pk} = 0.86$

This is less than 1, so the process is incapable.

*NOTE* $C_p$ would be $\frac{T}{6\sigma}$

$= \frac{0.004}{2.58 \times .0009}$

$C_p = 1.72$ (indicating capability)

It is obvious that the process needs to be reset to come closer to design nominal.

**SUMMARY**

Once a process is found to be in statistical control, we have to determine how capable it is in producing parts to customer requirements. Capability is relayed by calculating capability indices $C_p$ and $C_{pk}$.

If the process is centred on nominal then we can use $C_p$ (in such a case $C_{pk}$ will be the same).

If the setting is not on nominal we must use $C_{pk}$.

For the process to be capable $C_p$ and $C_{pk}$ must be greater than 1.00 and preferably greater than 1.33.

The objective is therefore to get $C_p$ and $C_{pk}$ as high as possible, thus indicating a small process spread and hence grater freedom for movement within the specification limits.

High values for $C_p$ and $C_{pk}$ (and hence high capability) can only be brought about by decreasing process variability (as indicated by $\overline{R}$).

In Japan, companies are frequently achieving $C_p$ values in excess of 8.0.

# 7 Control Charts for Attributes

**Attribute Control Charts**

When a chart shows the number of articles conforming and the number of articles failing to conform to any specified requirements then it is said to be an attribute control chart.

The attribute control charts are classified as

(*a*) *p*-chart or Fraction defective chart

(*b*) *np*-chart or Number defective chart

(*c*) *c*-chart or Defects per unit chart

(*d*) U-chart

***p*-chart**

If the attribute data is collected for the number of defective articles produced in a given period and a control chart is used to present the attribute data is known as *p*-chart.

The fraction defective of each sample is denoted by *p*

$$p = \frac{d}{n} = \frac{\text{No. of defectives in a sample}}{\text{No. of items inspected in a sample}}$$

The standard fraction defective.

$$\bar{p} = \frac{\Sigma d}{\Sigma n} = \frac{\text{Total no. of defectives in all samples}}{\text{Total no. of items inspected in all samples}}$$

If $\bar{p}$ represents the fraction defective of the process then the probability of occurrence of '*r*' defectives in a sample is given by the Binomial Distribution.

$$P(r) = {}^{n}C_{r}\ P^{r}\ q^{n-r}$$

The standard deviation = $\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$

Steps to be followed to construct *p*-chart.

1. Collect about 25 samples of size *n*. If '*n*' is constant then it is called a constant *p*-chart. If '*n*' is varying then it is called variable *p*-chart.
2. Calculate the fraction defective of each sample

$$p = \frac{d}{n}$$

3. Compute the standard fraction defective

$$\bar{p} = \frac{\Sigma d}{\Sigma n}$$

4. Compute the control limits

$$\text{UCL} = \bar{p} + 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{LCL} = \bar{p} - 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{CL} = \bar{p}$$

If $n$ is constant then only one UCL and LCL is computed. If '$n$' is varying then compute UCL and LCL for each sample.

5. Compare the fraction defective $p$ with its control limits. If all the samples are falling within the limits then homogenisation is not necessary.
6. Otherwise delete the corresponding subgroups which are crossing the control limits and calculate $\bar{p}_1$.
7. Again compute the control limits for $\bar{p}_1$.
8. Prepare a chart with the fraction defective on the vertical axis and subgroup No. on the horizontal axis.
9. Draw the UCL and LCL on the chart and locate the points of $p$ with respect to subgroup no.

Purpose of setting the lower control limits on the $p$ chart or attribute control chart.

When the points fall below the LCL, one of the six factors are usually responsible for extra good quality.

(*a*) Unexpected improvement in the process.
(*b*) Faulty inspection.
(*c*) Error in charting or plotting.
(*d*) Chance variation.
(*e*) Use of very good raw material.
(*f*) Good training to the operator.

**SOLVED PROBLEMS**

**Problem 1.** *The inspection results in a machine shop based on sampling size of 50 are given below.*

(*a*) *Calculate the control limits for the p-chart using 3σ limits.*

(*b*) *Plot the data and offer your comments on the behaviour of the process.*

(*c*) *What standard fraction defective would you recommend for the future period.*

| *Sample No.* | *No. of defectives (d)* | *Fraction defective* $p = d/n$ |
|---|---|---|
| 1 | 6 | 0.12 |
| 2 | 3 | 0.06 |
| 3 | 1 | 0.02 |
| 4 | 2 | 0.04 |

| | | |
|---|---|---|
| 5 | 12 | 0.24 |
| 6 | 6 | 0.12 |
| 7 | 4 | 0.08 |
| 8 | 7 | 0.14 |
| 9 | 1 | 0.02 |
| 10 | 8 | 0.16 |
| 11 | 3 | 0.06 |
| 12 | 7 | 0.14 |
| 13 | 1 | 0.02 |
| 14 | 15 | 0.3 |
| 15 | 4 | 0.08 |
| 16 | 18 | 0.36 |
| 17 | 3 | 0.06 |
| 18 | 2 | 0.04 |
| 19 | 6 | 0.12 |
| 20 | 7 | 0.14 |

The standard fraction defective

$$\bar{p} = \frac{\Sigma d}{\Sigma n}$$

$$= \frac{116}{20 \times 50}$$

$$= 0.116$$

Control limits for $p$-chart

$$\text{UCL} = \bar{p} + 3 \times \sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$$

$$= 0.116 + 3 \times \sqrt{\frac{0.116(1-0.116)}{50}}$$

$$= 0.2518$$

$$\text{LCL} = \bar{p} - 3 \times \sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$$

$$= 0.116 - 3 \times \sqrt{\frac{0.116(1-0.116)}{50}}$$

$$= -0.0198 \sim 0.0$$

$$\text{CL} = \bar{p} = 0.116$$

It is seen from the data that 2 subgroups are crossing the UCL which indicates the presence of assignable causes so homogenisation is necessary.

$$\bar{p}_1 = \frac{116 - 15 - 18}{18 \times 50}$$

$$= 0.092$$

Again control limits

$$\text{UCL} = \bar{p}_1 + 3 \times \sqrt{\frac{\bar{p}_1 (1 - \bar{p}_1)}{n}}$$

$$= 0.092 + 3 \times \sqrt{\frac{0.092 (1 - 0.092)}{50}}$$

$$= 0.214$$

$$\text{LCL} = \bar{p}_1 = 3 \times \sqrt{\frac{\bar{p}_1 (1 - \bar{p}_1)}{n}}$$

$$= 0.092 - 3 \times \sqrt{\frac{0.092 (1 - 0.092)}{50}}$$

$$= - 0.03 \sim 0.0$$

$$\text{CL} = \bar{p}_1 = 0.092$$

Again one more subgroup is crossing the crossing limits

$$\bar{p}_2 = \frac{116 - 15 - 18 - 12}{17 \times 50}$$

$$= 0.0835$$

Again control limits

$$\text{UCL} = \bar{p}_2 + 3 \times \sqrt{\frac{\bar{p}_2 (1 - \bar{p}_2)}{n}}$$

$$= 0.0835 + 3 \times \sqrt{\frac{0.0835 (1 - 0.0835)}{50}}$$

$$= 0.2$$

$$\text{LCL} = \bar{p}_2 - 3 \times \sqrt{\frac{0.0835 (1 - 0.0835)}{50}}$$

$$= - 0.033$$

$$\sim 0.0$$

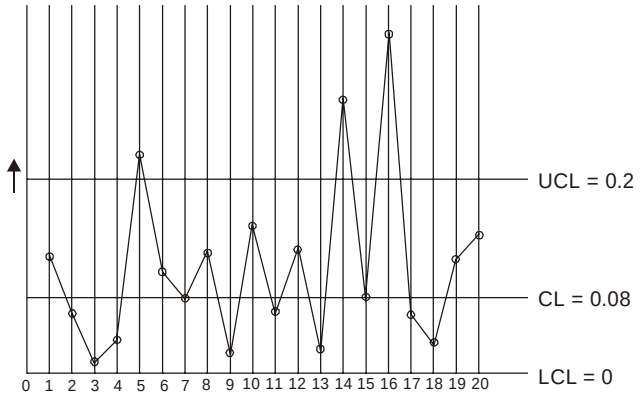
$$\text{CL} = \bar{p}_2 = 0.0835$$

Now all the points are falling within the control limits. The final values are

UCL = 0.2

LCL = 0.0

CL = 0.0835




Since three subgroups are crossing the UCL, the process is not in a state of statistical control.

The recommended value of $\bar{p}$ for the future period in $\bar{p}_2 = 0.0835$

**Problem 2.** *A firm buys screws in cartons containing several thousands. As a part of acceptance procedure 400 screws are selected and subjected to visual inspection for a certain non-conformaties. In a shipment of 10 cartons the percentage of rejected screws in the samples from each carton are 0.0, 0.75, 0, 2, 0.25, 0.25, 0 and 1.25. Construct an appropriate chart and determine whether the shipment of screws appear to exhibit statistical control with respect to the quality characteristic examined in this inspection.*

| *Sample No.* | *% defective* | *No. of defectives* $d$ | *Fraction defective* $p = d/n$ |
|---|---|---|---|
| 1 | 0.0 | 0 | 0.0 |
| 2 | 0.0 | 0 | 0.0 |
| 3 | 0.5 | 2 | 0.005 |
| 4 | 0.75 | 3 | 0.0075 |
| 5 | 0.0 | 0 | 0.0 |
| 6 | 2.0 | 8 | 0.02 |
| 7 | 0.25 | 1 | 0.0025 |
| 8 | 0.25 | 1 | 0.0025 |
| 9 | 0.0 | 0 | 0.0 |
| 10 | 1.25 | 5 | 0.0125 |

**Solution.** $\Sigma d = 20$

Standard fraction defective

$$\bar{p} = \frac{\Sigma d}{\Sigma n}$$

$$= \frac{20}{10 \times 400}$$

$$= 0.116$$

Control limits for $p$-chart

$$\text{UCL} = \bar{p} + 3 \times \sqrt{\frac{\bar{p}\,(1 - \bar{p})}{n}}$$

$$= 0.005 + 3 \times \sqrt{\frac{0.005\,(1 - 0.005)}{400}}$$

$$= 0.01558$$

$$\text{LCL} = \bar{p} - 3 \times \sqrt{\frac{\bar{p}\,(1 - \bar{p})}{n}}$$

$$= - 0.0055$$

$$\text{LCL} \sim 0.0$$

$$\text{CL} = \bar{p} = 0.005$$

It is seen from the data that one subgroup is crossing the UCL which indicates the presence of assignable causes, so homogenisation is necessary.

$$\bar{p}_1 = \frac{20 - 8}{9 \times 400} = 0.0033$$

Again control limits

$$\text{UCL} = \bar{p}_1 + 3 \times \sqrt{\frac{\bar{p}_1\,(1 - \bar{p}_1)}{n}}$$

$$= 0.0033 + 3 \times \sqrt{\frac{0.0033\,(1 - 0.0033)}{400}}$$

$$= 0.0119$$

$$\text{LCL} = \bar{p}_1 - 3 \times \sqrt{\frac{\bar{p}_1\,(1 - \bar{p}_1)}{n}}$$

$$= 0.0033 - 3 \times \sqrt{\frac{0.0033\,(1 - 0.0033)}{400}}$$

$$= - 0.005$$

LCL ~ 0.0

CL = $\bar{p}_1$ = 0.0033

Again one more subgroup is crossing the UCL

$$\bar{p}_2 = \frac{20 - 8 - 5}{8 \times 400}$$

= 0.00218

Again control limits

$$UCL = 0.00218 + 3 \times \sqrt{\frac{0.00218\,(1 - 0.00218)}{400}}$$

= 0.00917

LCL = 0.0

CL = $\bar{p}_2$ = 0.00218

Now all the points are falling within the control limits.

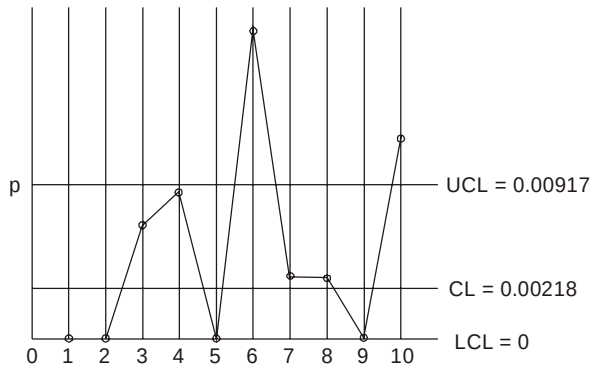
The final values are

UCL = 0.00917

LCL = 0.0

CL = 0.00218

The chart is platted below for the final values.




The shipment of screws does not exhibit in a state of statistical control with respect to the quality characteristic examined.

**Problem 3.** *The following data is noticed on inspection by a preventive maintenance group with 65 fuse contact installations.*

| *S.G. No.* | *No. of items inspected* | *No. of defective* (*d*) | *Fraction defective* $p = d/n$ |
|---|---|---|---|
| 1 | 65 | 21 | 0.323 |
| 2 | 65 | 20 | 0.307 |
| 3 | 65 | 19 | 0.292 |

| | | | |
|---|---|---|---|
| 4 | 65 | 20 | 0.307 |
| 5 | 65 | 20 | 0.307 |
| 6 | 65 | 22 | 0.338 |
| 7 | 65 | 23 | 0.353 |
| 8 | 65 | 29 | 0.446 |
| 9 | 65 | 20 | 0.307 |
| 10 | 65 | 21 | 0.323 |
| 11 | 65 | 28 | 0.430 |
| 12 | 65 | 25 | 0.384 |
| 13 | 65 | 6 | 0.092 |
| 14 | 65 | 20 | 0.307 |
| 15 | 65 | 18 | 0.276 |

Construct a *p*-chart and offer your comment on the behaviour of the process.

$\Sigma d = 312$

Standard fraction defective $\bar{p} = \frac{\Sigma d}{\Sigma n}$

$$\bar{p} = \frac{312}{65 \times 15}$$

$$= 0.32$$

Control limits for *p*-chart.

$$\text{UCL} = \bar{p} + 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.32 + 3 \times \sqrt{\frac{0.32(1-0.32)}{65}}$$

$$= 0.493$$

$$\text{LCL} = \bar{p} - 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.32 - 3 \times \sqrt{\frac{0.32(1-0.32)}{65}}$$

$$\text{LCL} = 0.146$$

$$\text{CL} = \bar{p} = 0.32$$

It is seen from the data that one subgroup is crossing the LCL, so the homogenisation is necessary

$$\bar{p}_2 = \frac{312 - 6}{65 \times 14} = 0.33$$

Now control limits

$$UCL = \bar{p}_1 + 3 \times \sqrt{\frac{\bar{p}_1 (1 - \bar{p}_1)}{n}}$$

$$= 0.033 + 3 \times \sqrt{\frac{0.33 (1 - 0.33)}{65}}$$

$$UCL = 0.504$$

$$LCL = \bar{p}_1 - 3 \times \sqrt{\frac{\bar{p}_1 (1 - \bar{p}_1)}{n}}$$

$$= 0.155$$

$$CL = \bar{p}_1 = 0.33$$

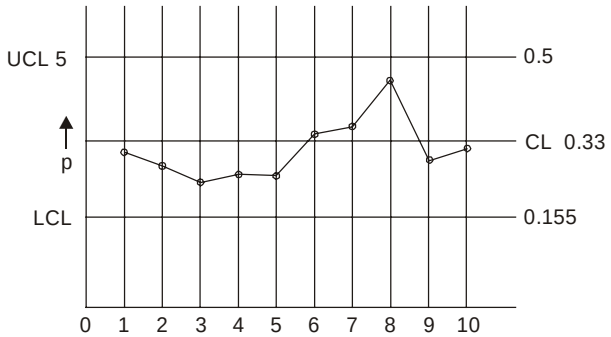
Now all the points are falling within the control limits.

The final values are

UCL = 0.504

LCL = 0.155

CL = 0.33




Since one subgroup is crossing the LCL. The process is under improvement.

**Problem 4.** *A produced is given 100% inspection as it is manufactured. The samples are summarised as follows.*

| *Sample No.* | *No. of items inspected* ($n$) | *No. of defectives* ($d$) | *Fraction defective* $p = d/n$ | *UCL* | *LCL* |
|---|---|---|---|---|---|
| 1 | 48 | 5 | 0.104 | 0.1592 | 0.0 |
| 2 | 36 | 5 | 0.138 | 0.1748 | 0.0 |
| 3 | 50 | 0 | 0.0 | 0.1571 | 0.0 |
| 4 | 47 | 5 | 0.106 | 0.1602 | 0.0 |
| 5 | 48 | 0 | 0.6 | 0.1592 | 0.0 |
| 6 | 54 | 3 | 0.055 | 0.1534 | 0.0 |

| | | | | | |
|---|---|---|---|---|---|
| 7 | 50 | 0 | 0.0 | 0.1571 | 0.0 |
| 8 | 42 | 1 | 0.023 | 0.1662 | 0.0 |
| 9 | 32 | 5 | 0.156 | 0.1819 | 0.0 |
| 10 | 40 | 2 | 0.05 | 0.1688 | 0.0 |

(*a*) Construct a suitable control chart and offer your comments.

(*b*) What value of $\bar{p}^{1}$ would you recommend for the future period.

**Solution.** $\Sigma d = 26$

$\Sigma n = 447$

The standard fraction defective

$$\bar{p} = \frac{\Sigma d}{\Sigma n} = \frac{26}{447} = 0.058$$

Specimen calculations for subgroup No. 1

$$\text{UCL} = \bar{p} + 3 \times \sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$$

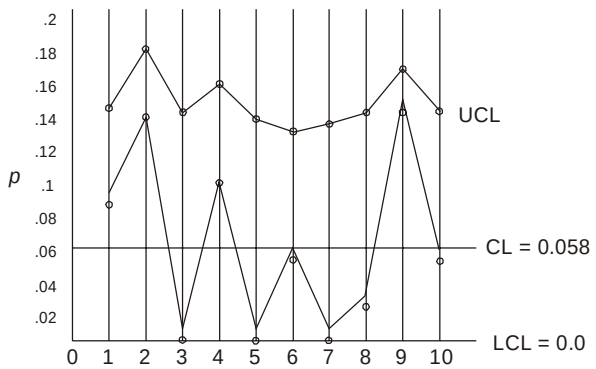
$$= 0.058 + 3 \times \sqrt{\frac{0.058\,(1-0.058)}{48}}$$

$$= 0.1592$$

$$\text{LCL} = 0.058 - 3 \times \sqrt{\frac{0.058\,(1-0.058)}{48}}$$

$$= -0.04 \sim 0.0$$

Similarly UCL and LCL is calculated and is given in the table for other subgroups.




Since all the points are falling within the control limits. The process is in a state of statistical control.

The recommended values of $p^{1}$ for the future period is $\bar{p} = 0.058$.

**Problem 5.** *The table below shows the no. of items inspected and rejected. Plot the data on an appropriate control chart and offer your comments on the behaviour of the process.*

| *Sample No.* | *No. of items inspected*($n$) | *No. of items rejected*($d$) | *Fraction defective* $p = d/n$ | *UCL* | *LCL* |
|---|---|---|---|---|---|
| 1 | 960 | 130 | 0.135 | 0.150 | 0.087 |
| 2 | 1015 | 129 | 0.127 | 0.149 | 0.088 |
| 3 | 1020 | 147 | 0.144 | 0.149 | 0.088 |
| 4 | 910 | 94 | 0.097 | 0.150 | 0.087 |
| 5 | 955 | 106 | 0.110 | 0.150 | 0.087 |
| 6 | 975 | 82 | 0.084 | 0.150 | 0.087 |
| 7 | 1015 | 116 | 0.144 | 0.149 | 0.088 |
| 8 | 1000 | 95 | 0.095 | 0.149 | 0.088 |
| 9 | 1015 | 126 | 0.124 | 0.149 | 0.088 |
| 10 | 900 | 136 | 0.151 | 0.151 | 0.086 |
| 11 | 1250 | 148 | 0.118 | 0.146 | 0.091 |
| 12 | 1250 | 108 | 0.086 | 0.146 | 0.091 |
| 13 | 1015 | 104 | 0.102 | 0.149 | 0.088 |
| 14 | 950 | 102 | 0.107 | 0.150 | 0.087 |
| 15 | 1010 | 138 | 0.136 | 0.149 | 0.088 |
| 16 | 960 | 95 | 0.098 | 0.150 | 0.087 |
| 17 | 600 | 56 | 0.093 | 0.158 | 0.079 |
| 18 | 895 | 109 | 0.121 | 0.151 | 0.086 |
| 19 | 1015 | 136 | 0.133 | 0.149 | 0.088 |
| 20 | 960 | 192 | 0.213 | 0.150 | 0.087 |
| | $\Sigma n = 19720$ | $\Sigma d = 2349$ | | | |

**Solution.** $\bar{p} = \dfrac{\Sigma d}{\Sigma n} = \dfrac{2349}{19720} = 0.119$

Specimen calculation for subgroup No. 1

$$\text{UCL} = \bar{p} + 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

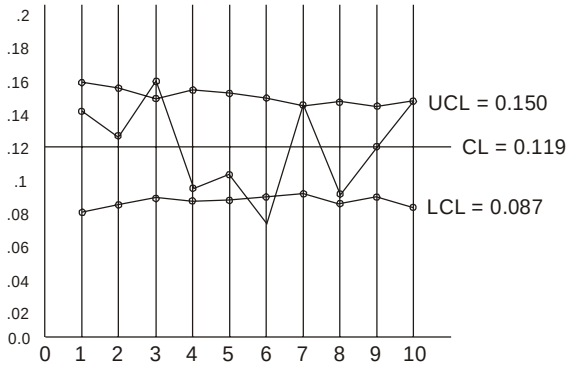
$$= 0.119 + 3 \times \sqrt{\frac{0.119(1-0.119)}{960}}$$

$$= 0.150$$

$$\text{LCL} = 0.119 - 3 \times \sqrt{\frac{0.119(1-0.119)}{960}}$$

$$= 0.087$$

Similarly UCL and LCL are calculated for other subgroup and are given in the table above.




It is seen that one subgroup is crossing the UCL which indicates the presence of assignable causes. So the process is not in a state of statistical control.

Two subgroups are falling below the L.C.L. which shows that the process is under improvement.

**Problem 6.** *Lots of pushing resews from a single supplier where 100% inspection is done and the results are as follows.*

| *Lot No.* | *Batch quality inspected* ($n$) | *Nos. Rejected* ($d$) | $p = d/n$ | *UCL* | *LCL* |
|---|---|---|---|---|---|
| *1* | *6000* | *183* | *0.0305* | *0.0233* | *0.013* |
| *2* | *6000* | *131* | *0.0218* | *0.0233* | *0.013* |
| *3* | *10000* | *185* | *0.0185* | *0.022* | *0.0149* |
| *4* | *9000* | *23* | *0.0025* | *0.0224* | *0.0139* |
| *5* | *2725* | *92* | *0.033* | *0.0258* | *0.0105* |
| *Total* | *33725* | *614* | | | |

*Draw an appropriate control chart and offer your comment on behaviour of the process.*

**Solution.** $\bar{p} = \dfrac{\Sigma d}{\Sigma n} = \dfrac{614}{33725} = 0.0182$

Specimen calculations for reading no. 1.

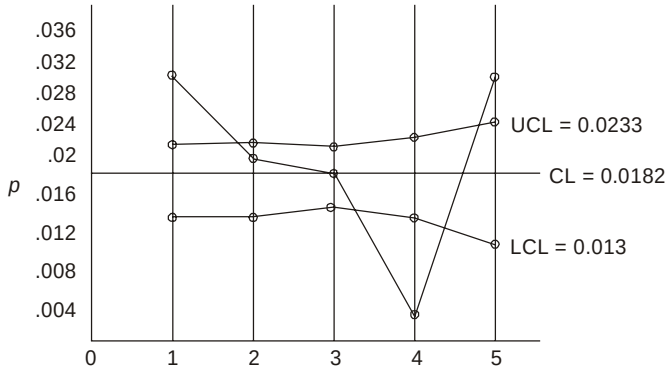
$$\text{UCL} = \bar{p} + 3 \times \sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$$

$$= 0.0182 + 3 \times \sqrt{\frac{0.0182\,(1-0.0182)}{6000}}$$

$$= 0.0233$$

$$\text{LCL} = 0.0182 - 3 \times \sqrt{\frac{0.0182\,(1-0.0182)}{6000}}$$

$$= 0.013$$




It is seen that subgroup No. 1 and 5 are crossing the UCL which indicates the presence of assignable causes so the process is not in a state of statistical control. The subgroup no. 4 is falling below LCL which indicates that the process is under improvement. Find the reasons and maintain the same consistently.

**Problem 7.** *A p-chart is used to analyse the monthly record for 100% inspection of a certain radio transmitting tubes. The total no. inspected this month are 2196 and the total no. of defectives are 158. Compute* $\bar{p}$ *and also the individual* $3\sigma$ *limits for the production figure of the following 3 days and state whether the fraction defective fell within the control limit each day.*

| *Day* | *No. of items inspected* ($n$) | *No. of defectives* $d$ | *Fraction defective* $p = d/n$ | *UCL* | *LCL* |
|---|---|---|---|---|---|
| *1* | *54* | *8* | *0.148* | *0.1773* | *0.0* |
| *2* | *162* | *24* | *0.148* | *0.1327* | *0.011* |
| *3* | *213* | *39* | *0.183* | *0.1249* | *0.018* |

**Solution.** $\bar{p} = \dfrac{\Sigma d}{\Sigma n} = \dfrac{158}{2196} = 0.0719$

Specimen calculations for 1st day

$$\text{UCL} = \bar{p} + 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.0719 + 3 \times \sqrt{\frac{0.0719(1-0.0719)}{54}} = 0.1773$$

$$\text{LCL} = \bar{p} - 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.0719 - 3 \times \sqrt{\frac{0.0719(1-0.0719)}{54}} = -0.033 \sim 0.0$$

It is clear from the table that subgroups 2 and 3 are not falling within the control limits.

**Problem 8.** *Table given below shows the inspection results of screws. Plot a suitable control chart and offer your comments on the behaviour of the process.*

| *Lot No.* | *No, of items inspected*$(n)$ | *Rejected* $d$ | $p = d/n$ | *UCL* | *LCL* |
|---|---|---|---|---|---|
| *1* | *135* | *11* | *0.0814* | *0.152* | *0.011* |
| *2* | *162* | *19* | *0.117* | *0.146* | *0.0174* |
| *3* | *140* | *9* | *0.064* | *0.151* | *0.0125* |
| *4* | *155* | *14* | *0.09* | *0.148* | *0.016* |
| *5* | *188* | *9* | *0.049* | *0.142* | *0.022* |
| *6* | *166* | *16* | *0.096* | *0.145* | *0.0182* |
| *7* | *138* | *10* | *0.072* | *0.152* | *0.012* |
| *8* | *144* | *12* | *0.0834* | *0.15* | *0.0135* |
| *9* | *161* | *11* | *0.068* | *0.1468* | *0.0172* |
| *10* | *158* | *16* | *0.101* | *0.147* | *0.0166* |

**Solution.** $\Sigma d = 127$

$\Sigma n = 1547$

$$\bar{p} = \frac{\Sigma d}{\Sigma n} = \frac{127}{1547} = 0.082$$

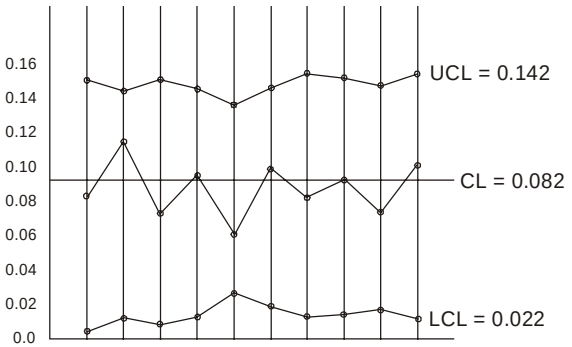
UCL and LCL for reading No. 5.

$$\text{UCL} = 0.082 + 3 \times \sqrt{\frac{0.082\,(1 - 0.082)}{188}}$$

$$= 0.142$$

$$\text{LCL} = 0.082 - 3 \times \sqrt{\frac{0.082\,(1 - 0.082)}{188}}$$

$$= 0.022$$




Since all the points are falling within the control limits so the process is in the state of statistical control.

**Problem 9.** *In a manufacturing unit of certain transformers the units are required to meet number of specifications (approximately 200) with which they are produced and subjected to inspection daily.*

*At the end 230 units have been rejected out of 4,150 produced and inspected.*

(*a*) *Determine the 3σ control limits on a daily process.*

(*b*) *Only one point has crossed the control limits. One that day 30 nonconforming items were found in 200 units inspected. What aimed value of P′ and C.L's would you recommend for the future.*

**Solution.** $\bar{p} = \frac{\Sigma d}{\Sigma n} = \frac{230}{4150} = 0.055$

$n = 200$

(*a*) Control limits for *p*-chart

$$UCL = \bar{p} + 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.055 + 3 \times \sqrt{\frac{0.055(1-0.055)}{200}}$$

$$= 0.1$$

$$LCL = 0.005 - 3 \times \sqrt{\frac{0.055(1-0.055)}{200}}$$

$$= 0.006$$

$$CL = 0.055$$

(*b*) $\bar{p}_1 = \frac{230 - 30}{4150 - 200} = 0.0506$

$$UCL = 0.0506 + 3 \times \sqrt{\frac{0.0506(1-0.0506)}{200}}$$

$$= 0.096$$

$$LCL = 0.0037$$

$$CL = 0.05$$

The recommended control limits for the future process are,

$$UCL = 0.096$$

$$LCL = 0.0037$$

$$CL = 0.05$$

Determination of single set of control limits when sub-group size is varying.

The max. sub-group size = $\bar{n}$ + 25% $\bar{n}$

The min. sub-group size = $\bar{n}$ – 25% $\bar{n}$

Check if there is any sub-group crossing the maximum and minimum subgroups size. For those subgroups which are crossing the control limits, calculate the individual control limits and for the remaining subgroups single set of control limits holds good.

**Problem 10.** *Table below gives the results of daily inspection of vacuum tubes. The standard fraction defective established at the start of month is 0.04. The estimated daily average production was 1,600.*

(*a*) *Establish a single set of control limits.*

(*b*) *Plot a control chart and take separate C. Is wherever they are needed.*

(*c*) *What value of standard fraction defective would you recommend for the future.*

| *Day* | *No. of items inspected* ($n$) | *No. of items rejected* ($d$) | *Fraction defective* $p = d/n$ |
|---|---|---|---|
| *1* | *1400* | *62* | *0.044* |
| *2* | *1950* | *41* | *0.021* |
| *3* | *900* | *27* | *0.030* |
| *4* | *1400* | *50* | *0.035* |
| *5* | *1395* | *63* | *0.045* |
| *6* | *2000* | *57* | *0.028* |
| *7* | *1300* | *64* | *0.049* |
| *8* | *1350* | *68* | *0.050* |
| *9* | *1500* | *73* | *0.048* |
| *10* | *1540* | *75* | *0.048* |

**Solution.** $\bar{p} = 0.04$, $\bar{n} = 1600$

Max. S.G. Size $= \bar{n} + 25\% \ \bar{n}$

$= 1600 + 25\%$ of $1600$

$= 1600 + 400$

$= 2000$

Minimum size of subgroup $= \bar{n} - 25\% \ \bar{n}$

$= 1600 - 25\% \ 1600$

$= 1600 - 400$

$= 1200$

(*a*) The single set of control limits are

$$\text{UCL} = \bar{p} + 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.04 + 3 \times \sqrt{\frac{0.04(1-0.04)}{1600}}$$

$= 0.054$

LCL $= 0.025$

CL $= 0.04$

The individual control limits for the 3rd day

$$UCL = 0.04 + 3 \times \sqrt{\frac{0.04\,(1 - 0.04)}{900}}$$

$$= 0.059$$

$$LCL = 0.0204$$

$$CL = \bar{p} = 0.04$$

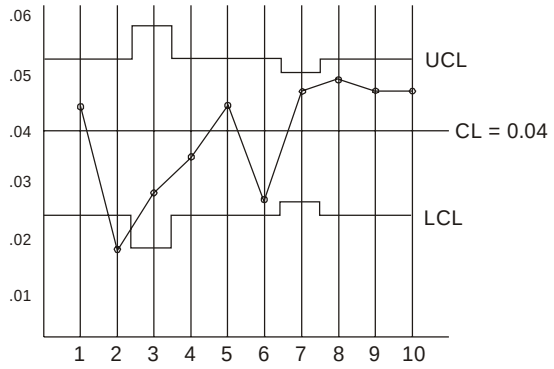
and for the 6th day

$$UCL = 0.04 + 3 \times \sqrt{\frac{0.04\,(1 - 0.04)}{2000}}$$

$$= 0.053$$

$$LCL = 0.026$$

$$CL = 0.04$$




Since all the points are falling below UCL the process is in a state of statistical control. The subgroup No. 2 is falling below LCL which indicates that the process is under improvement. The recommended value of $\bar{p}$ for the future period is $\bar{p} = 0.04$.

***np*-chart**

When the subgroup size is constant, the chart constructed for the actual no. of defectives rather than the fraction defectives is called *np*-chart.

**Advantages**

(1) *np*-chart is easier for operating personnel to understand.

(2) Inspection results are posted directly to chart without any calculations.

Control limits for *np*-chart are :

$$UCL = n\bar{p} + 3\sqrt{\frac{n\bar{p}\,(1 - \bar{p})}{n}}$$

$$\text{LCL} = n\bar{p} - 3\sqrt{\frac{n\bar{p}(1-\bar{p})}{n}}$$

$$\text{CL} = n\bar{p}\,.$$

**SOLVED PROBLEMS**

**Problem 1.** *A manufacturer uses a injection moulding to produce a plastic insulation barrier. He inspects 100 barriers daily picked randomly from the production and determines the no. of defects by visual inspection. He wishes to use the data accumulated during a 10 day period to construct an attribute chart. The results of inspection are shown below.*

(*a*) *Plot np-chart and offer your comments*

(*b*) *What control limits would you recommend for the future period.*

| *Lot No.* | *No of items rejected* |
|---|---|
| 1 | 6 |
| 2 | 14 |
| 3 | 18 |
| 4 | 10 |
| 5 | 2 |
| 6 | 20 |
| 7 | 18 |
| 8 | 5 |
| 9 | 12 |
| 10 | 8 |

**Solution.** $\Sigma d = 113$

$\Sigma n = 10 \times 100 = 1000$

$$\bar{p} = \frac{\Sigma d}{\Sigma n} = \frac{113}{1000} = 0.113$$

$$n\bar{p} = 100 \times 0.113 = 11.3$$

Control limits for *np*-chart

$$\text{UCL} = n\bar{p} + 3 \times \sqrt{n\bar{p}(1-\bar{p})}$$

$$= 11.3 + 3 \times \sqrt{11.3\,(1 - 0.113)}$$

$$= 20.79$$

$$\text{LCL} = n\bar{p} + 3 \times \sqrt{n\bar{p}(1-\bar{p})}$$

$$= 11.3 - 3 \times \sqrt{11.3\,(1 - 0.113)}$$

$$= 1.80$$

$$\text{CL} = n\bar{p} = 11.3$$

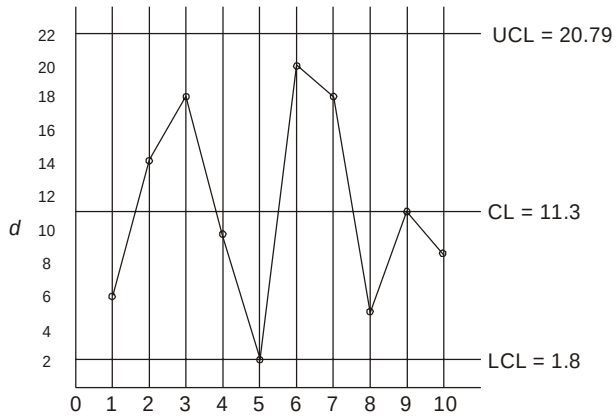
It is seen from the data that all the points are falling within the control limits so the final values are

UCL = 20.79

LCL = 1.8

CL = 11.3

The chart is plotted below :




The process is in a state of statistical control.

**Problem 2.** *Sample of 200 components taken at random each day are gauged as means of checking. Following results are obtained for 15 days in terms of fraction defective. Plot an np-chart and offer your comments.*

| *S.G. No.* | *Fraction defective* | *No. of defectives* |
|---|---|---|
| | *p* | *d* |
| *1* | *0.045* | *9* |
| *2* | *0.05* | *10* |
| *3* | *0.045* | *9* |
| *4* | *0.06* | *12* |
| *5* | *0.035* | *7* |
| *6* | *0.055* | *11* |
| *7* | *0.065* | *13* |
| *8* | *0.075* | *15* |
| *9* | *0.075* | *15* |
| *10* | *0.045* | *9* |
| *11* | *0.055* | *11* |
| *12* | *0.05* | *10* |
| *13* | *0.06* | *12* |
| *14* | *0.035* | *7* |
| *15* | *0.045* | *9* |

**Solution.** $\Sigma d = 159$

$\Sigma n = 15 \times 200 = 3000$

$$\bar{p} = \frac{159}{3000} = 0.053$$

$$n\bar{p} = 200 \times 0.053 = 10.6$$

Control limits for *np*-chart

$$UCL = n\bar{p} + 3 \times \sqrt{n\bar{p}(1-\bar{p})}$$

$$= 10.6 + 3 \times \sqrt{10.6\,(1 - 0.053)}$$

$$= 20.1$$

$$LCL = n\bar{p} - 3 \times \sqrt{n\bar{p}(1-\bar{p})}$$

$$= 10.6 - 3 \times \sqrt{10.6\,(1 - 0.053)}$$

$$= 1.09$$

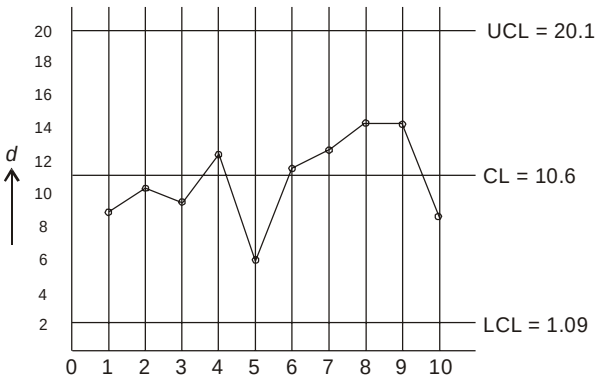
$$CL = n\bar{p} = 10.6$$

It is seen from data that all the points are falling within the control limits. So the values are

UCL = 20.1

LCL = 1.09

CL = 10.6




The process is in a state of statistical control.

**Example 3.** *The following are the inspection results of 20 lots of magnets. Each lot being 750. The no. of defective magnets are given below.*

| *S.G. No.* | *Defects (d)* | *S.G. No.* | *Defects* |
|---|---|---|---|
| *1* | *48* | *11* | *47* |
| *2* | *83* | *12* | *50* |
| *3* | *70* | *13* | *47* |

| | | | |
|---|---|---|---|
| *4* | *85* | *14* | *57* |
| *5* | *45* | *15* | *51* |
| *6* | *56* | *16* | *71* |
| *7* | *48* | *17* | *53* |
| *8* | *67* | *18* | *34* |
| *9* | *37* | *19* | *29* |
| *10* | *52* | *20* | *30* |

**Solution.** Construct a suitable chart and state whether the process is in control or not.

$$n = 750$$

$$\Sigma d = 1060$$

$$\Sigma n = 20 \times 750 = 15000$$

$$\bar{p} = \frac{\Sigma d}{\Sigma n} = \frac{1060}{15000} = 0.0706$$

$$n\bar{p} = 750 \times 0.0706 = 52.95$$

Control limtis for *np*-chart

$$\text{UCL} = n\bar{p} + 3 \times \sqrt{n\bar{p}(1-\bar{p})}$$

$$= 52.95 + 3 \times \sqrt{52.95(1-0.0706)}$$

$$= 73.99$$

$$\text{LCL} = n\bar{p} - 3 \times \sqrt{n\bar{p}(1-\bar{p})}$$

$$= 52.95 - 3 \times \sqrt{52.95(1-0.0706)}$$

$$= 31.90$$

$$\text{CL} = n\bar{p} = 52.95$$

It is seen from the data that two subgroups are crossing UCL and two subgroups are below LCL, so homogenisation is necessary.

$$\bar{p}_1 = \frac{1060 - 83 - 85 - 29 - 30}{16 \times 750} = 0.0694$$

$$n\bar{p}_1 = 750 \times 0.0694 = 52.05$$

$$\text{UCL} = 52.05 + 3 \times \sqrt{52.05(1-0.0694)}$$

$$= 72.92$$

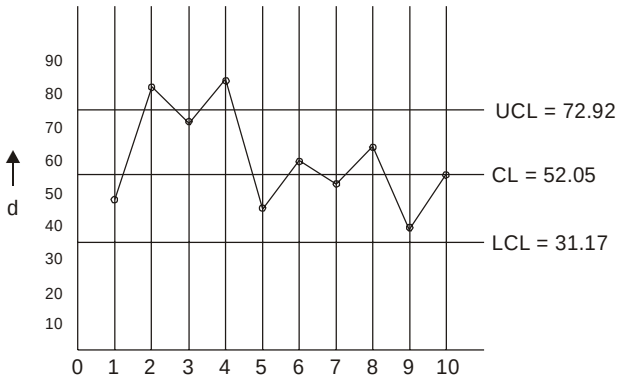
$$\text{LCL} = 52.05 - 3 \times \sqrt{52.05(1-0.0694)}$$

$$= 31.17$$

$$CL = n\bar{p}_1 = 52.05$$

Now all the points are falling within the control limits so the final values are

$$UCL = 72.92$$
$$LCL = 31.17$$
$$CL = 52.05$$




It is seen that subgroups nos. 2 and 4 are crossing the UCL, which indicates the presence of assignable causes so the process is not in a state of statistical control. The sub group no. 19 and 20 are falling below LCL which indicates that the process is under improvement. Find the reasons and maintain the same consistantly.

**Problem 4.** *An item is mode in lots of 200 each. Lots are given 100% inspection. The required sheet for the first 25 groups showed that a total of 75 items were defective.*

(*a*) *Determine the trial control limis for np-chart.*

(*b*) *Assume that all the points are falling within the control limits. What is your estimate of the process average fraction defective 'p'.*

(*c*) *If 'p' remains unchanged what is the probability that the 26th will contain exactly 7 defectives.*

(*d*) *That it will contain 7 or more defectives.*

**Solution.**

$$n = 200$$
$$K = 25$$
$$\Sigma d = 75$$
$$\Sigma n = 25 \times 2000 = 5000$$

$$\bar{p} = \frac{\Sigma d}{\Sigma n} = \frac{75}{5000} = 0.015$$

$$n\bar{p} = 200 \times 0.015 = 3$$

Control limits for *np*-chart

(*a*) $UCL = n\bar{p} + 3 \times \sqrt{n\bar{p}(1-\bar{p})}$

$= 3 + 3 \times \sqrt{3(1-0.015)}$

$= 8.15$

$LCL = n\bar{p} - 3 \times \sqrt{n\bar{p}(1-\bar{p})}$

$= 3 - 3 \times \sqrt{3(1-0.015)}$

$= -2.15$

$\sim 0.0$

$CL = n\bar{p} = 3.$

(*b*) Since all the points are falling within the control limits.

The estimated process average fraction defective.

$p^1 = \bar{p} = 0.015.$

(*c*) It follows a binomial distribution.

$p + q = 1$

$n = 200$

$q = 1 - p$

$= 1 - 0.015$

$= 0.985$

$n = 7$

$P(r) = {}^nC_r \; P^r q^{n-r}$

$= 200c_7 \; (0.015)^5 \; (0.985)^{193}$

$= 0.0211$

$= 2.11\%$ $\qquad \dfrac{200!}{7!193!}.$

(*d*) 7 or more defectives.

$= 1 - [P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)]$

It is difficult to calculate by Binomial distribution, so from poisson's table for

$np^1 = 3$

$c = 6$

the probability is = 0.966

$1 - 0.0966 = 0.034$

$= 3.4\%$

or Normal approximation to the binomial

$$Z = \frac{x - \mu}{\sigma} = m = np^1$$

$$\sigma = \sqrt{npq}$$

$$Z = \frac{6 - 3}{\sqrt{npq}} = \frac{6 - 3}{\sqrt{200 \times 0.015 \times 0.985}}$$

$$= 1.745$$

from tables

probability = 0.9554

1 – 0.9554 = 0.0446 = 4.46%.

**Problem 5.** *An electrical parts manufacturer requires that 100% inspection be made durng the first 6 months of production. A total of 600 items are found defective in the first so lots of new items manufactured in a lot of 300.*

(*a*) *Compute the trial control limits for np-chart.*

(*b*) *If the process average is increased by 4, what is the probability that the sent lot inspected will exceed the original UCL.*

**Solution.** $\Sigma n = 50 \times 300 = 15000$

$\Sigma d = 600$

$$\bar{p} = \frac{\Sigma d}{\Sigma n} = \frac{75}{5000} = 0.015$$

$$n\bar{p} = 200 \times 0.015 = 3$$

Control limits for *np*-chart

(*a*) $UCL = n\bar{p} + 3 \times \sqrt{n\bar{p}(1 - \bar{p})}$

$= 12 + 3 \times \sqrt{12(1 - 0.04)}$

$= 22.18$

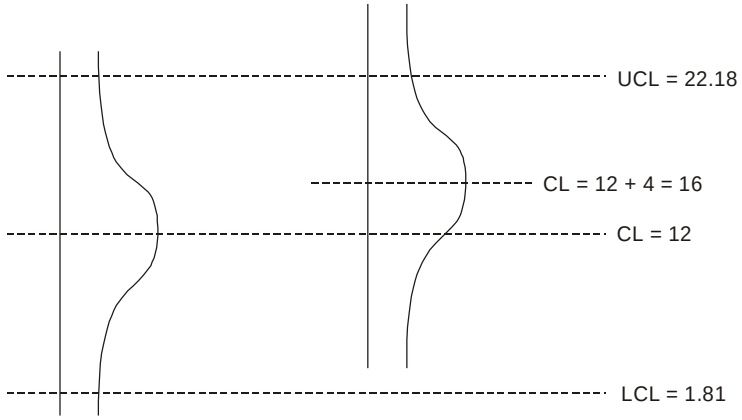
$LCL = n\bar{p} - 3 \times \sqrt{n\bar{p}(1 - \bar{p})}$

$= 12 - 3 \times \sqrt{12(1 - 0.04)}$

$= 1.81$

$CL = n\bar{p} = 12$

(*b*) New process average = 12 + 4 = 16




$Z = \frac{x - \mu}{\sigma}$ $\mu = 16$

$Z = \frac{22.18 - 16}{3.39}$ $\sigma = \sqrt{npq}$

$p = 0.04$

$z = 1 - 0.04$

$= 1.82$ $= 0.96$

from tables

probability = 0.9656 $\sigma = \sqrt{300 \times 0.04 \times 0.96}$

= 96.56% = 3.39

100 – 96.56 = 3.44%.

**C-Chart or U-chart**

When the subgroup size is unity : C-chart and when the subgroup size is > 1, uniform or non-uniform : U-chart.

Examples : Number of defects in a casting or number of imperfections in a certain area of cloth or Number of mistakes per page.

There are many cases in an industry where the number of defects is considered as important. The pattern of variation of the number of defects per unit is usually explained by poisson's distribution. If the average no. of defects per unit is usually 'μ', then the probability of occurrence of '*r*' defects in a sample is given by

$$P(r) = \frac{e^{-\mu}\,\mu^r}{r\,!}$$

The standard deviation = $\sigma = \sqrt{\mu}$

Steps involved in the construction of C-chart (when the subgroup size is unity)

| *S.G. No.* | *No. of items inspected* | *No. of defects per unit (c)* |
|---|---|---|
| 1 | 1 | $C_1$ |
| 2 | 1 | $C_2$ |
| 3 | 1 | $C_3$ |
| 4 | 1 | $C_4$ |
| ⋮ | ⋮ | ⋮ |

The average no. of defects per unit

$$\bar{C} = \frac{\Sigma c}{\Sigma n}$$

Where $\Sigma c$ is the total no. of defects in all the samples. $\Sigma n$ is the total no. of items inspected in all the samples.

**Step 1 :** Collect about 25 samples of size $n(n = 1)$ and count the number of defects in each sample.

**Step 2 :** Compute the average no. of defects per sample.

**Step 3 :** Determine the control limits using the standard value of $\bar{C}$.

$$UCL = \bar{C} + 3\sqrt{\bar{C}}$$

$$LCL = \bar{C} - 3\sqrt{\bar{C}}$$

$$CL = \bar{C}.$$

**Step 4 :** Compare the no. of defects in each sample with its control limits. If all the samples are falling within the control limits then homogenisation is not necessary otherwise homogenise the sample by deleting the samples which are crossing the control limits and determine $\bar{C}_1$.

**Step 5 :** Compute the revised control limits using $\bar{C}_1$. If LCL is –ve consider it as zero.

**Step 6 :** Plot a chart with the no. of defects per sample on vertical axis and the subgroup no. On the horizontal axis.

**Step 7 :** Interpret the result same as in *p*-chart.

**Applications of C and U chart**

1. Number of surface defects in a galvanised sheet.
2. Number of imperfections in a certain area of cloth.
3. Number of defective units in an air craft unit.
4. Number of mistakes per unit.

**SOLVED PROBLEMS**

**Problem 1.** *The following data refers to the no. of missing rivets on an aircraft body noticed during preventive maintenance schedule.*

(*a*) *Compute the control limits for a suitable control chart.*

(*b*) *Plot the data and offer your comments.*

(*c*) *What value of* $C^1$ *would you recommend for the future period.*

| *Air craft No.* | *No. of missing rivets* (*c*) |
|---|---|
| 1 | 8 |
| 2 | 15 |
| 3 | 15 |
| 4 | 19 |
| 5 | 9 |
| 6 | 15 |
| 7 | 9 |
| 8 | 12 |
| 9 | 21 |
| 10 | 13 |
| 11 | 21 |
| 12 | 16 |
| 13 | 9 |
| 14 | 23 |
| 15 | 15 |

**Solution.** $\Sigma n = 15$

$\Sigma c = 220$

$$\bar{C} = \frac{\Sigma c}{\Sigma n} = \frac{220}{15}$$

$= 14.66$

Control limits for C chart.

$$UCL = \bar{C} \times 3\sqrt{C}$$

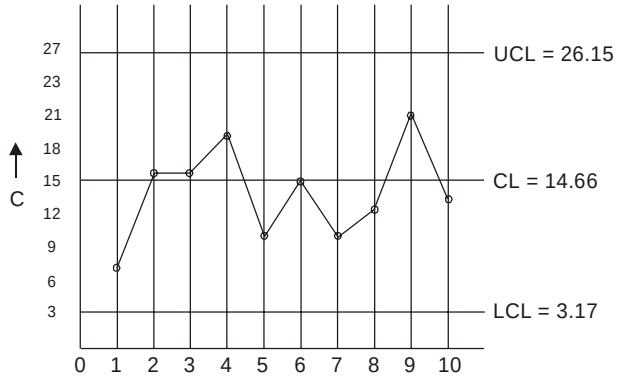
$$= 14.66 + 3\sqrt{14.66}$$

$= 26.15$

$LCL = 3.17$

$CL = 14.66$

It is seen from the data that all the points are falling within the control limits.




The process is in a state of statistical control. The recommended value of $C^1$ for the future period is $\overline{C}$.

**Problem 2.** *The following are the inspection results of 100 mtrs. of a piece of woollen cloth.*

(*a*) *Determine the control limits for C-chart.*

(*b*) *Plot the data and offer your comments.*

(*c*) *What value of* $C^1$ *would you suggest for the future period.*

| *Piece No.* | *No. of defects* (*c*) |
|---|---|
| *1* | *3* |
| *2* | *3* |
| *3* | *6* |
| *4* | *3* |
| *5* | *0* |
| *6* | *1* |
| *7* | *3* |
| *8* | *5* |
| *9* | *7* |
| *10* | *8* |

**Solution.** $\Sigma n = 10$

$\Sigma c = 39$

$$\overline{C} = \frac{\Sigma c}{\Sigma n} = \frac{39}{10} = 3.9$$

Control limits for C-chart

$$UCL = \overline{C} + 3\sqrt{\overline{C}}$$

$$= 3.9 + 3\sqrt{3.9}$$

$= 9.82$

$$LCL = \bar{C} - 3\sqrt{\bar{C}}$$

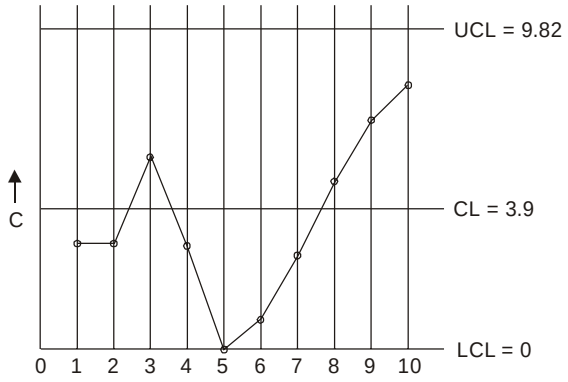
$$= 3.9 - 3\sqrt{3.9}$$

$$= -2.02$$

$$\sim 0.0$$

$$CL = 3.9$$

It is seen for the data that all the points are falling within the control limits.




The process is in a state of statistical control. The recommended value of $C^1$ for the future period is $\bar{C} = 3.9$.

**Problem 3.** *A textile manufacturer initiates the use of C-chart to monitor the no. of imperfections found in a bale of cloth. Each is of same length, width and fibre composition. A total 191 imperfections were found in the last 25 bales inspected. The four highest and lowest counts were as follows.*

| *Highest* | *Lowest* |
|---|---|
| *22* | *4* |
| *19* | *4* |
| *14* | *5* |
| *12* | *5* |

(*a*) *Calculate the 3σ control limits*

(*b*) *Is the process in control*

(*c*) *If not what aimed value of $C^1$ and control limits would you suggest for the future period.*

**Solution.** $\Sigma c = 191$

$\Sigma n = 25$

$$\bar{C} = \frac{\Sigma c}{\Sigma n} = \frac{191}{25} = 7.64$$

Control limits for C-chart

$$UCL = \bar{C} + 3\sqrt{\bar{C}}$$

$$= 7.64 + 3\sqrt{7.64}$$

$$= 15.93$$

$$LCL = \bar{C} - 3\sqrt{\bar{C}}$$

$$= 7.64 - 3\sqrt{7.64}$$

$$= -0.65$$

$$\sim 0.0$$

$$CL = \bar{C} = 7.64$$

It is seen from the data that two subgroups are crossing the cross limits UCL. So homogenisation is necessary.

$$\bar{C}_1 = \frac{191 - 22 - 19}{23} = \frac{150}{23} = 6.52$$

Again control limits.

$$UCL = \bar{C}_1 - 3\sqrt{\bar{C}_1}$$

$$= 6.52 + 3\sqrt{6.52}$$

$$= 14.18$$

$$LCL = \bar{C}_1 - 3\sqrt{\bar{C}_1}$$

$$= 6.52 - 3\sqrt{6.52}$$

$$= -1.14$$

$$LCL = 0.0$$

$$CL = \bar{C}_1 = 6.52$$

Now all the points are falling within the control limits. But the process is not in a state of statistical control.

The recommended value of $C^1$ for the future period = $\bar{C}_1 = 6.52$ and C.LS.

$$UCL = 14.18$$

$$LCL = 0.0$$

$$CL = 6.52.$$

**Problem 4.** *The number of students failing in an university exam. was analysed college wise. The count of failures notices in 15 colleges are as follows.*

| *College Code* | *No. of failures* |
|---|---|
| *a* | *6* |
| *b* | *12* |
| *c* | *10* |
| *d* | *10* |

| | |
|---|---|
| *e* | *11* |
| *f* | *13* |
| *g* | *37* |
| *h* | *31* |
| *i* | *33* |
| *j* | *38* |
| *k* | *10* |
| *l* | *11* |
| *m* | *12* |
| *n* | *10* |
| *o* | *9* |

*Analyse the data through an appropriate control chart and check whether the assignable causes at colleges are responsible for high failures.*

***Solution.*** $\Sigma c = 15$

$\Sigma n = 253$

$$\overline{C} = \frac{\Sigma c}{\Sigma n} = \frac{253}{15} = 16.86$$

Control limits for C-chart

$$UCL = \overline{C}_1 + 3\sqrt{\overline{C}_1}$$

$$= 16.86 + 3\sqrt{16.86}$$

$$= 29.17$$

$$LCL = \overline{C}_1 - 3\sqrt{\overline{C}_1}$$

$$= 16.86 - 3\sqrt{16.86}$$

$$= 4.54$$

$$CL = \overline{C} = 16.86$$

It is seen from the data that 4 subgroups are crossing the UCL. Which indicates the presence of assignable causes. So the homogenisation is necessary.

$$C_1 = \frac{253 - 37 - 33 - 38}{11} = 10.36$$

Again control limits.

$$UCL = \overline{C}_1 + 3\sqrt{\overline{C}_1}$$

$$= 10.36 + 3\sqrt{10.36}$$

$$= 20.01$$

$$LCL = \overline{C}_1 - 3\sqrt{\overline{C}_1}$$

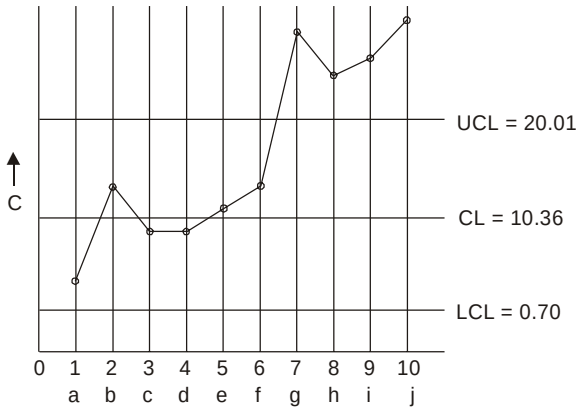
$= 10.36 - 3\sqrt{10.36}$

$= 0.70$

$CL = \bar{C}_1 = 10.36$

Now all the points are falling within the control limits.

The chart is plotted below.




It is seen that 4 subgroups are crossing the UCL which indicates that this high failure is due to the assignable causes at colleges which may be lack of teachers or any other reason.

**Problem 5.** *A cloth manufacturing company is maintaining 100% inspection of all the finished fabric. During the past 8 months data from the inspection it indicates that the average no. of defects per unit of fabric in 8.*

(*a*) *Determine the 3σ control limits for the C-chart.*

(*b*) *With these control limits what is the probability that the process is out of control when it is really not.*

**Solution.** $\bar{C} = 8$

$UCL = \bar{C} + 3\sqrt{\bar{C}}$

$= 8 + 3\sqrt{8}$

$= 16.48$

$LCL = \bar{C} - 3\sqrt{\bar{C}}$

$= 8 - 3\sqrt{8}$

$= - 0.48$

$\sim 0.0$

$CL = 8$

The probability that the process is out of control when it is really not is from the poisson tables for

$$\overline{C} = C^1 = 8$$

Number of defects = UCL = 16.48 ~ 17.0(*c*)

The probability is

$$0.998$$

$$1 - 0.998 = 0.002$$

$$= 0.2\%$$

This is called the type 1 error because of the chance causes.

**Problem 6.** *A sheet metal manufacturer records the no. of defects found on each sheet of metal produced. From the inspection it has been found that the average no. of defects per sheet is 9.*

(*a*) *Determine the control limits for C-chart.*

(*b*) *With there control limits what is the probability that the process is out of control when it is really not.*

**Solution.** $\overline{C} = 9$

Control limits for C-chart.

$$UCL = \overline{C} + 3\sqrt{\overline{C}}$$

$$= 9 + 3\sqrt{9}$$

$$= 12.0$$

$$LCL = 9 - 3\sqrt{9}$$

$$= 0.0$$

$$CL = 9$$

The probability that the process is out of control when it is really not.

From poisson's tables for

$$\overline{C} = C^1 = 9$$

No. of defects = UCL = 12

The probability = 0.876

$$1 - 0.876.$$

**Problems on U-chart**

**Problem 1.** *Listed below are the cloths produced on a daily basis in a small textile mill and the corresponding number of imperfections found in their bales is as follows.*

(*a*) *Use the data to estimate* $\overline{U}$.

(*b*) *Determine the control limits and plot the data.*

(*c*) *What value of* $U^1$ *would you recommend for the future period.*

| *S.G. No.* | *No. of bales inspected* | *No. of imperfections* | *U = c/n* | *UCL* | *LCL* |
|---|---|---|---|---|---|
| *1* | *20* | *27* | *1.35* | *2.17* | *0.59* |
| *2* | *20* | *23* | *1.15* | *2.17* | *0.59* |
| *3* | *20* | *30* | *1.5* | *2.17* | *0.59* |
| *4* | *21* | *28* | *1.33* | *2.14* | *0.61* |
| *5* | *22* | *29* | *1.32* | *2.13* | *0.63* |
| *6* | *22* | *31* | *1.41* | *2.13* | *0.63* |
| *7* | *23* | *37* | *1.60* | *2.11* | *0.645* |
| *8* | *23* | *29* | *1.26* | *2.11* | *0.645* |
| *9* | *23* | *36* | *1.56* | *2.11* | *0.645* |
| *10* | *21* | *27* | *1.28* | *2.14* | *0.61* |

**Solution.** $\bar{U} = \frac{\Sigma c}{\Sigma n} = \frac{297}{215} = 1.38$

Specimen calculation for first reading

$$UCL = \bar{U} + 3\sqrt{\frac{\bar{U}}{n}}$$

$$= 1.38 + 3\sqrt{\frac{1.38}{20}}$$

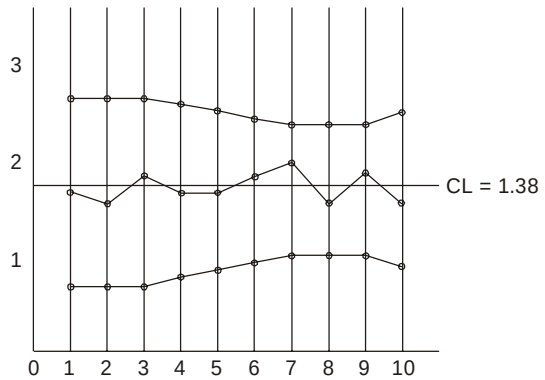
$$= 2.17$$

$$LCL = \bar{U} - 3\sqrt{\frac{\bar{U}}{n}}$$

$$= 1.38 - 3\sqrt{\frac{1.38}{20}}$$

$$= 0.59$$

Similarly UCL and LCL is calculated for all other subgroups.



Since all the points are falling within the control limits, the process is in a state of statistical control.

The recommended value of $U^1$ for the future period is

$$\overline{U} = 1.38.$$

**Problem 2.** *Determine the control limits for the following data using suitable control chart. Plot the data and offer your comments.*

| *Sub Group No.* | *Sample size* ($n$) | *No. of defectives* ($c$) | $U = c/n$ |
|---|---|---|---|
| *1* | *10* | *45* | *4.5* |
| *2* | *10* | *51* | *5.1* |
| *3* | *10* | *36* | *3.6* |
| *4* | *10* | *48* | *4.8* |
| *5* | *10* | *46* | *4.6* |
| *6* | *10* | *25* | *2.5* |
| *7* | *10* | *33* | *3.3* |
| *8* | *10* | *46* | *4.6* |
| *9* | *10* | *32* | *3.2* |
| *10* | *10* | *42* | *4.2* |

**Solution.** $\Sigma n = 100$

$\Sigma c = 404$

$$\overline{U} = \frac{404}{100} = 4.04$$

$$USL = \overline{U} + 3 \times \sqrt{\frac{\overline{U}}{n}}$$

$$= 4.04 + 3 \times \sqrt{\frac{4.04}{10}}$$

$$= 5.94$$

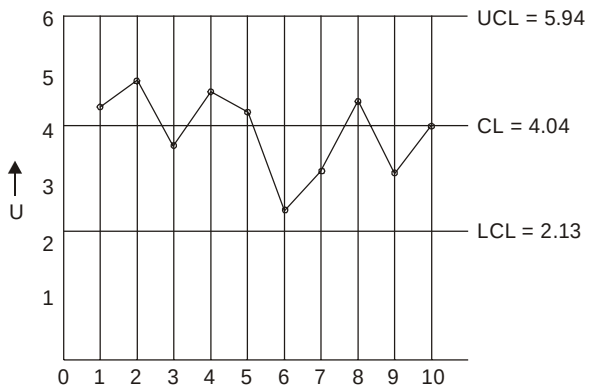
$$LCL = \overline{U} - 3 \times \sqrt{\frac{\overline{U}}{n}}$$

$$= 4.04 - 3 \times \sqrt{\frac{4.04}{10}}$$

$$= 2.13$$

$$CL = U = 4.04$$

Since all the points are within the C.LS. The chart is plotted below.




The process is in a state of statistical control.

**Problem 3.** *Determination of UCL and LCL for the U chart based on the probability limits.*

*A control chart for defects per unit (U) uses the probability limits corresponding to the probabilities of 0.975 and 0.025. The control line on the control chart is at* $\overline{U} = 2$. *The limits vary with the value of n. Determine the correct position of UCL and LCL when n = 5.*

**Solution.** $\overline{U} = 2$

$n = 5$

For the probabilities 0.975 and 0.025 from the normal tables the Z values are 1.96 and – 1.96 respectively.

The control limits for U-chart.

$$UCL = \overline{U} + Z\sqrt{\frac{\overline{U}}{n}}$$

$$= 2 + 1.96\sqrt{\frac{2}{5}} = 3.23$$

$$LCL = 2 - 1.96\sqrt{\frac{2}{5}}$$

$$= 0.76$$

$CL = \overline{U} = 2$

UCL = 3.23

LCL = 0.76

CL = 2.0

**Problem 4.** *A control chart for defects per units (U) uses the probability limits corresponding to the probabilities of 0.95 and 0.55. At* $\overline{U} = 1.5$, *the limits vary with the value of n. Determine the correct position of UCL and LCL when the sample size in 6 = 8.*

**Solution.** $\overline{U} = 1.5$

$n = 6$

For the probabilities 0.95 and 0.050, from the normal tables the Z values are

1.645 and – 1.645

The control limits for U-chart.

$$UCL = \overline{U} + Z\sqrt{\frac{\overline{U}}{n}}$$

$$= 1.5 + 1.645\sqrt{\frac{1.5}{6}}$$

$$= 2.3225$$

$$LCL = \overline{U} - Z\sqrt{\frac{\overline{U}}{n}}$$

$$= 1.5 - 1.645\sqrt{\frac{1.5}{6}}$$

$$= 0.6775$$

$$CL = \overline{U} = 1.5$$

So when $n = 6$, the C.L.S are

UCL = 2.3225

LCL = 0.6775

CL = 1.5

when $n = 8$

$$UCL = 1.5 + 1.645\sqrt{\frac{1.5}{8}}$$

$$= 2.2123$$

$$LCL = 1.5 - 1.645\sqrt{\frac{1.5}{8}}$$

$$= 0.7876$$

$$CL = \overline{U} = 1.5$$

**Type-I and Type-II errors with respect to the control charts**

Control limits are usually established at $3\sigma$ from the central line. They are used as a basis to judge whether there is an evidence of lack of control. The choice of $3\sigma$ limits is an economic one with respect to two types of errors that can occur. They are Type-I and Type-II errors.

Type-I error occurs when assuming that an assignable cause of variation, when really a chance cause is present. When the limits are set at $3\sigma$ the type-I error will occur at 0.27%. In other words when a point is outside the control limits, it is assumed to be due to assignable causes even though it would be due to chance causes of 0.27%.

Type-II error occurs when assuming that a chance cause of variation is present when really an assignable cause is present. In other words when a point is inside the control limits it is assumed to be because of the chance causes even though it would be due to the assignable causes.

**Difference between P-chart and NP-chart**

| **Data** | **P-chart** | **NP-chart** |
|---|---|---|
| Sample size | Varying | Constant |
| Plotted variable | Fraction defective | Number defecitve |
| Control limits | Vary with sample size | UCL, LCL are fined |
| Standard deviation | $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ | $\sqrt{n\bar{p}(1-\bar{p})}$ |
| Centre line | $\bar{p}$ | N $\bar{p}$ |

**Difference between C-chart and U-chart**

| Data | C-chart | U-chart |
|---|---|---|
| Sample size | Constant | Varying |
| Plotted variable | C | U = $c/n$ |
| Control limits | UCL, LCL, fined | Vary with sample size |
| Standard deviation | $\sqrt{\bar{C}}$ | $\sqrt{\bar{U}/n}$ |
| Centre line | $\bar{C}$ | $\bar{U}$ |

# 8 Acceptance Sampling

## INTRODUCTION

Acceptance sampling by Attributes is the most common type of sampling. A predetermined number of units (sample) from each lot is inspected by attributes. If the amount of defectives is less than or equal to the prescribed minimum the lot is accepted, if not the lot is rejected as it falls below standard.

A single sampling plan is determined by a lot size (N), the sample size ($n$) and the acceptance number ($c$). For example consider a sampling plan having N = 9000, $n$ = 300 and $c$ = 7. This means that a lot of 9000 units has 300 units inspected and if 7 or less defectives are found in 300 units of sample, the lot is accepted. If more than 7 defectives are found in the sample of 300 units the lot is rejected.

Acceptance sampling can be performed in a number of different situations where customer-producer relationships exist. The customer and producer can be two different companies, two plants within the same company or two departments within the same plant.

In any case there is always the problem associated with the acceptance of goods. Acceptance sampling of the product is most likely to be used in one of the following three situations :

1. When the test is destructive, sampling plan is necessary otherwise all the products will be destroyed by testing.
2. When the cost of 100% inspection is very high, sampling can save money.
3. When there are many similar items to be inspected, the sampling inspection will produce as good, if not better results than 100% inspection. This is true because the manual or 100% inspection causes fatigue and boredom and results in a very high rate of defective material.

**Advantages and Disadvantages of the sampling plan compared with 100% inspection**

**Advantages :**

1. More economical owing to fewer inspection.
2. Less handling damages during inspection.
3. Upgrading the inspection job from piece by piece decision to lot by lot.

**Disadvantages :**

1. There is a certain risk of accepting the bad lot and rejecting the good lot.
2. More time and effort is devoted to plan the documentation.
3. Less information is usually provided about the product.

**Comparison between 100% inspection and sampling inspection.**

| 100% Inspection | Sampling Inspection |
|---|---|
| 1, It is costly & time consuming<br>2. It is not practicable for man produced components.<br>3. It is not feasible where destructive test is applied.<br>4. 100% inspection is not 100% efficient because of fatigue and monotony. | 1. It is cheap, quick and easy.<br>2. It is practicable for man produced components.<br>3. It is the only alternative where destructive testing is involved.<br>4. Because of the low quantity involved, inspection is efficient. |

**Guidelines for the Formation of Lots**

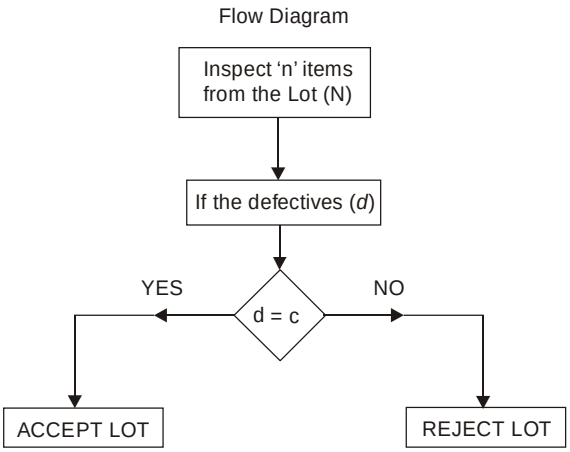
1. The lot should be homogenous that is all the products in the lot are produced by the same machine, same operator and same input material.
2. The lot should be as large as possible.
3. Lots should be suitable for material handling.
4. Lots should be confirmed to packing principle.

**TYPES OF SAMPLING PLANS**

1. Single sampling plan.
2. Double sampling plan.
3. Multiple sampling plan.
4. Sequential sampling plan.

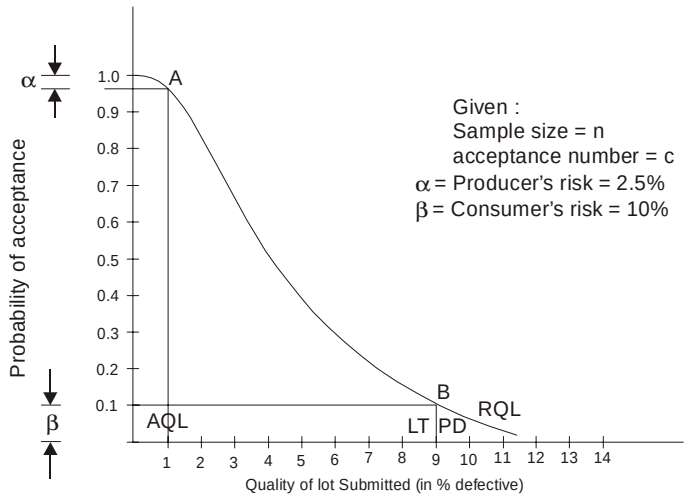
**Single Sampling Plan**

A single sampling plan is a plan, in which, one sample of specific size is taken to decide on the acceptability of the lot. The following are the parameters on which it operates : N = Lot size ; $n$ = sample size and $c$ = Acceptance number.



**OPERATING CHARACTERISTIC CURVE (O.C. CURVE)**

Operating Characteristic Curve is a plot of submitted lot quality versus the Probability of acceptance. It is an excellent evaluation technique of a sampling plan. In judging a particular sampling plan it is desirable to know the probability that the lot submitted with a certain percentage defective will be accepted or rejected. The following figure shows an OC curve.




The figure indicates a 100% probability of acceptance of submitted lot when the quality of lot is zero percent defective and a near 0% probability of acceptance for those lots having 12% defective. Point A and B on the OC curve refer respectively to the acceptable quality level (AQL) and the lot tolerance percent defective (LTPD). Point A is situated on the OC curve at an intersection of a horizontal drawn through the 97.5% probability point. While point B, is at the intersection of 10% horizontal line and the curve. Their probability percentages agree with AQL and LTPD acceptance probabilities that are common.

The corresponding quality of lots submitted for inspection in the figure shown is AQL 1% and LTPD is 9.1%. The OC curve permits the determination of the probability of acceptance for lots of varying quality. The OC Curve depicts the graphical relationship between arrangement of lot quality levels and the corresponding probability of acceptance and is based on the poisson's distribution.

The following two parameters are needed to calculate and plot an OC Curve : *n* = Sample size and *c* = Acceptance number.

Assume a range of values of percent defectives for the lot submitted for inspection and calculate the expected number of defectives by multiplying by it with the sample size '*n*'.

**Type A and Type B OC curves**

| Type A | Type B |
|---|---|
| 1, The lot size in finite. | 1. The lot size is infinite. |
| 2. The hypergeometric distribution will give theoretically correct result. | 2. The binomial distribution will give theoretically correct result. |
| 3. Binomial & Poisson's distribution will give approximate result. | 3. Poisson's distribution will give approximate result. |
| 4. Sample size is less than 1/10th of the lot size. | 4. Sample size is greater than or equal to 1/10th of the lot size. |
| 5. It will be used for the evaluation of consumer's risk. | 5. It will be used for the evaluation of producer's risk. |

**Note.** If the question is specified as plot the type A O.C. curve then the probability of acceptance must be calculated using hypergeometric distribution. If it is specified as type B O.C. curve then the probability of acceptance must be calculated using the binomial distribution.

**AQL (Acceptance Quality Level)**

The AQL of a sampling plan represents a level of quality of the incoming lots which in the opinion of the customer is acceptable to him at anytime. The customer is always willing to buy the lots of AQL quality.

**LTPD (Lot Tolerance Percent Defective)**

The LTPD of a sampling plan represents a level of quality of the incoming lots which in the opinion of the customer represent a quality of limiting worseness and hence is not acceptable to him at anytime.

**Producer's Risk ($\alpha$)**

Producer's risk of an acceptance sampling plan is defined as the probability of rejection of a lot of AQL quality. The risk should be kept as low as possible. The producer can decrease this risk by producing the material at a better quality level.

**Consumer's Risk ($\beta$)**

Consumer's risk of an acceptance sampling plan is defined as the probability of acceptance of a lot of LTPD quality.

**AOQL (Average Outgoing Quality Level)**

The AOQL of an acceptance rectification sampling plan is defined as a long run limiting value of AOQ (Average Out going Quality) that can occur with the usage of such sampling plan, irrespective of the quality of submitted lots.

**Chracteristics of a Good Sampling Plan**

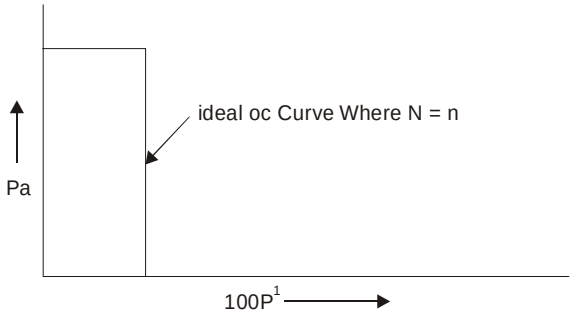
To protect the producer against having any lot rejected when his product is in a state of control and satisfactory as to level and uniformity.

1. To protect the consumer against the acceptance of a bad lot.
2. To give a long run protection to the consumers.

3. To encourage the producer to maintain his quality.
4. To minimise the cost of sampling inspection and administration.
5. To provide information for the quality of the product.

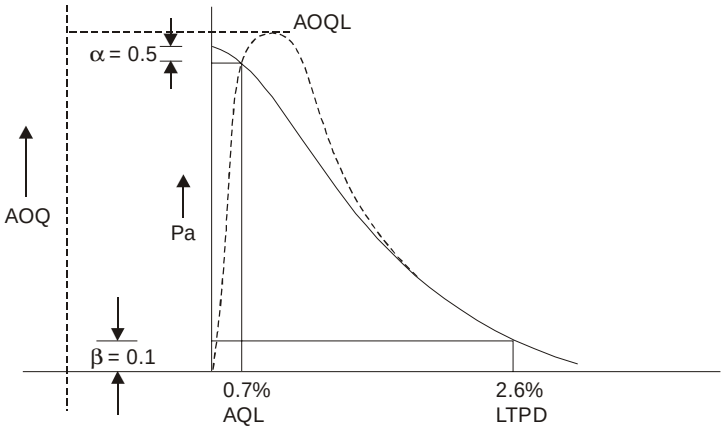
**Relation between the Consumer and Producer**

When the acceptance sampling is used, there is a conflicting interest between the consumer and the producer, the producer wants all his good lots to be accepted and the consumer wants all bad lots to be rejected. Only an ideal sampling plan which has an OC curve, that is, a vertical line can satisfy both the producer and the consumer. An ideal OC curve is shown below.




IDEAL OC CURVE

The above can be only achieved by 100% inspection. Therefore the sampling inspection carries a risk of rejecting a good lot and accepting a bad lot. The producer risk ($\alpha$) is the probability of rejection of good lot. The risk is frequently given as 0.05. But it can range from 0.01 to 0.1. Since $\alpha$ is expressed in terms of probability of rejection, it cannot be located on the OC Curve unless specified in terms of probability of acceptance. Thus the probability of acceptance = $1 - \alpha$ (where $\alpha = 0.05$). Curve is for N = 4000, $n$ = 300, $c$ = 4. The Pa is 0.95.

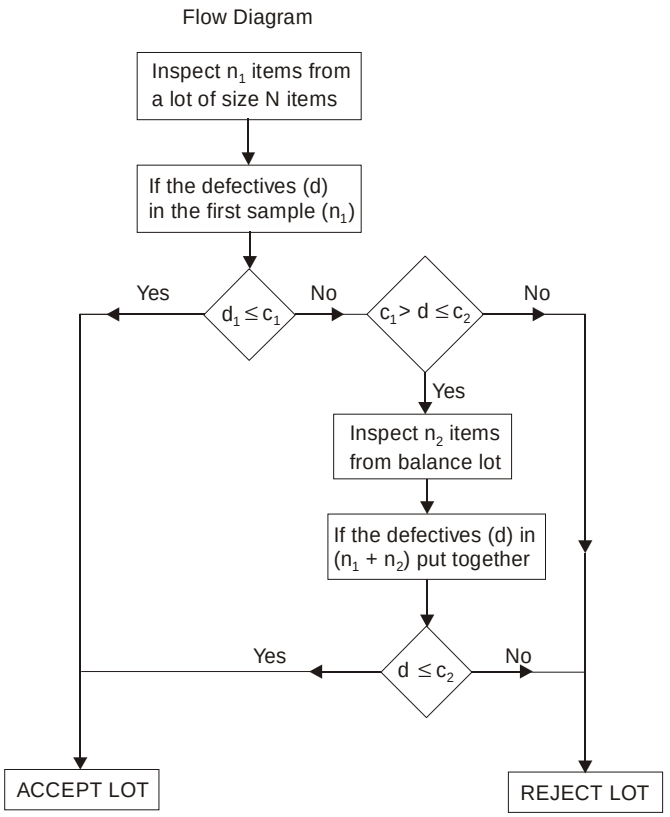



Associated with producer's risk is ($\alpha$) the numerical definition of a good quality which is called the Acceptance quality level. The AQL is the maximum percentage defective that can be considered to be accepted. For the sampling plan, N = 4000, $n$ = 300, $c$ = 4 AQL = 0.7% for $\alpha = 0.05$. In other words the product 0.7% defective (good quality) will have a rejcetion of probability of 0.05. The consumer's risk ($\beta$) is the probability of accepting the

bad lot. The risk is frequently given as 0.1. Since β is expressed in terms of probability of acceptance no conversion is necessary. Associated with the consumer's risk is the numerical definition of the bad quality called RQL or LTPD. It is the percent defective in the lot which can be tolerated by the consumer. LTPD in this case is 2.6% for a β value of 0.10, lots that are 2.6% defective (bad quality) will have an acceptance probability of 0.1.

**DOUBLE SAMPLING PLAN**

A double sampling plan is a plan in which a maximum of two samples are checked before deciding upon the acceptability of the lot.




Where N = Lot size

$n_1$ = Sample size in the I sample

$n_2$ = Sample size for the II sample

$C_1$ = Acceptance number for the I sample

$C_2$ = Acceptance number for both the samples ($n_1 + n_2$).

**Acceptance-Rectification plan**

An acceptance-rectification plan involves acceptance of goods on the basis of sampling and on the basis of 100 percent inspection, if sampling inspection does not accept. However all the goods dispatched by the producer will reach the consumer. Further the

quality of goods reaching the consumer stores will be better than the goods originally shipped by the producer.

**Terminology**

**Average Outgoing Quality (AOQ)**

The average outgoing quality of an acceptance sampling plan is defined as the long-run average quality of goods that are accepted to the consumer stores. The AOQ of a sampling plan is given by AOQ = P $P_a$.

AOQ Curve : The AOQ Curve of an acceptance-rectification plan is plot of AOQ versus incoming lot quality.

**Average Outgoing Quality Limit (AOQL)**

The AOQL of an acceptance-rectification plan is defined as the AOQ that can occur irrespective of the value of incoming quality.

**Average Total Inspection (ATI)**

The ATI of acceptance-rectification plan is defined as the average amount of inspection effort that will occur with usage of such sampling plan. ATI for single sampling plan is given

$$ATI = nP_a + N(1 - P_a)$$

ATI for double sampling plan

$$ATI = n_1 P_{aI} + (n_1 + n_2) \times PaII + N[1 - P_a \text{ combined}].$$

**Sequential Sampling Plan**

Sequential sampling is similar to multiple sampling except that the sequential sampling can theoretically continue indefinitely. In practice the plan is truncated after the number inspected is equal to three times the number inspected by a corresponding single sampling plan. Sequential sampling, which is used for cortely or destructive tests, usually has a subgroup size of 1, thereby making it an item-by-item plan.

Item-by-item sequential sampling is based on the concept of the sequential probability ratio test, (SPRT). Figure illustrates the sampling plan technique. The "stepped" line shows the number detective for the total number inspected and is updated with the inspection results of each item. If the cummulative results equal or are greater than the upper line, the lot is rejected. If the cummulative results equal or are less than the lower line, the lot is accepted. If neither decision is possible, another item is inspected. Thus, if the 20th sample is found to be defective, the cummulative number of defective will be five. Since the five defectives exceeds the rejection line for 20 inspections, the lot is rejected, as shown by the dashed line in figure.

The sequential sampling plan is defined by the producer's risk, $\alpha$ and its process quality P$\alpha$, and by the consumer's risk, $\beta$, and its process quality, P$\beta$, Using these requirements, the equations (slope intercept form) can be determined for the acceptance line and rejection line using the following formulae.

$$ha = \frac{\log\left(\frac{1-\alpha}{\beta}\right)}{\log\left(\frac{P\beta}{P\alpha}\right) + \log\left(\frac{1-P\alpha}{1-P\beta}\right)}$$

$$hr = \frac{\log\left(\frac{1-\beta}{\alpha}\right)}{\log\left(\frac{P\beta}{P\alpha}\right)+\log\left(\frac{1-P\alpha}{1-P\beta}\right)}$$

$$S = \frac{\log\left(\frac{1-P\alpha}{1-P\beta}\right)}{\log\left(\frac{P\beta}{P\alpha}\right)+\log\left(\frac{1-P\alpha}{1-P\beta}\right)}$$

$da = -ha + sn$

$dr = hr + sn$

where

S = Slope of the lines

$hr$ = Intercept for the rejection line

$ha$ = Intercept for the acceptance line

$P\beta$ = Fraction defective for consumer's risk

$P\alpha$ = Fraction defective for producer's risk

$\beta$ = Consumer's risk

$\alpha$ = Producer's risk

$da$ = number of defection for acceptance

$dr$ = number of defection for rejection

$n$ = number of units inspected.

**Comparison between attribute and variable sampling plans**

| Data | Attribute sampling plan | Variable sampling plan |
|---|---|---|
| Type of item | Each item is classified as defective or non-defective (Gauging) | Measurement must be taken on each item. |
| Cost | Cheap | Costly |
| Sample size | Large | Small |
| Skill | Less | More |
| Assumption | None | Follows Normal distribution |
| Parameters used | N, $n$, $c$, $r$ | M, K, $\sigma$ |
| Calculations | Decision criteria does not involve calculations | Decision criteria involves calculation of mean and SD |

**PROBLEMS ON ATTRIBUTE SAMPLING PLANS**

**Notations used**

N = Lot size

$n$ = Sample size

$n_1$ = Size of first sample

$n_2$ = Size of second sample

$c$ = Acceptance number

$c_1$ = Acceptance number 1

$c_2$ = Acceptance number $(n_1 + n_2)$

Pr = Producer's risk

Cr = Consumer's risk

P′ = Fraction defective

$P_a$ = Probability of acceptance

P*a*I = Probability of acceptance with one sample

P*a*II = Probability of acceptance with two samples

**Procedure for Plotting Graphs**

(OC Curve and AOQ Curve)

**Step 1.** Assume that the lot contains various defectives as 0.005, 0.01, 0.002, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08.

**Step 2.** Determine the probability of acceptance ($P_a$) for all the above values of P′.

**Step 3.** Determine AOQ for all the values of P′.

**Step 4.** Tabulate the assumed P′ values, 100P′ values, $n$P′ values and $P_a$ values as shown below.

**Step 5.** Plot the OC Curve taking 100P′ values on 'X' axis and $P_a$ on 'Y' axis.

**Step 6.** Plot AOQ curve taking 100P′ values on 'X' axis and AOQ values on 'Y' axis.

**Note :**

1. The procedure is followed for all the problems.
2. The curves (OC curve and AOQ curve) shown in this book are not to scale.

**SOLVED PROBLEMS**

**Problem 1.** *A single sampling plan is as follows : N = 5000 ; n = 80 ; c = 2*

(*a*) *Plot the O.C. curve for the above plan*

(*b*) *What is the producer's risk if AQL is 1.5% ?*

(*c*) *What is the consumer's risk if LTPD is 4.5% ?*

(*d*) *What is the ATI of the above plan at 1.25% defective of the incoming lot ?*

(*e*) *Plot the AOQ curve and determine the AOQL.*

**Solution.** Given data : N = 5000 ; $n$ = 80 ; $c$ = 2

The probability of acceptance, $P_a = P(0) + P(1) + P(2)$

Specimen calculation :

Consider P′ = 0.0005

100P′ = 100 × 0.0005 = 0.05

$n$P′ = 80 × 0.05 = 0.4

we know that as per poisson's

$$P(r) = \frac{e^{-np}\,(np)^r}{r\,!}$$

$$P(0) = \frac{e^{0.4}(0.4)^0}{0!} = 0.670$$

$$P(1) = \frac{e^{-0.4}(0.4)^1}{1!} = 0.268$$

$$P(2) = \frac{e^{-0.4}(0.4)^2}{2!} = 0.054$$

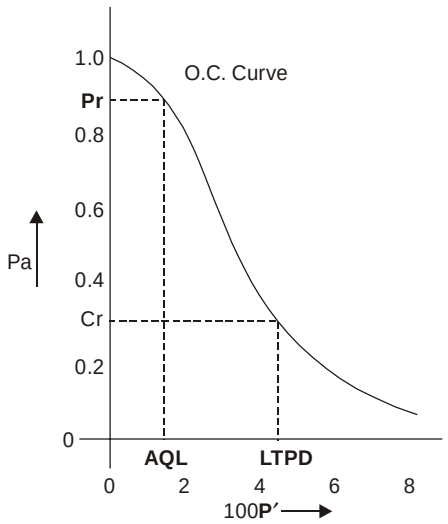
Therefore, $P_a = P(0) + P(1) + P(2)$

$= 0.670 + 0.268 + 0.054 = 0.992$

Similarly $P_a$ is calculated and tabulated for all the assumed values of P′.

| *Assumed values of P′* | *100P′* | *nP′* | $P_a$ | *AOQ* ($100P' \times P_a$) |
|---|---|---|---|---|
| 0.0005 | 0.5 | 0.4 | 0.992 | 0.496 |
| 0.01 | 1.0 | 0.8 | 0.952 | 0.952 |
| 0.02 | 2.0 | 1.6 | 0.783 | 1.566 |
| 0.03 | 3.0 | 2.4 | 0.569 | 1.707 |
| 0.04 | 4.0 | 3.2 | 0.379 | 1.516 |
| 0.05 | 5.0 | 4.0 | 0.238 | 1.19 |
| 0.06 | 6.0 | 4.8 | 0.142 | 0.852 |
| 0.07 | 7.0 | 5.6 | 0.082 | 0.574 |
| 0.08 | 8.0 | 6.4 | 0.046 | 0.368 |

(*a*) Operative Characteristic curve (OC curve)




(*b*) Producer's risk when AQL values 1.5%.

From the graph (OC curve) when AQL = 1.5%, the producer's risk,

$Pr = 1 - 0.88 = .12 = 12\%$.

(*c*) Consumer's risk when LTPD is 4.5%

From the graph (OC curve) when LTPD = 4.5% the consumer's risk,

$C_r = 0.305 = 30.5\%$

(*d*) ATI given 1.25% defective lot

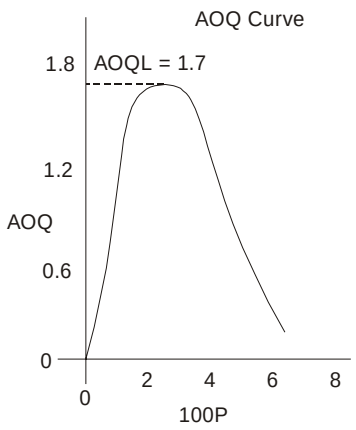
we know that ATI = $nP_a + N(1 - P_a)$

Given $P' = 0.0125$ therefore $nP^1 = 80 \times 0.0125 = 1$

When $nP' = 1$ and C = 2. The $P_a$ value from tables = 0.92

$ATI = 80 \times 0.92 + 5000 (1 - 0.92) = 473.6$

(*e*)




**Problem 2.** *A single sampling plan is as follows : N = 4000 ; n = 75 ; c = 2*

(*a*) *Plot the O.C. curve*

(*b*) *If AQL is 1.5% find Pr (Producer's risk) and if Cr (Consumer's risk) is 10%. Find LTPD.*

(*c*) *Plot the AOQ curve and determine AOQL*

(*d*) *Find the ATI of the above plan at 1.5% defective of the incoming lot.*

**Solution.** Given data : N = 4000, *n* = 75, *c* = 2

Probability of acceptance, $P_a = P(0) + P(1) + P(2)$

Specimen calculation for $nP' = 0.375$

$$P(r) = \frac{e^{-nP^1} (nP_1)^r}{r\,!}$$

$$P(0) = \frac{e^{-0.375} (0.375)^0}{0\,!} = 0.687$$

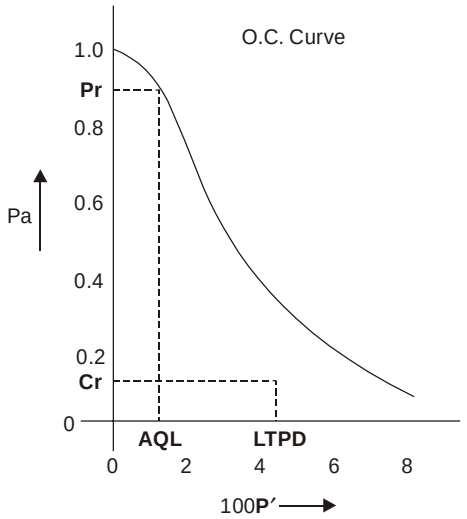
$$P(1) = \frac{e^{-0.375} (0.375)^1}{1\,!} = 0.257$$

$$P(2) = \frac{e^{-0.375} (0.375)^2}{2\,!} = 0.048$$

$$P_a = P(0) + P(1) + P(2) = 0.687 + 0.258 + 0.048 = 0.993$$

| *Assumed values of P′* | *100P′* | *nP′* | $P_a$ | *AOQ* ($100P' \times P_a$) |
|---|---|---|---|---|
| 0.005 | 0.5 | 0.375 | 0.993 | 0.496 |
| 0.01 | 1.0 | 0.75 | 0.959 | 0.952 |
| 0.02 | 2.0 | 1.5 | 0.809 | 1.618 |
| 0.03 | 3.0 | 2.25 | 0.609 | 1.827 |
| 0.04 | 4.0 | 3.0 | 0.423 | 1.692 |
| 0.05 | 5.0 | 3.75 | 0.277 | 1.385 |
| 0.06 | 6.0 | 4.5 | 0.173 | 1.038 |
| 0.07 | 7.0 | 5.25 | 0.105 | 0.735 |
| 0.08 | 8.0 | 6.0 | 0.062 | 0.496 |

(*a*)

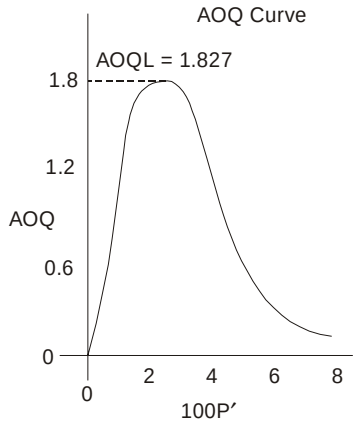



(*b*) For AQL 1.5%, the producer's risk is given by

$$Pr = 1 - .885 = 11.5\%$$

For consumer's risk 10%, the LTPD = 7.1%

(*c*) From AOQ curve, AOQL = 1.827



(*d*) ATI = $nP_a$ = N(1 – $P_a$)

Given P = 0.015 ; therefore $nP_1$ = 75 × .015 = 1.125

$P_a$ = P(0) + P(1) + P(2)

$$P(0) = \frac{e^{-(1.125)}(1.25)^0}{0} = 0.325$$

$$P(1) = \frac{e^{-(1.125)}(1.25)^1}{1} = 0.365$$

$$P(2) = \frac{e^{-(1.125)}(1.25)^2}{2} = 0.205$$

$P_a$ = 0.325 + 0.365 + 0.205 = 0.895

ATI = 75 × 0.895 × 4000 (1 – 0.895) = 487.125

**Note.** In the following problems the OC curve and AOQ curve are not represented graphically. The readers are requested to plot the curves for the data given. However, the solutions are given for all the problems.

**Problem 3.** *A single sampling plan is as follows : N = 5000 ; n = 100 ; c = 3*

(*a*) *Plot the O.C. curve*

(*b*) *Determine the AQL and LTPD for Pr = 10% and Cr = 15% respectively.*

(*c*) *What is the ATI of the above plan for 1.5% defective of the incoming lot.*

**Solution.** Given data N = 5000 ; *n* = 100 ; *c* = 3

Probability of acceptance, $P_a$ = P(0) + P(1) + P(2) + P(3)

| *Assumed values of P′* | *100P′* | *nP′* | $P_a$ | *AOQ* ($100P' \times P_a$) |
|---|---|---|---|---|
| 0.005 | 0.5 | 0.5 | 0.998 | 0.499 |
| 0.01 | 1.0 | 1.0 | 0.981 | 0.981 |
| 0.02 | 2.0 | 2.0 | 0.875 | 1.714 |
| 0.03 | 3.0 | 3.0 | 0.647 | 1.941 |
| 0.04 | 4.0 | 4.0 | 0.433 | 1.732 |
| 0.05 | 5.0 | 5.0 | 0.165 | 1.325 |
| 0.06 | 6.0 | 6.0 | 0.151 | 0.906 |
| 0.07 | 7.0 | 7.0 | 0.082 | 0.574 |
| 0.08 | 8.0 | 8.0 | 0.042 | 0.336 |

(*b*) From graph for producer's risk of 10% AQL = 1.7% and for consumer's risk of 15% LTPD = 6.1%

(*c*) We know that ATI = $nP_a$ + N(1 – $P_a$)

Given *p* = 1.5% = 0.015, therefore *n*P′ = 100 × 0.015 = 1.5

$P_a$ for *n*P′ = 1.5 and *c* = 3

$P_a$ = 0.934

ATI = 100 × 0.0934 + 5000 (1 – 0.934) = 423.4.

**Problem 4.** *A single sampling plan has the following parameters : N = 3000 ; n = 150 ; c = 1. If an acceptance-rectification plan is used what will be the AOQL of the above plan ?*

**Solution.** Given data N = 3000 ; $n$ = 150 ; C = 1

Probability of acceptance, $P_a$ = P(0) + P(1)

| *Assumed values of P′* | *100P′* | *nP′* | $P_a$ | *AOQ* $(100P' \times P_a)$ |
|---|---|---|---|---|
| 0.005 | 0.5 | 0.75 | 0.827 | 0.413 |
| 0.01 | 1.0 | 1.5 | 0.558 | 0.558 |
| 0.02 | 2.0 | 3.0 | 0.199 | 0.398 |
| 0.03 | 3.0 | 4.5 | 0.061 | 0.183 |
| 0.04 | 4.0 | 6.0 | 0.017 | 0.068 |
| 0.05 | 5.0 | 7.5 | 0.004 | 0.02 |
| 0.06 | 6.0 | 9.0 | 0.001 | 0.006 |
| 0.07 | 7.0 | 10.5 | 0.000 | 0.000 |
| 0.08 | 8.0 | 12.0 | 0.000 | 0.000 |

Frame graph AOQL = 0.558.

**Problem 5.** *A single sampling plan is as follows : n = 150, c = 2*

(*a*) *Plot the curve showing AOQ verses incoming quality (100P′).*

(*b*) *Hence determine the AOQL of the above plan.*

**Solution.** Given data $n$ = 150, $c$ = 2

Probability of acceptance, $P_a$ = P(0) + P(1) + P(2)

| *Assumed values of P′* | *100P′* | *nP′* | $P_a$ | *AOQ* $(100P' \times P_a)$ |
|---|---|---|---|---|
| 0.005 | 0.5 | 0.75 | 0.959 | 0.479 |
| 0.01 | 1.0 | 1.5 | 0.809 | 0.809 |
| 0.02 | 2.0 | 3.0 | 0.423 | 0.846 |
| 0.03 | 3.0 | 4.5 | 0.173 | 0.159 |
| 0.04 | 4.0 | 6.0 | 0.062 | 0.248 |
| 0.05 | 5.0 | 7.5 | 0.0202 | 0.101 |
| 0.06 | 6.0 | 9.0 | 0.006 | 0.036 |
| 0.07 | 7.0 | 10.5 | 0.002 | 0.014 |
| 0.08 | 8.0 | 12.0 | 0.01 | 0.008 |

From graph AOQL = 0.846.

**Problem 6.** *A single sampling inspection scheme for a large lot of mass produced items states that from each lot take and inspect a random sample of 50. If 3 or more defectives are found, inspect the whole lot and remove all the defectives. If less than 3 are found to be defective, accept the lot without further inspection.*

(*a*) *Obtain an equation fior probability of acceptance in terms of fraction defective P′.*

(*b*) *Evaluate $P_a$ for P′ of 0.01, 0.02, 0.03, 0.04, 0.05, 0.07, 0.1, 0.15, 0.2, 0.3.*

(*c*) *Plot the O.C. curve & AOQ curve*

(*d*) *Estimate the Pr at P of 2.5% and Cr at 1 of 6%.*

**Solution.** The probability of acceptance, $P_a = P(0) + P(1) + P(2)$

Using poisson's approximation to binomial

$$P(0) = \frac{e^{-50P'}(50P^1)^0}{0!}$$

$$P(1) = \frac{e^{-50P'}(50P_1)^1}{1!}$$

$$P(2) = \frac{e^{-50P'}(50P_2)^2}{2!}$$

(*a*) $P_a = e^{-50P'}\left(1 + 50P' + \frac{(50P)^2}{2}\right)$

(*b*)

| *Assumed values of P′* | *100P′* | *nP′* | $P_a$ | *AOQ* ($100P' \times P_a$) |
|---|---|---|---|---|
| 0.01 | 1 | 0.5 | 0.986 | 0.986 |
| 0.02 | 2 | 1.0 | 0.920 | 1.84 |
| 0.03 | 3 | 1.5 | 0.809 | 2.427 |
| 0.04 | 4 | 2.0 | 0.677 | 2.708 |
| 0.05 | 5 | 2.5 | 0.543 | 2.725 |
| 0.07 | 7 | 3.5 | 0.321 | 2.247 |
| 0.1 | 10 | 5.0 | 0.125 | 1.25 |
| 0.15 | 15 | 7.5 | 0.020 | 0.3 |
| 0.2 | 20 | 10.0 | 0.003 | 0.06 |
| 0.3 | 30 | 15.0 | 0.000 | 0.00 |

(*d*) Pr at 2.5% = 1 – 0.86 = 0.14 = 14%

Cr at 6% = 0.38 = 38%.

**Problem 7.** *The lot size is 2000 in a certain AOQL inspection procedure. The desired AOQL of 1% can be obtained with any one of the sampling plans*

| *PLAN 1* | *PLAN II* | *PLAN III* |
|---|---|---|
| *n = 36* | *n = 80* | *n = 140* |
| *c = 0* | *c = 1* | *c = 2* |

*which plan will you select considering both sampling inspection and screening of rejected lot if a large number of lots of 0.5% defective are submitted.*

**Solution.** Consider the plan I

$N = 2000$, $n = 36$, $c = 0$

$P' = 0.5\% = 0.005$

$P_a = P(0)$

$nP' = 36 \times 0.005 = 0.18$

Using the Poisson's distribution

$$P(0) = \frac{e^{-0.18}(0.18)^0}{0!} = 0.835$$

$P_a = 0.835$

$ATI = nP_a + N(1 - P_a)$

$= 36 \times 0.835 + 2000\ (1 - 0.835) = 360.06$

Consider Plan II

$N = 2000,\ n = 80,\ c = 1$

$P' = 0.5\% = 0.005$

$P_a = P(0) + P(1)$

$nP' = 80 \times 0.005 = 0.4$

$$P(0) = \frac{e^{-0.4}(0.4)^0}{0!} = 0.670$$

$$P(1) = \frac{e^{-0.4}(0.4)^1}{1!} = 0.268$$

$P_a = 0.938$

$ATI = nP_a + N(1 - P_a)$

$= 80 \times 0.938 + 2000\ (1 - 0.938) = 199.04$

Consider the Plan III

$N = 2000,\ n = 140,\ c = 2$

$P - 0.5\% = 0.005$

$P_a = P(0) + P(1) + P(2)$

$nP' = 140 \times 0.005 = 0.7$

$P_a = 0.965$

$ATI = nP_a + N(1 - P_a)$

$= 140 \times 0.965 + 2000\ (1 - 0.965) = 205.1$

Since ATI is minimum for plan II, select plan II.

**Problem 8.** *A consumer receives lots of 2000 items and uses a single sampling plan to accept or reject the lots. The plan calls for 200 items to be inspected and the lot is to be accepted if 2 or less defective are found.*

(*a*) *Plot the O.C. curve. Assume the lot contains 0.5%, 1.0%, 2.0%, 3%, 4%, 5%, 6%, 7% and 8% defective*

(*b*) *What is the Pr when AQL, is 1.5% ?*

(*c*) *What is LTPD at Cr is 20% ?*

(*d*) *If all the lots are screened, what is the AOQL of the above plan ?*

**Solution.** Given data $N = 2000$ ; $n = 200$ ; $c = 2$

Probability of acceptance $P_a = P(0) + P(1) + P(2)$

| *Assumed values of P′* | *100P′* | *nP′* | $P_a$ | *AOQ* ($100P' \times P_a$) |
|---|---|---|---|---|
| 0.005 | 0.5 | 1.00 | 0.920 | 0.46 |
| 0.01 | 1.0 | 2.00 | 0.627 | 0.677 |
| 0.02 | 2.0 | 4.00 | 0.238 | 0.476 |
| 0.03 | 3.0 | 6.00 | 0.062 | 0.186 |
| 0.04 | 4.0 | 8.00 | 0.014 | 0.056 |
| 0.05 | 5.0 | 10.00 | 0.003 | 0.015 |
| 0.06 | 6.0 | 12.00 | 0.001 | 0.006 |
| 0.07 | 7.0 | 14.00 | 0.000 | 0.00 |
| 0.08 | 8.0 | 16.00 | 0.000 | 0.00 |

(*b*) From graph for AQL 1.5%, the producer's risk is given by

$$Pr = 1 - 0.41 = 0.59 = 59\%$$

(*c*) For Cr 20%, the LTPD = 2.2% (From OC curve)

(*d*) From the AOQ curve, the AOQL = 0.677.

**Problem 9.** *Given below are the two different types of sampling plans. Assume that the lot size is very very large in comparison with the sample size.*

*Plan A – n = 100 : c = 2*

*Plan B – n = 175 : c = 3*

(*a*) *For each plan draw the O.C. curve*

(*b*) *What is the producer's risk Pr at P = 0.8% ?*

(*c*) *What is the Cr at P = 4% ?*

(*d*) *State giving the reasons for your answer which of the two plans would you consider if you were to be a producer and a consumer.*

**Solution.** Consider Plan A – Given data : $n = 100$, $c = 2$

Probability of acceptance, $Pa = P(0) + P(1) + P(2)$

| *Assumed values of P′* | *100P′* | *nP′* | $P_a$ |
|---|---|---|---|
| 0.005 | 0.5 | 0.5 | 0.986 |
| 0.01 | 1.0 | 1.0 | 0.920 |
| 0.02 | 2.0 | 2.0 | 0.677 |
| 0.03 | 3.0 | 3.0 | 0.423 |
| 0.04 | 4.0 | 4.0 | 0.238 |
| 0.05 | 5.0 | 5.0 | 0.125 |
| 0.06 | 6.0 | 6.0 | 0.062 |
| 0.07 | 7.0 | 7.0 | 0.030 |
| 0.0 | 8.0 | 8.0 | 0.014 |

Consider plan B – Given data : $n = 175$, $c = 3$

Probability of acceptance, $Pa = P(0) + P(1) + P(2) + P(3)$

*Assumed*

| $P'$ | $100P'$ | $nP'$ | $P_a$ |
|---|---|---|---|
| 0.005 | 0.5 | 0.875 | 0.987 |
| 0.01 | 1.0 | 1.75 | 0.899 |
| 0.02 | 2.0 | 3.5 | 0.536 |
| 0.03 | 3.0 | 5.25 | 0.231 |
| 0.04 | 4.0 | 7.0 | 0.082 |
| 0.05 | 5.0 | 8.75 | 0.025 |
| 0.06 | 6.0 | 10.5 | 0.007 |
| 0.07 | 7.0 | 12.25 | 0.001 |
| 0.08 | 8.0 | 14.0 | 0.000 |

From graph (OC curves)

For plan A, Pr at P = .8% is 1 – 0.95 = 0.5 = 5%

and Cr at P = 4% is 0.24 = 24%

For plan B, Pr = 1 – 0.94 = 0.06 = 6%

and Cr = 0.082 = 8.2%

Being a producer will select plan A as Pr is less for plan A.

Being a consumer will select plan B as Cr is less for plan B.

**Problems on Double Sampling Plan**

**Problem 1.** *A sampling plan is as follows*

$N = 3000$

$n_1 = 55$, $c_1 = 0$

$n_2 = 120$, $c_2 = 2$

$100P' = 1$, $P' = 0.01$

*Calculate Pa, ATI, AOQ, ASN.*

**Solution.** P($a$) combined = P($a$)I + P($a$)II

P($a$)I = P(0)I

P($a$)II = P(1)I × P(1 or less)II + P(2)I × P(0)II

$n_1 P' = 55 \times 0.1 = 0.55$ $n_2 P' = 120 \times 0.01 = 1.2$

P(0)I = 0.576 P(0) = 0.301

P(1) = 0.317 P(1) = 0.361

P(2) = 0.087

P($a$) = P(0)I = 0.576

P$a$)II = 0.317 × (0.361 + 0.301) + 0.087 × 0.087 × 0.301

= 0.236

P($a$) combined = P($a$)I + P($a$)II

= 0.576 + 0.236

= 0.812

ATI = $n_1$ P($a$)I + ($n_1$ + $n_2$) P($a$)II + N(1 – P$a$)

= 55 × 0.576 + (55 + 120) × 0.236 + 3000 (1 – 0.812)

= 636.98

AOQ without rectification

$$AOQ = 100P' \times Pa$$
$$= 1 \times 0.812 = 0.812\%$$

AOQ with rectification

$$AOQ = \frac{[(N - n_1)\, P(a)I + (N - n_1 - n_2)\, P(a)II]\, P'}{N}$$

$$= \frac{[(3000 - 55)\, 0.576 + (3000 - 55 - 120)\, 0.236]\, 0.01}{3000}$$

$$= 0.0078$$
$$= 0.78\%$$

$$ASN = n_1 + n_2\,(1 - P_1)$$

$$P_I = P(a)I + P(r)I$$

$P(r)I$ = Probability of rejection of lot after the first lot or sample

$$= 1 - P(2 \text{ or less})I$$
$$= 1 - (0.087 + 0.317 + 0.576)$$
$$= 0.02$$

$$P_1 = 0.576 + 0.02$$
$$= 0.596$$

$$ASN = 55 + 120\,(1 - 0.596)$$
$$= 103.48.$$

**Problem 2.** *A double sampling plan is as follows :*

| | | |
|---|---|---|
| $N = 5000$ | | |
| $n_1 = 150$ | $c_1 = 3$ | $r_1 = 6$ |
| $n_2 = 200$ | $c_2 = 8$ | $r_2 = 9$ |
| $100P' = 1.5$ | $P' = 0.015$ | |

*Calculate Pa, ATI, AOQ, ASN.*

**Solution.** $P(a)I = P(0) + P(1) + P(2) + P(3)$

$P(a)II = P(4)I \times P(4 \text{ or less})II + P(s)I + P(3 \text{ or less})II$

Since $r_1$ is 6, we cannot inspect a second sample if number of defectives are 6 or more in the first sample,

| | |
|---|---|
| $n_1P' = 150 \times 0.015 = 2.25$ | $n_2\,P' = 200 \times 0.015 = 3.0$ |
| $P(0)I = 0.105$ | $P(0) = 0.049$ |
| $P(1)I = 0.237$ | $P(1)II = 0.224$ |
| $P(3)I = 0.200$ | $P(3)II = 0.224$ |
| $P(4)I = 0.112$ | $P(4)II = 0.168$ |
| $P(5)I = 0.0506$ | |

$$P(a)I = 0.105 + 0.237 + 0.266 + 0.2 = 0.808$$

$$P(a)II = 0.112\,(0.168 + 0.224 + 0.224 + 0.149 + 0.049) + 0.0506\,(0.224 + 0.224 + 0.149 + 0.049)$$
$$= 0.124$$

$P(a)$ combined = 0.808 + 0.124 = 0.932

$$ATI = n_1 P(a)I + (n_1 + n_2) P(a)II + N(1 - Pa)$$
$$= 150 + 0.808 + (150 + 200)\ 0.124 + 5000\ (1 - 0.932)$$
$$= 504.6$$

AOQ without rectification

$$AOQ = 100P' \times Pa = 1.5 \times 0.932 = 1.398\%$$

AOQ with rectification

$$AOQ = \frac{[(N - n_1)\ P(a)I + (N - n_1 - n_2)\ P(a)II]\ P'}{N}$$

$$= \frac{[(5000 - 150)\ 0.808 + (5000 - 150 - 200)\ 0.124]\ 0.015}{5000}$$

$$= 0.013 = 1.3\%$$

$$ASN = n_1 + n_2\ (1 \times P_1)$$

$$P_1 = P(a)I + P(r)I$$

$$P(r)I = 1 - P(5 \text{ or less})I$$
$$= 1 - 0.9706 = 0.0294$$

$$P_1 = 0.808 + 0.0294 = 0.837$$

$$ASN = 150 + 200(1 - 0.837) = 182.6.$$

**Problem 3.** *A double sampling plan is as follows :*

$N = 5000$

$n_1 = 100,$ $\quad c_1 = 3$

$n_2 = 200,$ $\quad c_2 = 5$

$100\ P' = 1.2 \quad \Rightarrow \quad P' = 0.012$

*Calculate Pa, ATI, AOQ and ASN of the above plan.*

**Solution.** $P(a)$ combine = $P(a)I + P(a)II$

$P(a)I = P(0)I + P(1)I + P(2)I + P(3)I$

$P(a)II = P(4)I \times P(1 \text{ or less})\ II + P(5)I \times P(0)II$

$n_1 P' = 100 \times .012 = 1.2$ $\quad n_2 P' = 200 \times .012 = 2.4$

$P(0)I = 0.301$ $\quad P(0)II = 0.907$

$P(1)I = 0.361$ $\quad P(1)II = 0.217$

$P(2)I = 0.216$

$P(3)I = 0.086$

$P(4)I = 0.026$

$P(5)I = 0.006$

$P(a)I = 0.301 + 0.361 + 0.216 + 0.086$
$= 0.964$

$P(a)II = 0.301 + 0.361 + 0.216 + 0.086$
$= 0.964$

$P(a)II = 0.026\ [0.217 + 0.0907] + 0.006\ [0.0707]$
$= 0.026 \times 0.3077 + 0.006 \times 0.0907$
$= 0.0085$

P($a$) combined = 0.0964 + 0.085
= 0.9725

ATI = $n_1$ P($a$)I + ($n_1$ + $n_2$) P($a$)II + N(1 – P$a$)
= 100 × 0.964 + (100 + 200) 0.0085 + 5000 (1 – 0.9725)
= 96.4 + 2.55 + 137.5
= 236.45

AOQ without rectification

AOQ = 100P′ × Pa = 1.2 × 0.9725 = 1.167%

AOQ with rectification

$$AOQ = \frac{[(N - n_1)\,P(a)I + (N - n_1 - n_2)\,P(a)II]\,P'}{N}$$

$$= \frac{[(5000 - 100)\,0.964 + (5000 - 100 - 200)\,0.0085]\,0.0120}{5000}$$

= 0.011 = 1.1%

ASN = $n_1$ + $n_2$(1 – $P_1$)

$P_1$ = P($a$)I + P($r$)I

P($r$)I = Probability of rejection of lot after the first sample
= 1 – P(5 or less)I
= 1 – 0.996 = 0.004

$P_1$ = 0.964 + 0.004 = 0.968

ASN = 100 + 200(1 – 0.968)
= 106.4.

**Problem 4.** *A double sampling plan is as follows :*

*N = 6000*

$n_1$ = 250, $c_1$ = 1

$n_1$ = 250, $c_2$ = 4

100 P′ = 2 ⇒ P′ = 0.02

*Calculate Pa, ATI, AOQ, ASN.*

**Solution.** P($a$) combined = P($a$)I + P($a$)II

P($a$)I = P(0) + P(1)I

P($a$)II = P(2)I × P(2 or less)II + P(3)I × P(1 or less)II + P(4)I × P(0)II

$n_1$P′ = 250 × 0.02 = 5 $n_2$ P′ = 200 × 0.02 = 4

P(0)I = 0.0067 P(0)II = 0.018

P(1)I = 0.0033 P(1)II = 0.073

P(2)I = 0.084 P(2)II = 0.146

P(3)I = 0.140

P(4)I = 0.175

P($a$)I = 0.0067 + 0.0033 = 0.01

P($a$)II = 0.084 × (0.146 + 0.073 + 0.018)
+ 0.14 × (0.073 + 0.018) + 0.175 × 0.018
= 0.036

$P(a)$ combined = 0.01 + 0.036

= 0.046

ATI = $n_1$ P($a$)I + ($n_1$ + $n_2$)P($a$)II + N(1 – P$a$)

= 250 × 0.01 + (250 + 200) 0.036 + 6000 (1 – 0.046)

= 5742.7

AOQ without rectification

AOQ = 100P′ × P$a$ = 2 × 0.046 = 0.092%

AOQ with rectification

$$AOQ = \frac{[(N - n_1)\, P(a)I + (N - n_1 - n_2)\, P(a)II]\, P'}{N}$$

$$= \frac{[(5000 - 100)\, 0.964 + (5000 - 100 - 200)\, 0.0085]\, 0.0120}{5000}$$

= 0.00085 = 0.085%

ASN = $n_1 + n_2(1 - P_1)$

$P_1$ = P($a$)I + P($r$)I

= 1 – P(4 or less)I

= 1 – 0.409 = 0.591

$P_1$ = 0.01 + 0.591 = 0.601

ASN = 250 + 200(1 – 0.601)

= 329.8.

**Problem 5.** *A double sampling plan is as follows :*

*N = 4000*

$n_1$ = 150, $c_1$ = 4

$n_2$ = 200, $c_2$ = 7

$P^1$ = 0.015

*Calculate Pa, ATI, AOQ, ASN of the above plan.*

**Solution.** P($a$) combined = P($a$)I + P($a$)II

P($a$)I = P(0)I + P(1)I + P(2)I + P(3)I + P(4)I

P($a$)II = P(5)I × P(2 or less)II + P(6)I × P(1 or less)II + P(7)I × P($a$)II

$n_1$P′ = 150 × .015 = 2.25

$n_2$P′ = 200 × 0.015 = 3

P(0)I = 0.105 P(0)II = 0.049

P(1)I = 0.237 P(1)II = 0.149

P(2)I = 0.266 P(2)II = 0.224

P(3)I = 0.2

P(4)I = 0.112

P(5)I = 0.0506

P(6)I = 0.018

P(7)I = 0.006

P($a$)I = 0.105 + 0.237 + 0.266 + 0.2 + 0.112

= 0.92

$$P(a)II = 0.0506\ (0.224 + 0.149 + 0.049) + 0.018\ (0.149 + 0.049) + 0.006 \times 0.049$$
$$= 0.025$$

$$P(a) \text{ combined} = 0.92 + 0.025$$
$$= 0.945$$

$$ATI = n_1\ P(a)I + (n_1 + n_2)\ P(a)II + N(1 - Pa)$$
$$= 150 \times 0.92 + (150 + 200)\ 0.025 + 4000\ (1 - 0.945)$$
$$= 366.75$$

AOQ without rectification

$$AOQ = 100P^1 \times Pa = 100 \times 0.015 \times 0.945$$
$$= 1.42\%$$

AOQ with rectification

$$AOQ = \frac{[(N - n_1)\ P(a)I + (N - n_1 - n_2)\ P(a)II]\ P'}{N}$$

$$= \frac{[(4000 - 150)\ 0.92 + (4000 - 150 - 200)\ 0.025]\ 0.015}{4000}$$

$$= 0.0136 = 1.36\%$$

$$ASN = n_1 + n_2(1 - P_1)$$
$$P_1 = P(a)I + P(r)I$$
$$P(r)I = 1 - P(7 \text{ or less})I$$
$$= 1 - 0.9946 = 0.054$$
$$P_1 = 0.92 + 0.0054 = 0.9254$$
$$ASN = 150 + 200(1 - 0.9254)$$
$$= 164.92.$$

**Problem 6.** *A double sampling plan is as follows :*

$N = 4000$

| | | |
|---|---|---|
| $n_1 = 100,$ | $c_1 = 1,$ | $r_1 = 4,$ |
| $n_2 = 150,$ | $c_2 = 7,$ | $r_2 = 8$ |

(*a*) *Determine probability of acceptance at 1.25% defective*

(*b*) *What is the ATI, AOQ, ASN of the above plan ?*

**Solution.** $P' = 1.25/100 = 0.0125$

$P(a)$ combined $= P(a)I + P(a)II$

$P(a)I = P(0)I + P(1)I$

$P(a)II = P(2)I \times P(5 \text{ or less})II + P(3)I \times P(4 \text{ or less})II$

Since $r_1 = 4$, we cannot inspect a second sample if number of defectives are 4 or more in the first sample.

$$n_1P' = 100 \times 0.0125 = 1.25$$
$$n_2P' = 150 \times 0.0125 = 1.875$$

P(0)I = 0.282 P(0)II = 0.153
P(1)I = 0.358 P(1)II = 0.287
P(2)I = 0.223 P(2)II = 0.269
P(3)I = 0.093 P(3)II = 0.168
P(4)II = 0.079
P(5)II = 0.029

$P(a)I = 0.282 + 0.358 = 0.64$

$P(a)II = 0.223\ (0.153 + 0.287 + 0.269 + 0.168 + 0.079 + 0.029)$
$+ 0.093 + (0.153 + 0.287 + 0.269 + 0.169 + 0.079)$
$= 0.223 \times 0.985 + 0.093 \times 0.956 = 0.308$

$P(a)$ combined $= 0.64 + 0.308 = 0.948$

$ATI = n_1\ P(a)I + (n_1 + n_2)\ P(a)II + N(1 - Pa)$
$= 100 - 0.64 + (100 + 150) \times 0.308 + 4000(1 - 0.948) = 349$

AOQ without rectification

$AOQ = 100P' \times Pa = 1.25 \times 0.948 = 1.285\%$

AOQ with rectification

$$AOQ = \frac{[(N - n_1)\ P(a)I + (N - n_1 - n_2)\ P(a)II]\ P'}{N}$$

$$= \frac{[(4000 - 100)\ 0.64 + (4000 - 100 - 150)\ 0.308]\ 0.00125}{4000}$$

$= 0.0114 = 1.14\%$

$ASN = n_1 + n_2(1 - P_1)$

$P_1 = P(a)I + P(r)I$

$P(r)I = 1 - P(3 \text{ or less})I$
$= 1 - 0.956 = 0.044$

$P_1 = 0.64 + 0.044 = 0.684$

$ASN = 100 + 150(1 - 0.684) = 147.4.$

**Problem 7.** *A double sampling plan is as follows :*

$n_1 = 150,$ $c_1 = 2,$ $r_1 = 5$
$n_2 = 200,$ $c_2 = 7,$ $r_2 = 8$
$100P^1 = 1.0,$ $P^1 = 1/100 = 0.01$

*What is the Pa, ATI, AOQ, ASN of the above plan ?*

***Note.*** *If lot size is not given assume minimum 10 times of the sample size*

$N = (150 + 200)10 = 3500.$

**Solution.** $P(a)$ combined $= P(a)I + P(a)II$

$P(a)I = P(0)I + P(1)I + P(2)I$

$P(a)II = P(3)I \times P(4 \text{ or less})II + P(4)I \times P(3 \text{ or less})II$

Since $r_1 = 5$, we cannot inspect a II sample if number of defectives are 5 or more in the first sample.

$n_1P^1 = 150 \times .01 = 1.5$

$n_2 P^1 = 200 \times .01 = 2$

| | |
|---|---|
| P(0)I = 0.223 | P(0)II = 0.049 |
| P(1)I = 0.334 | P(1)II = 0.149 |
| P(2)I = 0.251 | P(2)II = 0.224 |
| P(3)I = 0.125 | P(3)II = 0.224 |
| P(4)I = 0.047 | P(4)II = 0.168 |

P($a$)I = 0.223 + 0.334 = 0.251 = 0.808

P($a$)II = 0.125 × 0.814 + 0.047 × 0.646 = 0.132

P($a$) combined = 0.808 + 0.132 = 0.9678

ATI = $n_1$ P($a$)I + ($n_1$ + $n_2$) P($a$)II + N(1 – P$a$)

= 150 × 0.808 + (150 + 200) 0.132 + 3500(1 – 0.94) = 377.4

AOQ without rectification

AOQ = 100P$^1$ × P$a$ = 1 × .94 = 0.94%

AOQ with rectification

$$AOQ = \frac{[(N - n_1)\, P(a)I + (N - n_1 - n_2)\, P(a)II]\, P^1}{N}$$

$$= \frac{[(3500 - 150)\, 0.808 + (3500 - 150 - 200)\, 0.132]\, 0.01}{3500}$$

= 0.00892 = 0.892%

ASN = $n_1 + n_2(1 - P_1)$

$P_1$ = P($a$)I + P($r$)I

P($r$)I = 1 – P(4 or less)I

= 1 – 0.98 = 0.02

$P_1$ = 0.808 + 0.02 = 0.828

ASN = 150 + 200(1 – 0.828) = 184.4.

**Problem 8.** *A double sampling plan is as follow. A producer and a consumer agree to accept and reject the lots based on a double sampling plan. Under this plan a first sample of 50 items is inspected. The lot is accepted if no defective is found, and is rejected if 3 or more defectives are found ; otherwise a second sample of 100 is drawn and the lot is accepted if the combined number of defectives does not exceed 2. Determine the producer's risk for a fraction defective of 0.01.*

$n_1 = 50$, $c_1 = 0$, $r_1 = 3$

$n_2 = 100$, $c_2 = 2$,

$P^1 = 0.01$

**Solution.** P($a$) combined = P($a$)I + P($a$)II

P($a$)I = P(0)I

P($a$)II = P(1)I × P(1 or less)II + P(2)I × P(0)II

$n_1 P^1 = 50 \times 0.01 = 0.5$ $n_2P^1 = 100 \times .01 = 1$

P(0)I = 0.606 P(0)II = 0.367

P(1)I = 0.303 P(1)II = 0.367

P(2)I = 0.075

P(*a*)I = 0.606

P(*a*)II = 0.303(0.367 + 0.367) + 0.075 × 0.367 = 0.249

P(*a*) combined = 0.606 + 0.249 = 0.855

Producer's risk = 1 – P(*a*)

= 1 × 0.855 = 0.145 = 14.5%.

**Problem 9.** *A double sampling plan calls for a first sample of 25 items to be inspected. If no defectives are found the lot is accepted. If 3 or more defectives are found the lot is rejected otherwise a second sample of 100 is drawn and the lot is accepted if the combined number of defectives does not exceed two.*

(*a*) *Determine the producer's risk for a fraction defective of 0.01*

(*b*) *Determine the consumer's risk for a lot fraction defective of 0.08*

(*c*) *What is the ATI of the above plan when the incoming lots are containing 2% defective. Assume the lot size is very 2 large in comparison with the sample size.*

$N = (25 + 100)10 = 1250$

$n_1 = 25$, $c_1 = 0$, $r_1 = 3$

$n_2 = 100$, $c_2 = 2$.

**Solution.** P(*a*) combined = P(*a*)I + P(*a*)II

P(*a*) I = P(0)I

P(*a*)II = P(1)I × P(1 or less)II + P(2)I × P(0)II

(*a*) $P^1 = 0.001$

$n_1P^1 = 25 \times .01 = 0.25$ $n_2 P^1 = 100 \times .01 = 1$

P(0)I = 0.778 P(0)II = 0.367

P(1)I = 0.194 P(1)II = 0.367

P(2)I = 0.024

P(*a*)I = 0.778

P(*a*)II = 0.194 (0.367 + 0.367) + 0.024 × 0.367 = 0.151

P(*a*) combined = 0.778 + 0.151 = 0.929

Producer's risk = 1 – P*a* = 1 – 0.929 = 0.071 = 7.1%

(*b*) $P^1 = 0.08$

$n_1 P^1 = 25 \times .08 = 2$ $n_2 P^1 = 100 \times .08 = 8$

P(0)I = 0.135 P(0)II = 0.00033

P(1)I = 0.018 P(1)II = 0.0026

P(2)I = 0.018

P(*a*)I = 0.135

P(*a*)II = 0.018 (0.00033 + 0.0026) + 0.018 × 0.00033 = 0.00005

P(*a*) combined = 0.135 + 0.00005 = 0.13505

Consumer's risk = 13.5%

(*c*) $P^1 = 0.02$

$n_1 P^1 = 25 \times .02 = 0.5$

$n_2 P^1 = 100 \times .02 = 2$

P(0)I = 0.606 P(0)II = 0.135

P(1)I = 0.303 P(1)II = 0.270

P(2)I = 0.075

P(*a*)I = 0.606

P(*a*)II = 0.303 (0.135 + 0.270) + 0.075 × .135 = 0.133

P(*a*) combined = 0.606 + 0.133

= 0.739

ATI = $n_1$ P(*a*)I + ($n_1$ + $n_2$) P(*a*)II + P(1 – P*a*)

= 25 × 0.606 + (25 + 100) 0.133 + 1250 (1 – 0.739)

= 358.025.

## EXERCISE PROBLEMS

**1.** A double sampling plan is as follows : N = 5000 : $n_1$ = 150 ; $n_2$ = 300 ; $c_1$ = 2 and $c_2$ = 6. Lot size is 1.5% defective.

(*a*) Calculate the probability of acceptance.

(*b*) What is the ATI of the above plan ?

(*c*) What is the AOQ of the above plan ?

(*d*) What is the ASS or ASN of the above plan ?

**2.** A triple sampling plan is as follows : inspect a sample size of two randomly from a lot, if both good, accept the lot, if both bad, reject the lot, if one good and one bad take a II sample of size 2, if both good accept the lot, if both bad reject the lot, if one good and one bad take a III sample of size 2, if both good accept the lot otherwise reject the lot.

(*a*) Derive an expression for the probability of acceptance in terms of incoming quality $P^1$.

(*b*) What is the AOQ if the incoming lots are containing 2% defective ?

(*c*) What is the probability of acceptance if the incoming lots are containing 3% defective ?

(*d*) Plot the O.C. curve for the above plan.

**3.** A double sampling plan is as follows

$n_1$ = 100, $c_1$ = 1

$n_2$ = 50, $c_2$ = 3

(*a*) Calculate the probability of acceptance at fraction defective 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07 and 0.08.

(*c*) Plot the O.C. curve and determine the AQL and LTPD values for producer's risk 0.05 and consumer's risk 0.1.

**4.** A double sampling plan is as follows

$N = 3000$

$n_1 = 50,$ $c_1 = 3$

$n_2 = 100,$ $c_2 = 4$

(*a*) Calculate the probability of acceptance at fraction defective of 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07 and 0.08.

(*b*) Plot the O.C. curve and determine the AQL and LTPD for producer's risk of 10% and consumer's risk of 30% respectively.

## PROBLEMS ON SEQUENTIAL SAMPLING PLAN

**Problem 1.** *A sequential plan defined by*

$\alpha = 0.05,$ $\beta = 0.1$

$P\,\alpha = 0.06,$ $P\,\beta = 0.2$

(*a*) *Determine the equation for the acceptance and rejection.*

(*b*) *Using these equation establish a table of acceptance number rejection number and the number of items inspected. The table can be stopped when the rejection number is equal to 4.*

**Solution.**

$$ha = \frac{\log\left(\frac{1-0.05}{0.1}\right)}{\log\left(\frac{0.2}{0.06}\right)+\log\left(\frac{1-0.06}{1-0.2}\right)} = 1.64$$

$$hr = \frac{\log\left(\frac{1-0.1}{0.05}\right)}{\log\left(\frac{0.2}{0.06}\right)+\log\left(\frac{1-0.06}{1-0.2}\right)} = 2.11$$

$$S = \frac{\log\left(\frac{1-0.06}{0.2}\right)}{\log\left(\frac{0.2}{0.06}\right)+\log\left(\frac{1-0.06}{1-0.2}\right)} = 0.118$$

(*a*) (*i*) The acceptance line

$$da = -ha + Sn = -1.64 + 0.118n$$

(*ii*) The rejection line

$$dr = hr + Sn = 2.11 + 0.118n$$

| *No. of items inspected* | *Acceptance No.* $da = -ha + sn$ | *Rejection No.* $dr = hr + sn$ |
|---|---|---|
| 1 | – 1.522 | 2.228 |
| 2 | – 1.404 | 2.348 |
| 3 | – 1.286 | 2.464 |

| | | |
|---|---|---|
| 4 | – 1.168 | 2.582 |
| 5 | – 1.05 | 2.7 |
| 6 | – 0.932 | 2.828 |
| 7 | – 0.814 | 2.936 |
| 8 | – 0.696 | 3.054 |
| 9 | – 0.578 | 3.172 |
| 10 | – 0.46 | 3.29 |
| 11 | – 0.342 | 3.408 |
| 12 | – 0.224 | 3.526 |
| 13 | – 0.106 | 3.644 |
| 14 | 0.012 | 3.762 |
| 15 | 0.13 | 3.88 |
| 16 | 0.248 | 3.998 = 4.0 |

**Problem 2.** *A sequential plan is as follows :*

| *Fraction defective* | *Probability of acceptance* |
|---|---|
| $P_1 = 0.05$ | *0.9* |
| $P_2 = 0.12$ | *0.15* |
| *Pa = 0.05,* | *a = 1 – 0.9 = 0.1* |
| *Pb = 0.12,* | *b = 0.15.* |

*Draw the acceptance and rejection line and the table can be stopped when the rejection number is equal to 10.*

**Solution.**

$$= \frac{\log\left(\frac{1-0.1}{0.15}\right)}{\log\left(\frac{0.12}{0.05}\right)+\log\left(\frac{1-0.05}{1-0.12}\right)} = 1.88$$

$$hr = \frac{\log\left(\frac{1-0.15}{0.1}\right)}{\log\left(\frac{0.12}{0.05}\right)+\log\left(\frac{1-0.05}{1-0.12}\right)} = 2.24$$

$$S = \frac{\log\left(\frac{1-0.05}{1-0.12}\right)}{\log\left(\frac{0.12}{0.05}\right)+\log\left(\frac{1-0.05}{1-0.12}\right)} = 0.08$$

$da = – ha + sn = – 1.88 + 0.08n$

$dr = hr + sn = 2.24 + 0.08n$

| *No. of items inspected* | *Acceptance No.* $da = -ha + sn$ | *Rejection No.* $dr = hr + sn$ |
|---|---|---|
| 1 | – 1.8 | 2.32 |
| 2 | – 1.72 | 2.4 |
| 3 | – 1.64 | 2.48 |
| 4 | – 1.56 | 2.56 |
| 5 | – 1.48 | 2.64 |
| 6 | – 1.4 | 2.72 |
| 7 | – 1.32 | 2.8 |
| 8 | – 1.24 | 2.88 |
| 9 | – 1.16 | 2.96 |
| 10 | – 1.08 | 3.04 |
| 11 | – 1.0 | 3.12 |
| 12 | – 0.92 | 3.2 |
| 13 | – 0.84 | 3.28 |
| 14 | – 0.76 | 3.36 |
| 15 | – 0.68 | 3.44 |
| 16 | – 0.6 | 3.52 |
| 17 | – 0.52 | 3.6 |
| 18 | – 0.44 | 3.68 |
| 19 | – 0.36 | 3.76 |
| 20 | – 0.28 | 3.84 |
| 30 | 0.52 | 4.64 |
| 40 | 1.32 | 5.44 |
| 50 | 2.12 | 6.24 |
| 60 | 2.92 | 7.04 |
| 70 | 3.72 | 7.84 |
| 80 | 4.52 | 8.64 |
| 90 | 5.32 | 9.44 |
| 97 | 5.88 | 10.0 |

# 9 Reliability

**DEFINITION**

Reliability is the *probability* that a device will perform its intended *function satisfactory* without failure, for the stated period of *time*, under the specified *operating conditions*.

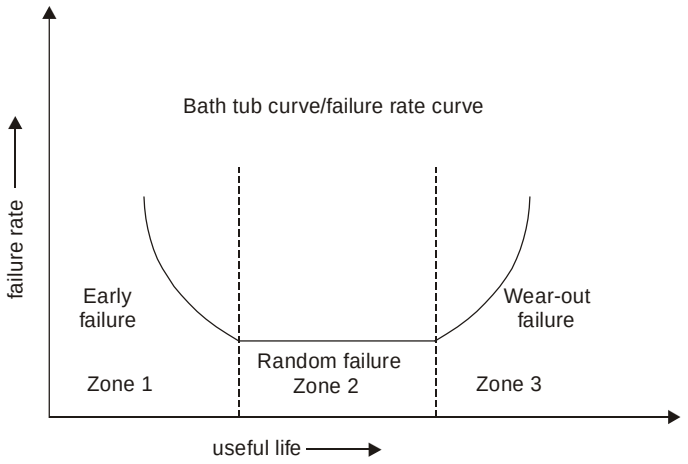
In the above definition, there are 4 factors which are essential to the concept of reliability.

(*a*) It is a probability.

(*b*) It is associated with time.

(*c*) Satisfactory performance is required.

(*d*) Operating environment should be defined.




**Zone 1 — Early Failure Period**

It is characterised by a high initial failure rate, gradually dropping off to a low failure rate. By the end of the failures in this zone are due to one or more of the foll assignable causes :

1. *Design* deficiency
2. *Manufacturing* error
3. *Raw material* defects
4. *Wrong maintenance* practices.

**Zone 2 — Random Failure Period**

It is characterised by a more or less constant failure rate. *This is the rate at which the normal usage of the product occurs without any expectation of failures.*

Failures in this zone are due to chance causes.

**Zone 3 — Wear Out Failure Period**

It is characterised by a gradual increase in failure rate.

Failures in this zone are due to one or more of the foll :

1. *Ageing*
2. Reduction in *physical strength* properties
3. *Wear and tear.*

**Relationship between the failure rate and the mean time between failures (MTBF), MTTR (Mean time to repair), MTTF (Mean time to failure)**

*MTBF (Repairable systems only) is defined as the mean time interval between successive failures*. It is denoted by θ. Related to the MTBF is the failure rate, which is denoted by λ. The failure rate is the reciprocal of MTBF.

$$\lambda = \frac{1}{\text{MTBF}} = \frac{1}{\theta}.$$

*If a large no. of terms are placed in the test of the same type and operated until each one fails, the mean time to failure* (*irrepairable system*), *i.e.*, MTTF is given by,

$$\text{MTTF} = \frac{\sum_{i=1}^{n} T_i}{n}$$ where $n$ = No. of items failed, T = tested duration.

**Prove that the Failure Rate is the Reciprocal of the MTBF**

**Proof.** Let us test '$n$' items each $t$ hours and the items which fail are repairable.

Suppose there are '$r$' failures,

$$\text{The failure rate, } \lambda = \frac{x\% \text{ of items failed}}{\text{Total test duration}} = \frac{r}{nt}$$

$$r = nt\,\lambda \qquad \ldots(1)$$

$$\text{MTBF} = \frac{1}{\lambda} = \frac{\text{Total test duration}}{x\% \text{ of items failed}} = \frac{nt}{r}$$

$$\frac{1}{\lambda} = \frac{nt}{nt\,\lambda} \quad \ldots\text{from (1)}$$

$$\frac{1}{\lambda} = \frac{1}{\lambda}.$$ Hence proved.

The reliability function of any system is given by,

$$R = e^{-\lambda t}$$

R = Prob. of survival ; $t$ = Tested duration.

**1.** *Determine the reliability of a system for a period of 100 hrs if the MTFB is 500 hrs.*

**Solution.** $R = e^{-\lambda t}$

**Data :** $t$ = 100 hrs ; MTBF = 500 hrs.

$$\text{WKT, } \lambda = \frac{1}{\text{MTBF}} = \frac{1}{500} = 0.002 \text{ failures/hr.}$$

$\therefore \quad R = e^{-(0.002 \times 100)}$.

R = 0.08187 is the prob. of survival of the system.

**2.** *A device has a failure rate of $5 \times 10^{-6}$ failures/hr.*

(*a*) *What is the reliability for an operating period of 100 hrs ?*

(*b*) *If 10,000 items are placed in the test, how many failures are expected in 100 hrs ?*

(*c*) *What is the MTBF ?*

(*d*) *What is the reliability of the system when operating time = MTBF ?*

(*e*) *If the useful life is 1 lakh hrs, what is the reliability for operating over its useful life ?*

**Data :** $\lambda = 5 \times 10^{-6}$ failures/hr ; $n$ = 10,000 ; $t$ = 100 ; $r$ = ?

(*a*) $R = e$

$R = e^{-(5 \times 10^{-6} \times 100)}$

R = 0.9995

(*b*) WKT, $r = nt\lambda$

$r = 10{,}000 \times 100 \times 5 \times 10^{-6}$

$r = 5$

(*c*) $\text{MTBF} = \frac{1}{\lambda} = \frac{1}{5 \times 10^{-6}} = 2{,}00{,}000$ hrs.

(*d*) $R = e^{-\lambda t}$

$R = e^{-(5 \times 10^{-6} \times 2{,}00{,}000)}$

R = 0.3678.

(*e*) $R = e^{-\lambda t}$

$R = e^{-(5 \times 10^{-6} \times 1{,}00{,}000)}$

R = 0.6065.

**3.** *A piece ground support equivalent has a specified mean time to failures of 100 hrs. What is its reliability for a mission time of 1 hr, 10 hrs, 50 hrs, 100 hrs, 200 hrs and 300 hrs. Graph these answers by plotting the mission time v/s reliability. Assume exponential distribution.*

**Solution. Data :** MTBF = 100 hrs.

$$\lambda = \frac{1}{\text{MTBF}} = \frac{1}{100} = 0.01 \text{ failures/hr.}$$

for $t$ = 1 hr, $R = e^{-\lambda t}$

$$R = e^{-(0.01 \times 1)} = 0.9900$$

for $t$ = 10 hr, $R = e^{-(0.01 \times 10)}$

$$R = 0.9048$$

for $t$ = 50 hr, $R = e^{-(0.01 \times 50)}$

$$R = 0.6065$$

for $t$ = 100 hr, $R = e^{-(0.01 \times 100)}$

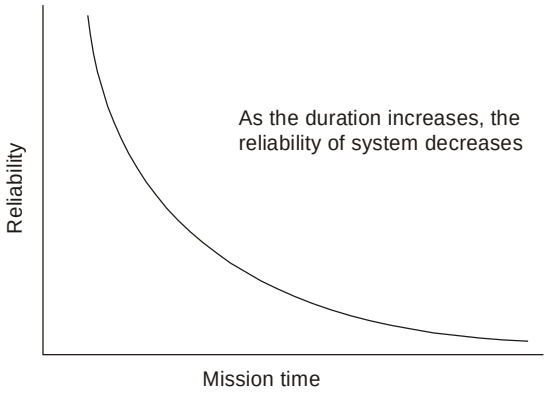
$$R = 0.3678$$

for $t$ = 200 hr, $R = e^{-(0.01 \times 200)}$

$$R = 0.1353$$

for $t$ = 300 hr, $R = e^{-(0.01 \times 300)}$

$$R = 0.04978.$$

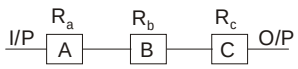



**Systems Reliability**

(1) System connected in series

(2) System connected in parallel

**System connected in series** follows a multiplication law of probability




If a system consists of 3 components A, B, C in series, then the reliability of the system,

$$R_S = R_A \cdot R_B \cdot R_C$$

**System Connected in Parallel**

Here the function of A can be done by B or vice versa. If the system consists of components A and B in parallel with reliability $R_A$ and $R_B$, then the reliability of the system,

$$R_S = [1 - (1 - R_A)(1 - R_B)]$$

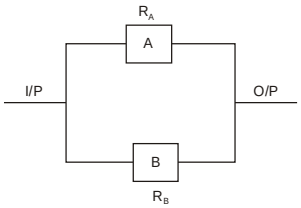
If *n* components are connected in parallel,

$$R_S = [1 - (1 - R_A)(1 - R_B) \ldots\ldots (1 - R_n)]$$

If $R_A = R_B = \ldots\ldots R$, then

$$R_S = [1 - (1 - R^n)]$$

Also $(1 - R_S) = (1 - R^n)$.




*Prove that the failure rate of the system = the sum of the failure rates of the components of the system.*

*Or*

*P.T. for a series system, the failure rates are additive :*

*i.e., P.T.* $\lambda_S = \lambda_A + \lambda_B + \ldots\ldots + \lambda_n$.

**Proof.** Let $R_S$, $R_A$, $R_B$ ..... be reliability of the system and its component parts A, B and C so on.

For independent components in series and exponentially distributed failure rate,

$$R_S = R_A \,.\, R_B \,.\, R_C \qquad \ldots(1)$$

If $\lambda_S$, $\lambda_A$, $\lambda_B$, $\lambda_C$ ..... are the failure rate of the system and its component parts for a *mean time '$t_m$'.*

From eqn. (1),

$$e^{-\lambda_{Stm}} = (e^{-\lambda_{Atm}}) \,.\, (e^{-\lambda_{Btm}}) \,.\, (e^{-\lambda_{Ctm}}) \ \ldots\ldots$$

$$e^{-\lambda_{Stm}} = e^{-\lambda_{tm}(\lambda_A + \lambda_B + \lambda_C + \ldots\ldots)}$$

$$e^{\lambda_S} = e^{(\lambda_A + \lambda_B + \lambda_C + \ldots\ldots)}$$

Since the bases are save, by comparing L.H.S. and R.H.S., we equal the powers,

$$\lambda_S = \lambda_A + \lambda_B + \lambda_C + \ldots\ldots$$

Hence proved.

**1.** *A system has 3 units with a failure rate of 1.5, 4, 3.8 failures per $10^6$ hrs.*

(*a*) *Find the MTBF of the system.*

(*b*) *Determine the reliability of the system for 10 hrs., if the components are connected in series.*

(*c*) *If the components are connected in parallel, R = ?*

**Solution. Data :** $\lambda_A = 1.5$ failures / $10^6$ hrs.

*i.e.,* $\lambda_A = 1.5 \times 10^{-6}$ failures / hr.

$\lambda_B = 4 \times 10^{-6}$ failures / hr. ; $\lambda_C = 3.8 \times 10^{-6}$ failures / hr.

(*a*) Assuming that components are in series,

$$\lambda_S = \lambda_A + \lambda_B + \lambda_C. \qquad \therefore \quad MTBF_{sys} = \frac{1}{\lambda_S} = 107526.8 \text{ hrs.}$$

$\lambda_S = (1.5 + 4 + 3.8) \times 10^{-6}$

$\lambda_S = 9.3 + 10^{-6}$ failures / hr.

failure rates can be added and not the MTBFs. To find MTBFs = $\frac{1}{\lambda_S}$

(*b*) $R_S = e^{-\lambda_s t}$

$R_S = e^{-(9.3 \times 10^{-6} \times 10)}$

$R_S = 0.9999.$

(*c*) If the components are connected in parallel,

$R_S = [1 - (1 - R_A)(1 - R_B)(1 - R_C)]$

$R_A = e^{-\lambda A t}$

$R_A = e^{-(1.5 \times 10^{-6} \times 10)} = 0.9999$

$R_B = e^{-(4 \times 10^{-6} \times 10)} = 0.9999$ } $R_A, R_B, R_C$ are equal

$R_A = e^{-(3.8 \times 10^{-6} \times 10)} = 0.9999$

$\therefore \quad R_S = [1 - (1 - 0.9999^3)]$

$R_S = 0.9997.$

**2.** *A **series** system has 3 independent parts A, B, C which have a MTBF of 100, 400, 800 hrs. reliability.*

*Find* (*a*) *MTBF of system.*

(*b*) *Failure rate of system in failures/million hrs.*

(*c*) *Failure rate of system in percent failures/1000 hrs.*

(*d*) *Reliability of system for 30 hrs.*

(*e*) *The increase in MTBF of A component to a 30% of increase in MTBF of system.*

**Solution. Data :** $MTBF_A = 100$ hrs. ; $MTBF_B = 400$ hrs. ; $MTBF_C = 800$ hrs.

$$\lambda_A = \frac{1}{100} = 0.01 \text{ failures/hr.}$$

$$\lambda_B = \frac{1}{400} = 0.0025 \text{ failures/hr.}$$

$$\lambda_C = \frac{1}{800} = 0.00125 \text{ failures/hr.}$$

(*a*) $MTBF_S = ?$

$\lambda_S = \lambda_A + \lambda_B + \lambda_C$

$\lambda_S = 0.01 + 0.0025 + 0.00125$

$\lambda_S = 0.01375$ failures/hr.

$$\therefore \quad MTBF_S = \frac{1}{\lambda_S} = \frac{1}{0.01375} = 72.727 \text{ hrs.}$$

(*b*) WKT, $\lambda_S = 0.01375$ failures/hr.

$\lambda_S = 0.01375 \times 10{,}00{,}000$ failures/million hr. (1 million = $10^6$)

$\lambda_S = 13750$ failures/million hr.

(*c*) $\lambda_S = 0.01375 \times 100 \times 1000\%$ failures/1000 hrs.

$\Downarrow \quad \Downarrow$

(%) (1000 hrs.)

$\lambda_S = 1375$ percent failures/1000 hrs.

(*d*) $R_S = e^{-\lambda s t}$

$R_S = e^{-(0.01375 \times 30)}$

$R_S = 0.6619.$

(*e*) $MTBF_{new_S} = 72.73 \times 1.30 = 94.536$ hrs.

$$\therefore \quad \lambda_{new_S} = \frac{1}{MTBF_{new_s}} = \frac{1}{94.536} = 0.0105 \text{ failures/hr.}$$

WKT, $\lambda_S = \lambda_A + \lambda_B + \lambda_C$.

$0.0105 + \lambda_A = 0.0025 + 0.00125.$

$\therefore \quad \lambda_A = 0.00682$ failures/hr.

$$\therefore \quad MTBF_A = \frac{1}{\lambda_A} = \frac{1}{0.0062} = 146.456 \text{ hrs} \approx 146 \text{ hrs.}$$

$\therefore$ **Increase in $MTBF_A$ = 146 – 100 = 46 hrs.**

$$\text{The \% increase in } MTBF_A = \frac{(146 - 100) \times 100}{100} = 46\%$$

**3.** *A system is composed of 10,000 parts. What average failure rate/part must be achieved to get a system's MTBF of 250 hrs. (Assume series with independent).*

**Solution. Data :** $n$ = 10,000 ; $\lambda$ = ? ; MTBF = 250 hrs.

$$\lambda = \frac{1}{MTBF} = \frac{1}{250} = 0.004 \text{ failures/hr}$$

$\lambda_S = \lambda_1 + \lambda_2 + .....$

$$\therefore \quad \text{Average/part} = \frac{0.004}{10,000} = 4 \times 10^{-7} \text{ failures/hr.}$$

**4.** *Determine the reliability of an equipment having an MTBF of 50 hrs for an operating period of 45 hrs. If the reliability has to be improved by 20%. What % change in MTBF is required ?*

**Solution. Data :** MTBF = 50 hrs ; $t$ = 45 hrs.

$R = e^{-\lambda t}$

$$\lambda = \frac{1}{MTBF} = \frac{1}{50} = 0.02 \text{ failures/hr.}$$

$R = e^{-(0.02 \times 45)}$

$R = 0.4066$

$R_{new} = 0.4066 \times 1.2 = 0.4879.$

$\therefore \quad R_{new} = e^{-(\lambda \text{ new } t)}$

$0.4879 = e^{-(\lambda \text{ new} \times 45)}$

$\ln 0.4879 = -(\lambda_{new} \times 45)$

$-0.7176 = -\lambda_{new} \times 45$

$$\therefore \quad \lambda_{new} = \frac{0.7176}{45} = 0.0159$$

$$\text{MTBF}_{new} = \frac{1}{\lambda_{new}} = \frac{1}{0.0159} = 62.89$$

$$\% \text{ change} = \frac{(62.89 - 50) \times 100}{50} = 25.78\%.$$

**5.** *The MTBF of a certain unit is 50 hrs. Calculate the reliability for 75 hrs of operating period. If the reliability of the unit is increased by 10%, 20%, 30%, 40%, 50%, calculate (a) the corresponding % changes in MTBF i.e., necessary (b) plot a graph % change in reliability v/s % change in MTBF.*

**Solution. Data :** MTBF = 50 hrs ; $t$ = 75 hrs.

$$\lambda = \frac{1}{\text{MTBF}} = \frac{1}{50} = 0.02 \text{ failures/hr.}$$

$$R = e^{-\lambda t} = e^{-(0.02 \times 75)} = 0.2231$$

(*a*) (*i*) If R = 1.1 × 0.2231 = 0.2454

*i.e.*, $0.2454 = e^{-(\lambda \times 75)}$

$ln\ 0.2454 = -(\lambda \times 75)$

$-1.4048 = -\lambda \times 75$

$\therefore \quad \lambda = 0.0187$ failures/hr. $\Rightarrow$ MTBF = 53.48 hrs.

(*ii*) If R = 1.2 × 0.2231 = 0.2677

*i.e.*, $0.2677 = e^{-(\lambda \times 75)}$

$ln\ 0.2677 = -(\lambda \times 75)$

$-1.3178 = -\lambda \times 75$

$\therefore \quad \lambda = 0.0176$ failures/hr. $\Rightarrow$ MTBF = 56.82 hrs.

(*iii*) Similarly if R = 1.3 × 0.2231 = 0.29

*i.e.*, $0.29 = e^{-(\lambda \times 75)}$

$ln\ 0.29 = -\lambda \times 75$

$-1.2378 = -\lambda \times 75$

$\therefore \quad \lambda = 0.0165$ failures/hr. $\Rightarrow$ MTBF = 60.61 hrs.

(*iv*) Similarly if R = 1.4 × 0.2231 = 0.3123

*i.e.*, $0.3123 = e^{-(\lambda \times 75)}$

$ln\ 0.3123 = -(\lambda \times 75)$

$-1.1637 = -\lambda \times 75$

$\therefore \quad \lambda = 0.0155$ failures/hr. $\Rightarrow$ MTBF = 64.52 hrs.

(*v*) Similarly if R = 1.5 × 0.2231 = 0.3347

*i.e.*, $0.3347 = e^{-(\lambda \times 75)}$

$ln\ 0.3347 = -(\lambda \times 75)$

$-1.0947 = -\lambda \times 75$

$\therefore \quad \lambda = 0.0146$ failures/hr. $\Rightarrow$ MTBF = 68.49 hrs.

$$\% \text{ change in MTBF } (i) = \frac{50 - 53.48 \times 100}{50} = -6.96\%$$

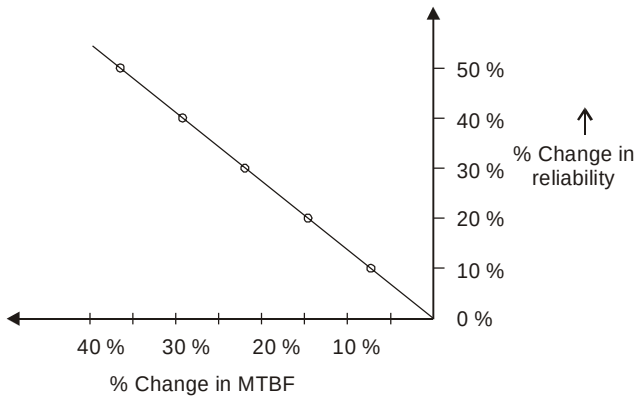
$$\% \text{ change in MTBF } (ii) = \frac{50 - 56.82 \times 100}{50} = -13.64\%$$

$$\% \text{ change in MTBF } (iii) = \frac{50 - 60.61 \times 100}{50} = -21.22\%$$

$$\% \text{ change in MTBF } (iv) = \frac{50 - 64.52 \times 100}{50} = -29.04\%$$

$$\% \text{ change in MTBF } (v) = \frac{50 - 68.49 \times 100}{50} = -36.98\%$$

(*b*)




**6.** *An electronic system consists of power supply whose failure rate is 30 failures/ $10^6$ hrs. A receiver whose failure rate is 25 failures/$10^6$ hrs and an amplifier whose failure rate is 20 failures/$10^6$ hrs in* ***series****. The equipment is to operate for 200 hrs. Determine its reliability.*

**Solution.** $\lambda_P = 30 \times 10^{-6}$ failures/hr.

$\lambda_r = 25 \times 10^{-6}$ failures/hr.

$\lambda_a = 20 \times 10^{-6}$ failures/hr.


Since the components are connected in series,

$\lambda_S = \lambda_p + \lambda_a + \lambda_r$

$\lambda_S = 75 \times 10^{-6}$ failures/hr.

$\therefore \quad R = e^{-\lambda t}$

$R = e^{-(75 \times 10^{-6} \times 200)}$

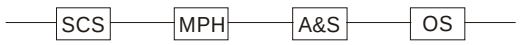
$R = 0.9851.$

**7.** *A cassette player has got 4 subsystems namely speed control system, the magnetic pick up head, amplification and sound system and other systems. For a satisfactory performance of a cassette player, all the 4 systems must perform satisfactorily. The MTBF of the various subsystems are :*

*$MTBF_{SCS}$ = 2000 hrs ; $MTBF_{MPH}$ = 2500 hrs ; $MTBF_{A\ and\ S}$ = 4000 hrs.*

*$MTBF_{OS}$ = 3000 hrs. Calculate the reliability of cassette player per 1500 hrs reception time. What is the mean time below failures of a cassette player ?*

**Solution.** The 4 subsystems should be connected in series.




$$\lambda_{SCS} = \frac{1}{MTBF} = \frac{1}{2000} = 5 \times 10^{-4} \text{ failures/hr}$$

$$\lambda_{MPH} = \frac{1}{2500} = 4 \times 10^{-4} \text{ failures/hr.}$$

$$\lambda_{A \text{ and } S} = \frac{1}{4000} = 2.5 \times 10^{-4} \text{ failures/hr.}$$

$$\lambda_{OS} = \frac{1}{3000} = 3.333 \times 10^{-4} \text{ failures/hr.}$$

Since the components are connected in series,

$$\lambda_S = (5 \times 10^{-4}) + (4 \times 10^{-4}) + (2.5 \times 10^{-4}) + (3.33 \times 10^{-4})$$

$$\lambda_S = 14.8 \times 10^{-4} \text{ failures/hr.}$$

(*a*) $R = e^{-\lambda_s t}$

$$R = e^{-(14.8 \times 10^{-4} \times 1500)}$$

$$R = 0.1086$$

(*b*) $MTBF_S = \frac{1}{\lambda_S} = \frac{1}{14.8 \times 10^{-4}} = 675.67$ hrs.

**8.** *An item is required to have a failure rate not > r than 0.1% per 1000 hrs of operation.*

(*a*) *Assuming a constant failure rate, what is the probability that one of these units will survive for a required 2000 hrs of service ?*

(*b*) *Determine the way acceptable failure rate where the probability of survival for a required 2500 hrs of operation is 0.999 ?*

**Solution. Data :**

$$\lambda = l = 0.1\% \text{ per } 1000 \text{ hrs.}$$

$$\lambda = 0.1 \times 10^{-2}/1000 \text{ hrs.} = 0.1 \times 10^{-2} \times 10^{-3} \text{ failures/hr.}$$

$$\lambda = 0.1 \times 10^{-5} \text{ failures/hr.}$$

(*a*) $R = e^{-\lambda t}$

$$= e^{-(0.1 \times 10^{-5} \times 2000)} = 0.998$$

(*b*) $$R = e^{-\lambda t}$$

$$0.999 = e^{-(\lambda \times 2500)}$$

$$-1.0005 \times 10^{-3} = -\lambda \times 2500$$

$$\lambda = 5 \times 10^{-7} \text{ failures/hr.}$$

**9.** *A participant in a motor rally is required to complete a mission time of 2500 kms. His vehicle can be imagined to have 3 subsystems namely fuel, ignition and other*

*systems. The mean time to repair (MTTR) of the 3 sub-systems are known to be 6000, 8000, 10000 kms respectively. Find the reliability of his completing the mission without repair.*

**Solution.** $MTTR_F = 6000$ kms

$MTTR_I = 8000$ kms $\qquad$ $MTTR_O = 10000$ kms.

$$\lambda_F = \frac{1}{MTTR_F} = \frac{1}{6000} = 1.66 \times 10^{-4} \text{ failures/kms.}$$

$$\lambda_I = \frac{1}{8000} = 1.25 \times 10^{-4} \text{ failures/km.}$$

$$\lambda_O = \frac{1}{10000} = 1 \times 10^{-5} \text{ failures/km.}$$

} Important

Since the components are connected in series,

$$\lambda_S = \lambda_F + \lambda_I + \lambda_O = 3.01 \times 10^{-4} \text{ failures/km.}$$

$$R = e^{-\lambda st} = e^{-(3.01 \times 10^{-4} \times 2500)}$$

$$R = 0.38.$$

**10.** *What is the failure rate of a piece of equipment if the prob. of survival is 88% for 900 hrs. of operating period ? Express the failure rate in terms of % failures/1000 hrs ?*

**Solution. Data.** $R = 0.88$ ; $t = 900$ hrs ; $\lambda = ?$

$$R = e^{-\lambda t}$$

$$0.88 = e^{-(\lambda \times 900)}$$

$$-0.1278 = -\lambda \times 900$$

$$\lambda = 1.42 \times 10^{-4} \text{ failures/hr.}$$

$$\lambda = 1.42 \times 10^{-4} \times 100 \times 1000\% \text{ failures/1000 hrs.}$$

$$\lambda = 14.2\% \text{ failures/1000 hrs.}$$

**11.** *The MTBF of an equipment is 1000 hrs. What is the failure rate expressed in (a) % failures/1000 hrs. (b) failures per $10^6$ hrs. (c) failures/hr. (d) Is MTBF a guaranteed failure-free period.* (**Ans.** No. It depends upon)

**Solution.** $$\lambda = \frac{1}{MTBF} = \frac{1}{1000} = 10^{-3} \text{ failures/hr.}$$

$$\lambda = 10^{-3} \times 100 \times 1000\% \text{ failures/1000 hrs.}$$

$$\lambda = 100\% \text{ failures/1000 hrs.}$$

$$\lambda = 10^{-3} \times 10^{-6} \text{ failures/} 10^6 \text{ hrs.}$$

$$\lambda = 10^{-3} \text{ failures/} 10^6 \text{ hrs.}$$

**12.** *A 750 hrs. life test is performed on 6 components. 1 component fails after 350 hrs. of operation. All others survive the test. Compute the failure rate.*

**Solution.** WKT, $$\lambda = \frac{\text{No. of items failed}}{\text{Total test duration}} = \frac{1}{\underset{\text{survival}}{(5 \times 750)} + \underset{\text{failed}}{(1 \times 350)}}$$

$$= 2.44 \times 10^{-4} \text{ failures/hr.}$$

**REDUNDANCY**

(Parallel connection increases the reliability when compared to series connective.)

One of the methods for improving the reliability of the system is by utilising the concept of redundancy. To enhance reliability of the system, quite often addition units are built into the system to perform the same function. In such a system, one component failure will not necessarily cause the system failure, since additional components are available to perform the same function.

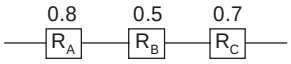
*Redundancy is defined as the characteristic of a system by virtue of which marginal component failures are prevented from causing system failures due to the presence of additional components.*

In order to increase the reliability of the system, select the component which has the least reliability and then arrange it parallelly.

$R_S = R_A \times R_B \times R_C$

$R_S = 0.8 \times 0.5 \times 0.7$

$R_S = 0.28.$

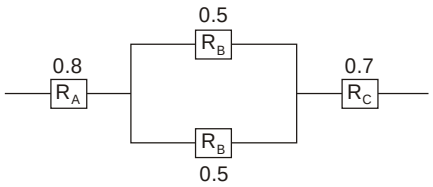



In order to increase the above system's reliability, select the component which has the least reliability (0.5) and arrange it parallelly.

$R_B' = 1 - (1 - R_B)(1 - R_B)$

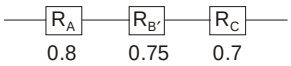
$R_B' = 1 - [(1 - 0.5)(1 - 0.5)]$

$R_B' = 0.75$




$\therefore \quad R_S = R_A \times R_B' \times R_C$

$R_S = 0.8 \times 0.75 \times 0.7 = 0.44.$




It is clear that the R is $\uparrow d$ from 0.28 to 0.44.

**Definition of Improvement Factor :**

In the case of '*n*' parallel redundancies, the improvement factor,

$$IF = \frac{1 - R}{1 - R_S}.$$

where $(1 - R)$ = Unreliability of each component.

$(1 - R_S)$ = Unreliability of the system.

If '*n*' components are connected in parallel with the same reliability as shown in fig. below :

$R_S = 1 - [(1 - R_A)(1 - R_B) \ldots\ldots (1 - R_n)]$

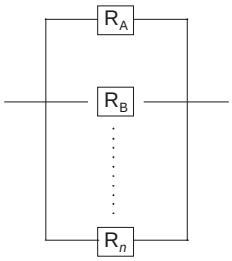
Since $R_A = R_B = \ldots\ldots R_n$

$R_S = 1 - [(1 - R)^n]$

$\therefore \quad 1 - R_S = (1 - R)^n$

$$\therefore \quad IF = \frac{1 - R}{1 - R_S}$$

$$IF = \frac{(1 - R)}{(1 - R)^n} = (1 - R)^{1 - n}$$




Let $(1 - R) = Q$,

then, $IF = \frac{Q}{Q^n}$

$$\boxed{IF = Q^{1 - n}}$$

**1.** *A system consists of 3 components A, B and C. The configuration of the system and the reliabilities of the elements are given below. Calculate the reliability of system* $R_A = 0.8$ ; $R_B = 0.7$ ; $R_C = 0.9$.

**Solution.**

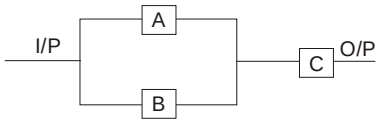
$R_{AB} = 1 - [(1 - R_A)(1 - R_B)]$

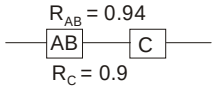
$R_{AB} = 1 - [(1 - 0.8)(1 - 0.7)]$

$R_{AB} = 0.94$

$R_S = R_{AB} \cdot R_C = 0.94 \times 0.9$

$R_S = 0.846.$



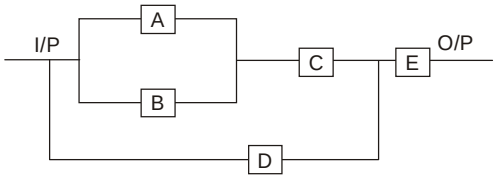

**2.** *Calculate the reliability of the configuration given below :*

$R_A = 0.7$ ; $R_B = 0.7$ ; $R_C = 0.9$

$R_D = 0.8$ ; $R_E = 0.9$.

**Solution.** $R_{AB} = 1 - [(1 - R_A)(1 - R_B)]$

$= 1 - [(1 - 0.7)(1 - 0.7)]$

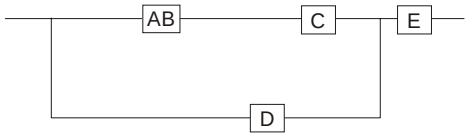



$R_{AB} = 0.91$

$R_{ABC} = R_{AB} \cdot R_C$

$= 0.91 \times 0.9$

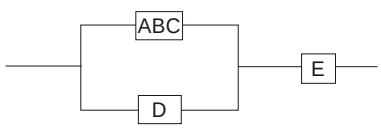
$= 0.819$




$R_{ABCD} = 1 - [(1 - R_{ABC})(1 - R_D)]$

$= 1 - [(1 - 0.819)(1 - 0.8)]$

$= 0.9638.$



$R_{ABCDE} = R_{ABCD} \cdot R_E$

$= 0.9638 \times 0.9$

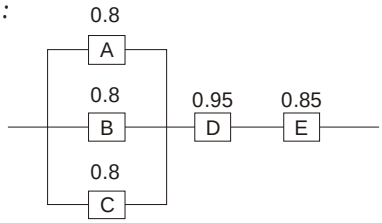
$= 0.8674.$


**3.** (*a*) *What is the reliability of system shown below :*

$P_A \times P_B \times P_C = 0.8$ (*Reliability*)

$P_D = 0.95$

$P_E = 0.85.$




(*b*) *How will the reliability improve further if the system F is also made parallel redundant ? Show the configuration of the system.*

**Solution.** (*a*) $R_{ABC} = 1 - [(1 - R_A)(1 - R_B)(1 - R_C)] = 1 - [(1 - R_A)^3]$

$R_{ABC} = 1 - [(1 - 0.8)^3]$

$R_{ABC} = 0.992$

$R_{ABCDE} = R_{ABC} \cdot R_D \cdot R_E = (0.992) \cdot (0.95) \cdot (0.85) = 0.801.$

(*b*) WKT, $R_{ABC} = 0.992$

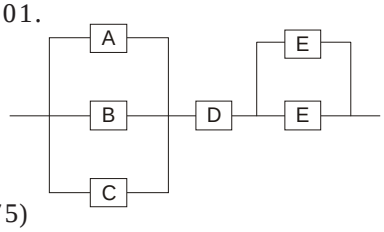
$R_D = 0.95$

$R_E = 1 - [(1 - 0.85)^2]$

$R_E = 0.9775$

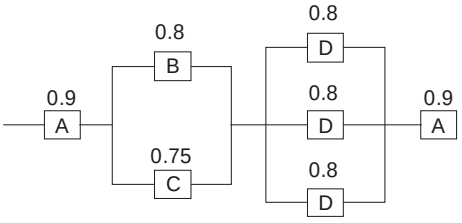
$\therefore \quad R_{ABCDE} = R_{ABC} \cdot R_D \cdot R_E = (0.992) \cdot (0.95) \cdot (0.9775)$

$= 0.9212.$




Improvement in R = 0.9212 – 0.801 = 0.1202.

**4.** *Determine the probability of success for the foll. System with all units operating.* $P_A$ *= 90% ;* $P_B$ *= 80% ;* $P_C$ *= 75% ;* $P_D$ *= 80%.*




**Solution.** $R_{BC} = 1 - [(1 - 0.8)(1 - 0.75)]$

$R_{BC} = 0.95$

$R_D = 1 - [(1 - 0.8)^3]$

$R_D = 0.992$

$\therefore \quad R_{ABCDA} = 0.9 \times 0.95 \times 0.992 \times 0.9$

$= 0.7633.$

**5.** *An electronic system consists of 5 subsystems with the foll MTBF :*

$SS_A = 12500 \qquad SS_C = 11000 \qquad SS_E = 15550$

$SS_B = 2830 \qquad SS_D = 9850$

*These 5 SS are arranged in series configuration. What is the probability of survival for an operating period of 800 hrs ?*

**Solution.**

$$\lambda_A = \frac{1}{MTBF_A} = \frac{1}{12500} = 0.00008 \text{ failures/hr.}$$

$$\lambda_A = \frac{1}{MTBF_B} = \frac{1}{2830} = 353.3569 \times 10^{-6} \text{ failures/hr.}$$

$$\lambda_C = \frac{1}{MTBF_C} = \frac{1}{11000} = 9.0909 \times 10^{-5} \text{ failures/hr.}$$

$$\lambda_D = \frac{1}{MTBF_D} = \frac{1}{9850} = 1.0152 \times 10^{-4} \text{ failures/hr.}$$

$$\lambda_E = \frac{1}{MTBF_E} = \frac{1}{15550} = 6.4309 \times 10^{-5} \text{ failures/hr.}$$

$\therefore \quad \lambda_S = \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E = 6.9009 \times 10^{-4}$ failures/hr.

$R_S = e^{-\lambda st}$

$R_S = e^{-(6.9009 \times 10^{-4} \times 800)}$

$R_S = 0.5757.$

**6.** *A step down transformer, rectifier, filter, comprise a series of system. The following are the failure rates of these components.*

*Transformer* = *1.56% failures/1000 hrs.*

*Rectifier* = *2% failures/1000 hrs.*

*Filter* = *1.7% failures/1000 hrs.*

*The equipment is to operate for 1500 hrs. What is the probability of survival of the system ?*

**Solution.** $\lambda_t = 1.56 \times 10^{-5}$ failures/hr.

$\lambda_r = 2 \times 10^{-5}$ failures/hr.

$\lambda_f = 1.7 \times 10^{-5}$ failures/hr.

$\lambda_S = \lambda_t + \lambda_r + \lambda_f$

$\lambda_S = 5.26 \times 10^{-5}$ failures/hr.

$R = e^{-\lambda_s t}$

$R = e^{-(5.26 \times 10^{-5} \times 1500)}$

$R = 0.9241.$

**7.** *Determine the reliability of the system for 20 hrs. of operating period. The configuration is given below. The failure rate/hr. are also given.*

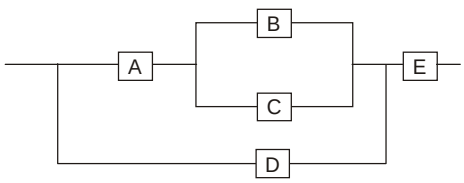
$\lambda_A = 0.01$

$\lambda_B = 0.015$

$\lambda_C = 0.02$

$\lambda_D = 0.02$

$\lambda_E = 0.025$



**Solution.** $R_A = e^{-\lambda_A t} = e^{-(0.01 \times 20)} = 0.8187$

$R_B = e^{-\lambda_B t} = e^{-(0.015 \times 20)} = 0.7408$

$R_C = e^{-\lambda_C t} = e^{-(0.02 \times 20)} = 0.6703$

$R_D = e^{-\lambda_D t} = e^{-(0.02 \times 20)} = 0.6703$

$R_E = e^{-\lambda_E t} = e^{-(0.025 \times 20)} = 0.6065$

$R_{BC} = 1 - [(1 - R_B)(1 - R_C)]$

$R_{BC} = 1 - [(1 - 0.7408)(1 - 0.6703)]$

$R_{BC} = 0.9145$

$R_{ABC} = R_A \cdot R_{BC}$

$R_{ABC} = 0.8187 \times 0.9145$

$R_{ABC} = 0.7487$

$R_{ABCD} = 1 - [(1 - R_{ABC})(1 - R_D)]$

$R_{ABCD} = 1 - [(1 - 0.7487)(1 - 0.6703)]$

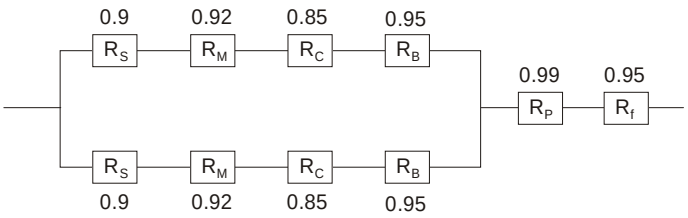
$R_{ABCD} = 0.9172$

$R_{ABCDE} = R_{ABCD} \times R_E$

$R_{ABCDE} = 0.9172 \times 0.6065$

$R_{ABCDE} = R_S = 0.5562.$

**8.** *In a rerolling 2 blowers are connected to reheat the furnace using pulverized coal. One of the blowers is a standby equipment. The blower system includes, stator motor, coupling and blower which have the reliabilities of 0.9, 0.92, 0.85 and 0.95 respectively. The pulverised coal system has a reliability of 0.99 and the furnace 0.95, all over an operating period of 150 hrs. Draw the system configuration and determine the reliability of the total system for 150 hrs. of operation ?*




**Solution.** $R_{\text{blower 1}} = R_S \cdot R_M \cdot R_C \cdot R_B$

$= 0.9 \times 0.92 \times 0.85 \times 0.95 = 0.6686.$

Similarly $R_{\text{blower 1}} = R_{\text{blower 2}} = 0.6686$

Since the 2 blowers are connected in parallel,

$R_{\text{blower}} = 1 - [(1 - R_{b1})(1 - R_{b2})]$

$= 1 - [(1 - 0.6686)^2]$

$= 0.8902$

$R_{\text{system}} = R_{\text{blower}} \cdot R_P \cdot R_f$

$= 0.8902 \times 0.99 \times 0.95 = 0.8372.$

# Experimental Design

## INTRODUCTION TO DESIGN OF EXPERIMENTS

Industrial organisations are constantly faced with the problem of decision making regarding product design, parts specifications, quality improvement, dominant factors affecting quality, cost reduction, import substitution, etc., for economic prosperity or even for survival in the increasingly competitive market. In all such problems, we are confronted with several alternatives which could be visualised from technological background or by referring to available literature on the subject, the problem being to choose out of all these (possibly quite numerous or even infinite) alternatives, the one which satisfies the requirement at minimum cost. How then do we take the right decision ?

The problem would have been simple to solve if sufficient research has already been carried out in the particular field and the information gathered thereby is available in a form which could be readily utilised for making the decision without recourse to any bunches.

However, experience shows that the general situation is quite different as the decision makers are not usually aware of the precise outcome associated with trying out the various alternatives, at best only having some broad ideas about what is likely to happen actually. In most of the cases, even this much is not known.

In all such cases, experiment has to be carried out either to discover something about a particular design or to compare the effect of several conditions on the phenomena under study. An experiment has been defined as "a trial or special observation made to confirm or disprove something doubtful, especially one under conditions determined by the experimenter ; an act or operation undertaken in order to discover some unknown principle or effect or to test, establish or illustrate some suggested or known truth". In brief, the purpose of experimentation is to ensure that the experimenter obtains the data relevant to the test of decision making in as economical a way as possible.

## TERMINOLOGY USED IN DESIGN OF EXPERIMENTS

### Design of Experiments

The primary reason for using statistically designed experiments is to obtain a maximum amount of information from minimum amount of expenditure. The fundamental purpose of designed experiment is to determine a course of action. This action results from conclusions drawn from an experiment.

In order to apply the methods of a statistically designed experiment, it is necessary to understand the terminology used. Some of the important terms are explained in the following list.

**EXPERIMENT**

A planned set of operations which leads to a corresponding set of observations. The purpose of experimentation is to ensure that the experimenter obtains the data relevant to the task of decision making in an economical way.

**FACTOR**

A feature of the experimental conditions which may be varied from one trial to another.

**LEVELS OF A FACTOR**

The alternate values of a factor considered in the experiment are called its levels.

**TREATMENT COMBINATION**

The set of levels of all factors including in a trial in experiment is called a treatment combination.

**OUTCOME (OR RESPONSE)**

The numerical result of a trial based on a given treatment combination is called outcome (or) response.

**MAIN EFFECT**

The change in the average response produced by a change in the level of the factor is called "Main Effect" of that factor.

**INTERACTION**

If the effect of one factor is different at different levels of another factor, the two factors are said to interact (or) to have interaction.

The interaction between two factors A and B is termed as "First Order interaction" (or) "Two factor interaction" and is denoted by $A \times B$.

If the interaction between two factors A and B, is different at different levels of a third factor C, then there is said to be interaction among the three factors. This is referred to as "second order interaction" (or) "three factor interaction" and is denoted by $A \times B \times C$.

**EXPERIMENTAL UNIT**

An experimental unit is one item to which a single treatment combination is applied in one replication of the basic experiment.

**REPLICATION**

Replication is a repetition of the whole experiment in order to estimate experimental error, increase precision (detect smaller changes).

**EXPERIMENTAL ERROR**

The failure of two identically treated experimented units to give the same value.

**STEPS IN DESIGNING AN EXPERIMENT**

1. Statement of the problem.
2. Formulation of hypothesis.
3. Planning of the experiment.
   (*a*) Choosing an appropriate experimental technique.
   (*b*) Examination of possible outcomes to make sure that the experiment provides the required information.

(c) Consideration of possible results from the point of view of the statistical analysis.

4. Collection of data, after performing the experiment according to the plan.
5. Statistical analysis of the data.
6. Drawing conclusions with appropriate level of significance.
7. Verification (or) evaluation of results (conclusions).
8. Drawing final conclusions and recommendations.

These eight steps are present in any experiment whether it has been effectively designed (or) not.

A good experiment has several requirements. It must possess no systematic error, have small standard or experimental error, and a wide range of validity, and allow proper statistical analysis to apply.

The standard and systematic errors which we wish to minimize can be caused by tolerance in the actual experimentation, observation of responses measurement, variation in experimental materials, (or) other extraneous factors. A good experiment minimizes these by

1. Using more homogeneous materials
2. Using information provided by related variables
3. Careful experimentation
4. Using more efficient design.

**Principles of an Experimental Design**

There are three general principles that are essential to a statistically designed experiment. They are :

(a) Randomization

(b) Replication

(c) Local control

(a) **Randomization.** If separate experimental runs (trials) are conducted in random order, then statistical analysis of responses can be conducted without danger of any bias to experimental unit.

(b) **Replication.** It is a repetition of the whole experiment in order to estimate experimental error (which in turn provides a means of testing interactions), increase precision (detect smaller changes).

(c) **Local controls.** Local controls means experimental planning, referring to the amount of blocking (or) balancing of grouped experimental runs among different settings of variables being studies.

The purpose of local controls is to make the experimental design more efficient, and guard against unforeseen surprises nullifying experimental adjustments.

To sum up replication makes statistical test possible randomization makes statistical test valid and local controls make the experiment more efficient.

**Different Factors of Experimentation**

1. *Recognition and statement of the problem.* This may seem to be a rather obvious point, but in practice it is often not simple to realize that a problem requiring

experimentation exists, and to develop a clear and genearlly accepted statement of this problem. It is necessary to develop all ideas about the objectives of the experiment. A clear statement of the problem often contributes substantially to a better understanding of the phenomena and the final solution of the problem.

2. *Choice of factors and levels.* The experimenter must select the independent variables or factors to be investigated in the experiment. For example, in the hardness testing experiment described previously, the single factor is quenching media. The factors in an experiment may be either quantitative or qualitative. If they are quantitative, thought should be given as to how these factors are to be controlled at the desired values and measured. We must also select the values or levels of the factors to be used in the experiment. These levels may be chosen specifically, or selected at random from the set of all possible factor levels.
3. *Selection of a response variable.* In choosing a response or dependent variable, the experimenter must be certain that the response to be measured really provides information about the problem under study. Thought must also be given to how the response will be measured, and the probable accuracy of those measurements.
4. *Choice of experimental design.* This step is of primary importance in the experimental process. The experimenter must determine the difference in true response he wishes to detect and the magnitude of the risks he is willing to tolerate so that an appropriate sample size (number of replicates) may be chosen. He must also determine the order in which the data will be collected and the method of randomization to be employed . It is always necessary to maintain a balanced between statistical accuracy and cost. Most recommended experiemntal designs are both statistically efficient and economical, so that the experimenter's efforts to obtain statistical accuracy usually result in economic efficiency. A mathematical model for the experiment must also be proposed, so that a statistical analysis of the data may be performed.
5. *Performing the experiment.* This is the actual data collection process. The experimenter should carefully monitor the progress of the experiment to ensure that it is proceeding according to the plan. Particular attention should be paid to randomization, measurement, accuracy, and maintaining as uniform an experimental environment as possible.
6. *Data analysis.* Statistical methods should be employed in analysing the data from the experiment. Numerical accuracy is an important concern here, although present day computers have largely relieved the experimenter from this problem, and simultaneously reduced the computational burden. Graphical methods are also frequently useful in the data analysis process.
7. *Conclusions and recommendations.* Once the data have been analysed, the experimenter may draw conclusions or inferences about his results. The statistical inference must be physically interpreted, and the practical significance of these findings evaluated. Then recommendations concerning these findings must be made. These recommendations may include a further round of experiments, as experimentation is usually an iterative process, with one experiment answering some questions and simultaneously posing others. In presenting

his results and conclusions to others, the experimenter should be careful to minimize the use of unnecessary statistical terminology, and phrase his information as simple as possible. The use of charts and graphs is a very effective way to present important experimental results to management.

**Some Important Designs**

(*i*) ***Completely Randomised Designs.*** This is the simplest design where the various treatments are allotted at random to different experimental units. If a specific number of treatments are to be allotted to a specific number of units, this procedure gives each one of the units the same chance of receiving any treatment. This design is quite efficient when the experimental units are highly homogeneous.

(*ii*) ***Randomised Block Designs.*** When several experimental treatments are to be compared it is clearly desirable that all other conditions shall be kept as nearly constant as is practicable. The number of tests to be carried out may be too great for all to be carried out under similar conditions but it is frequently possible to carry out one complete set of trials at a time under uniform conditions, these conditions being different from set to set.

(*iii*) ***Incomplete Randomised Block Design.*** Sometimes experimental conditions will not permit blocks large enough to include every treatment, so that all the treatments cannot be tested under uniform conditions, nevertheless, by suitably designed experiments, valid and efficient comparisons between the treatments can be made without being disturbed by the difference between blocks. Such designs are termed as Incomplete Randomised Block Designs.

(*iv*) ***Latin Sources.*** Randomised Block Designs amount to a sub-divisions of the experimental conditions into blocks of relatively uniform conditions. Sometimes this sub-division can be effected in more than one way : for instance, in a Multiple Plant the various units may differ in performance, and in addition there may exist a trend in time, as in certain electrolytic and caatlytic processes. The most efficient design for the comparison of different experimental treatments is then the Latin Square.

(*v*) ***Factorial Design.*** A design where all possible combinations of levels of factors are tried is known as a Factorial Design. For example, a factorial design wtih 2 factors each at two levels involves 4 trials ($2^2$) and 2 factors each at three levels involves 9 trials ($3^2$). In general if there are $m$ levels for each of $n$ factors, the factorial design involves ($m^n$) trials. Factorial experiments enable evaluation of interactions between factors and also provide more efficient estimates.

(*vi*) ***Fractional Factorial Designs.*** One disadvantage with factorials is that it calls for a large number of experiments especially when there are a number of factors with two levels each require $2^5 = 32$ experiments. It is possible to reduce the number of experiments and still estimate most of the important effects. This is achieved by fractional factorial experiments. Of course, by carrying out fractional factorials, some information is lost. But when there are several factors, higher order interactions are generally not of importance and in some cases hard to interpret. Hence information on those higher order interactions are deliberately lost to reduce the number of experiments.

In the above example of 5 factors each at two levels, all the main effects and first order interactions can be estimated with half factorial of $\frac{1}{2} \times 2^5 = 16$ trials. All the main effects can be estimated with a quarter factorial of 8 trials. Such designs are known as Fractional Factorial Designs.

(*vii*) ***Orthogonal Array (OA).*** OA design constitutes one particular type of the fractional factorial design. A special feature of the OA design as developed by Dr. G Taguchi (1959) of Japan is the associated concept of line graph which enables a scientist or an engineer to design and analyse complicated experiments without requiring sophisticated statistical known how in the construction of designs requiring knowledge of Galois fields. Prof. Taguchi has further developed a number of techniques for enabling the OA design to meet the needs of various practical situations such as :

— Studying the effect of various factors having different number of levels.

— analysing nested factorial effects when nested factors coexist with some other common factors (partial hierarchical design)

— estimating all the main effects along with a few desired lower order interaction of technological interest, etc.

In short, these techniques are so unique with good reproducibility of experimental results, so easy and flexible in the assignment of factors and analysis that the methods are leaping from industry to industry in a short period.

**Quality Control**

Quality control plays a vital role in all walks of life starting from the household to big engineering and service industries. If a company aspires to be world class, its leaders must understand, digest, disseminate and guide implementation of simple and powerful tools like off-line quality control techniques. The best and important powerful tool to achieve the world class quality to meet the present day global competition is the Design Of Experiments (DOE).

Quality is defined as fitness for use. As per Taguchi the quality of a product is the minimum loss imparted by the product to the society from the time the product is shipped. The quality control is an activity of ensuring manufacturing of good quality products which satisfy the customers needs or requirement.

**Taguchi's Quality Philosophy**

Dr. Genichi Taguchi is one of the pioneers of the present day quality control movement. Taguchi has revolutionized the main very idea of quality control by the introduction of Robust Deign concepts. Taguchi has envisaged a new method of conducing the design of experiments that is called 'Orthogonal Array' (OA). These standard arrays stipulate the way of conducting the minimum number of experiments that could give the full information of all the factors that affect the performance of the parameters. The crux of the Orthogonal Method lies in choosing the level combinations of the input design variables for the experiments.

In 1950 Taguchi joined the newly founded Electrical Communications Laboratory of the Nippon Telephone and Telegraph Company with the purpose of increasing the productivity of its R and D activities by training engineers in effective techniques. He stayed for more than 12 years, during which period he began to develop his methods. In the early

1970's Taguchi developed the concept of the Quality Loss Function. He published two other books in the 1970s and the third (current) edition of Design of Experiments. By the late 1970s Taguchi had an impressive record in Japan having won the Deming application prize in 1960 and Deming awards for literature on quality in 1951 and 1953.

Following his 1980 visit to the United States, more and more American manufacturers implemented Taguchi's methodology. Despite an adverse reaction among American statisticians at the methods, and possibly at the way they were being marketed, major US companies became involved in the method including Xerox, Ford and ITT.

Taguchi's quality philosophy comprises of the following seven important points :

- An important dimension of the quality of a manufacture product is the total loss generated by that product to society.
- In a competitive economy, continuous quality improvement and cost reduction are necessary for staying in business.

Measurable characteristics can be classified into four types :

- **Nominal-The-Best.** A characteristic with a specific target value.
  Examples : Dimension, Clearance, Viscosity etc.
- **Smaller-The-Better.** Here the ideal target value is zero.
  Examples : Wear, Shrinkage, Deterioration etc.
- **Larger-The-Better.** The ideal target value is infinity.
  Examples : Strength, Life, Fuel efficiency etc.
- **Attribute Characteristics.** Based on the visual inspection.
  Examples : Appearance, Taste, Good/bad, etc.

### Evaluation of Quality Loss

There are two methods to evaluate the quality loss : One is the traditional interpretation of quality loss and the other is Taguchi's interpretation of quality loss.

### Traditional Interpretation of Quality Loss

Traditionally the quality is viewed as a step function as shown in Fig. 10.1 This view assumes a product is uniformly good between the specifications and are shipped or utilized in subsequent assemblies.

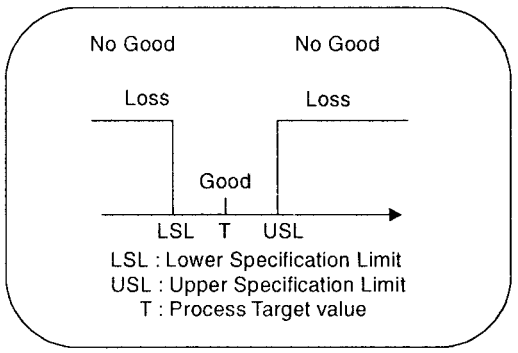



**Fig. 10.1.** Step function.

Taguchi compares specification limits to pass/fail criterion similar to the student examination. He feels that there is not much difference between a student scoring 60% and declared as first class and another student scoring 59% and being declared as second class. Similarly with respect to a product or process functional characteristic a product which barely meeting LSL is accepted and that falls just below the LSL is rejected. He emphasizes that the loss incurred by a product which falls close to LSL and which falls just below the LSL is almost same. For example to assemble car door with tight fit for good appearance many of the parts are to be assembled. If one part is nearer to the USL and the mating part is very close to the LSL then these two parts may not fit well together. This results in a special work to get everything to fit. If all the parts are manufactured around the target value this problem does not arise.

The problem with most of the traditional measures of quality (network rate, scrap rate, *Cp*, *Cpk*, etc.) is that by the time we get these figures the product is already either in production or in the hands of customer.

It is necessary to evaluate the quality at the earliest stage of product/process design rather than the manufacturing stage. To achieve this, Taguchi's optimization techniques are to be applied to minimize the variation in the quality characteristic of a product/process which improves the quality and minimizes the loss.

**Taguchi's Interpretation of Quality Loss Function (TQLF)**

The objective of TQLF is the quantitative evaluation of loss resulting from the function variation of the output quality characteristic from the target value.

For each output characteristic there exists some function which uniquely defines the relationship between the economic loss and the deviation of the output characteristic from its target value. Taguchi had found the quadratic representation of QLF to be an efficient and effective way to assess the loss due to the deviation of the output characteristic from its target value. To establish Taguchi's QLF, consumer tolerance and the customer loss are the two important points to be considered. Taguchi's quadratic loss function is shown in Fig 10.2. It is important to note that :

- Conformation to specification limits is inadequate measure of quality or loss due to poor quality.
- Quality loss results in customer dissatisfaction.
- Quality loss is a financial loss.
- Quality loss can be related to product characteristic.
- The quality loss function is an excellent tool for evaluating loss at the early stage of product/process development.

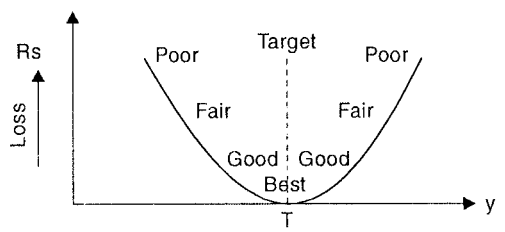



**Fig. 10.2.** Taguchi's Qadratic Representation of the QLF.

Taguchi quality loss function is applied for the three cases of the measurable quality characteristics viz., Nominal-The-Best (NTB), Smaller-The Better (STB) and Larger-The Better (LTB). The ideal value for NTB is the target mean (nominal value), for STB it is zero (minimum) and for LTB it is infinity (maximum).

**Signal-To-Noise Ratio (S/N)**

The S/N ratio developed by Taguchi is a statistical performance measure used to choose control levels that best cope up with noise. The S/N ratio takes both the mean and variability into account. In its simplest form, the S/N ratio is the ratio of the mean (signal) to the standard deviation (noise). The particular S/N ratio depends on the criterion for the quality characteristic to be optimized. Whatever the type of quality characteristic, the transformations are such that the S/N ratio is always interpreted in the same way, the larger the S/N ratio the better it is.

**Signal -to-Noise Ratio**

$$S/N = \frac{\text{Energy (or power) that is transformed into intended output}}{\text{Energy (or power) that is transformed into unintended output}}$$

$$S/N = \frac{\text{Useful output energy}}{\text{Harmful output energy}}$$

The higher the ratio, the higher the quality.

**Fig. 10.3.** Signal-to-noise Ratio.

**S/N ratios for Static Problem**

Some common types of static problems and the corresopnding S/N ratios are given in the Fig. 10.4 :

| *Problem Type* | *Range for the Observations* | *Ideal Value* | *Adjustment* | *S/N Ratio and Comments* |
|---|---|---|---|---|
| Smaller-The-Better type | $0 \le y < \infty$ | 0 | None | $\eta = -10 \log_{10}\left(\frac{1}{n}\sum_{=1}^{n} y_i^2\right)$ |
| Nominal-The-Best type | $0 \le y < \infty$ | Non zero, finite | Scaling | $\eta = 10 \log_{10} \frac{\mu^2}{\sigma^2}$<br>$\mu = \frac{1}{n}\sum_{i=1}^{n} y_i$<br>$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \mu)^2$ |

*(Contd.)*

| | | | | |
|---|---|---|---|---|
| Larger The-Better type | $0 \leq y < \infty$ | $\infty$ | None | $\eta = -10 \log_{10}\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{Y^2 i}\right)$ |
| Fraction defective | $0 \leq p \leq 1$ | 0 | None | $\eta = -10 \log_{10}\left(\frac{p}{1-p}\right)$ |

**Fig. 10.4.** S/N Ratios for Static Problems

**Classification of Factors (Parameters)**

A number of parameters can influence the quality characteristic or response of the product. These parameters can be classified into following three classes.

- Signal factors
- Noise factors
- Control factors

**Signal Factors.** These are the parameters set by the user or the operator of the product to express the intended value for the response of the product. For *e.g.*, the speed setting on a table fan is a signal factor for specifying the amount of breeze. The signal factors are selected by the design engineers based on the engineering knowledge of the product being developed.

**Noise Factors.** These are variables that affect system function, and are either uncontrollable or to expensive to control/change. There are three types of noise factors :

(*i*) **Outer noise**—environmental conditions, such as operating environment, storage environment, manufacturing environment.

(*ii*) **Inner noise**–Aging/deterioration.

(*iii*) **Between product noise**—Variability existing between products made in the manufacturing environment.

**Control Factors.** These are any design parameters of a system that engineers can specify by nominal values and maintain cost-effectively. In fact, it is the designer's responsibility to determine the best values of these parameters. Each control factor can take multiple values called levels. When the levels of certain control factors are changed the manufacturing cost does not change, however, when the levels of others are changed, the manufacturing cost also changes.

The Relationship between Loss and Noise Factors is shown in Fig. 10.5.

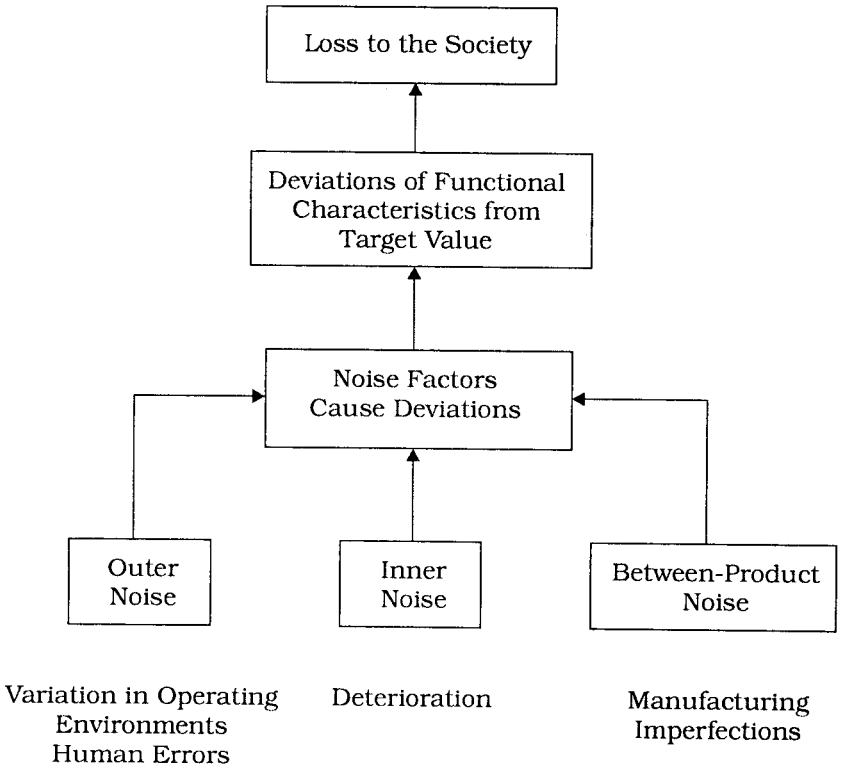



**Fig. 10.5.** The relationship between Loss and Noise Factors.

**Control Factors Vs. Noise Factors**

It is much more productive tio do parameter design at the early stages of a project rather than later. As a project moves downstream from R and D through the developmental cycle to the customer, more and more noise factors enter the picture. It is more effective to do parameter design at the upstream stage when more control factors are available, and design changes are less expensive.

The Relationship between Control Factors Vs Noise Factors are shown in the Fig. 10.6.

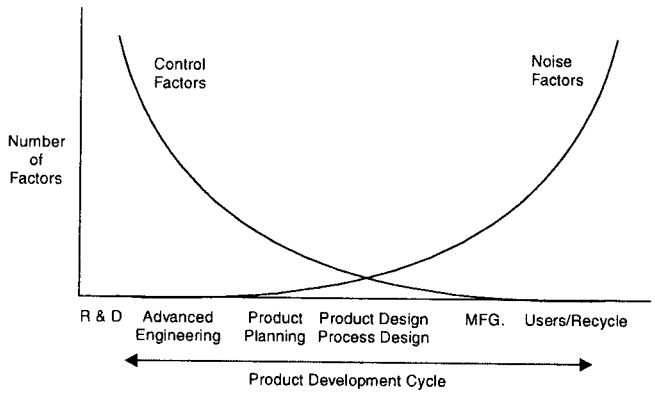



**Fig. 10.6.** Relationship between Control Factors and Noise Factors.

The primary reason for using statistically designed experiments is to obtain a maximum amount of information from minimum amount of expenditure. The fundamental tighter process specifications, better grade material and better equipment are often needed. Because of this, tolerance design usually increases production costs.

If we start with the lowest priced components and material in parameter design, we determine which must have tighter tolerances. However, we may minimize cost by experimenting to find tolerances that can be relaxed without appreciably affecting quality.

Tolerance Design is thus the application of design of experiments to make systematic changes in tolerances, in order to determine which factors contribute most to variation in the end product. Instead of tightening all tolerances in the system, the analysis tells us which tolerances to tighten and which can be relaxed. That is to say, we find the parameters with the highest contributing noise factors and tighten those tolerances. Lower contributing tolerances may be relaxed, thus minimizing cost. Tolerance Design often utilizes the Loss Function to obtain a trade-off between quality and cost. A flow chart showing various stages of Quality Engineering is shown in Fig. 10.7.

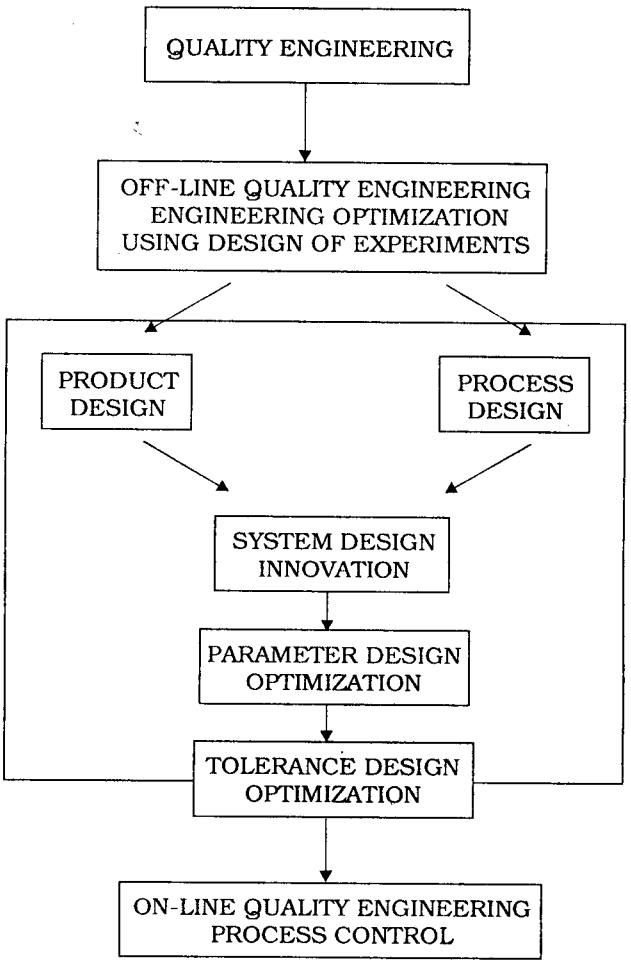



**Fig. 10.7.** Quality Engineering—Flow Chart.

**The Design Process**

1. **System Design**
   - Development of a system to function under an initial set of nominal conditions.
   - Requires technical knowledge from science and engineering.
   - Originality/Invention/Marketing Strategy.
2. **Parameter Design**
   - Determination of control factor levels so that the system is least sensitive to noise.
   - Involves use of orthogonal arrays and Signal-to-Noise Ratio.
   - Improves quality at minimal cost.
3. **Tolerance Design**
   - Specification of allowable ranges for deviations in parameter values.
   - Involves cause detection and removal of causes.
   - Typically increases product cost. However, cost may be minimized by experimenting to find tolerances that can be relaxed without adversely affecting quality.

**PARAMETER DESIGN**

1. **Objective**
   - To select the best combination of control factors so that the design is robust against noise.
2. **Strategy**
   - Design a product starting with low grade, low cost components or raw materials with broad tolerances.
   - Reduce variability without removing the cause of variation.

     —Removing the cause is usually expensive.

     —There is no cost increase in parameter design.

     —Identifying control and noise factors and treating them separately in design of experiments.
3. **Technique Used**
   - Taguchi's approach to Design of Experiments.
4. **Advantages**
   - Taguchi Methods deal with averages and variability.
   - Takes advantage of the interaction between control and noise factors in order to obtain "robustness."
   - Is applicable to product design, product improvement and process design and improvement.
   - Improves quality without a cost increase.

## TOLERANCE DESIGN

When Parameter Design is not sufficient for reducing the output variation we turn to **Tolerance Design.** Narrower tolerance ranges must be specified for those production factors whose variation imparts a large influence on the output variation. *To meet these tighter process specifications, better grade material and better equipment are often needed. Because of this, tolearnce design usually increase production costs.*

If we start with the lowest priced components and material in parameter design, we determine which must have tighter tolerances. However, we may minimize cost by experimenting to find tolerances that can be relaxed without appreciably affecting quality.

Tolerance Design is thus the application of design of experiments to make sytematic changes in tolerances, in order to determine which factors contribute most to variation in the end product. Instead of tightening all tolerances in the system, the analysis tells us which tolerances to tighten and which can be relaxed. That is to say, we find the parameters with the highest contributing noise factors and tighten those tolearnces. Lower contributing tolerances may be relaxed, thus minimizing cost.

*Tolerance Design often utilizes the Loss Function to obtain a trade-off between quality and cost.*

## TOLERANCE DESIGN

1. **Objectives**
   - To determine the allowable ranges of variation for product/process parameters once optimum settings have been determined by a parameter design experiment.
2. **Strategy**
   - To initially use wide tolerances, low cost materials and low cost components.
   - Using the best set of conditions as determined by the parameter design experiment, *determine the overall variation in our output characteristic.*

     —If the variation is within our requirements, keep the wide tolerances and low-cost materials/components.

     —If the variation is excessive, reduce it by selectively tightening tolerances and/or upgrading materials and components.
3. **Technique Used**
   - Determine the amount of total variation due to each of the factors by means of ANOVA (Analysis of Variance).

     —The factors which contribute to a large portion of the variability should be considered for tightened tolerances and upgraded quality.

# STATISTICAL QUALITY CONTROL TABLES

**THE BINOMIAL DISTRIBUTION** (Binomial Probability Sums)

| | | *P* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *n* | *r* | 0.10 | 0.20 | 0.25 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
| 5 | 0 | 0.5905 | 0.3277 | 0.2373 | 0.1681 | 0.0778 | 0.0312 | 0.0102 | 0.0024 | 0.0003 | 0.0000 |
| | 1 | 0.9185 | 0.7373 | 0.6328 | 0.5282 | 0.3370 | 0.1875 | 0.0870 | 0.0308 | 0.0067 | 0.0005 |
| | 2 | 0.9914 | 0.9421 | 0.8965 | 0.8369 | 0.6826 | 0.5000 | 0.3174 | 0.1631 | 0.0579 | 0.0086 |
| | 3 | 0.9995 | 0.9933 | 0.9844 | 0.9692 | 0.9130 | 0.8125 | 0.6630 | 0.4718 | 0.2627 | 0.0815 |
| | 4 | 1.0000 | 0.9997 | 0.9990 | 0.9976 | 0.9898 | 0.9688 | 0.9222 | 0.8319 | 0.6723 | 0.4095 |
| | 5 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 10 | 0 | 0.3487 | 0.1074 | 0.0563 | 0.0282 | 0.0060 | 0.0010 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| | 1 | 0.7361 | 0.3758 | 0.2440 | 0.1493 | 0.0464 | 0.0107 | 0.0017 | 0.0001 | 0.0000 | 0.0000 |
| | 2 | 0.9298 | 0.6778 | 0.5256 | 0.3828 | 0.1673 | 0.0547 | 0.0123 | 0.0016 | 0.0001 | 0.0000 |
| | 3 | 0.9872 | 0.8791 | 0.7759 | 0.6496 | 0.3823 | 0.1719 | 0.0548 | 0.0106 | 0.0009 | 0.0000 |
| | 4 | 0.9984 | 0.9672 | 0.9219 | 0.8497 | 0.6331 | 0.3770 | 0.1662 | 0.0474 | 0.0064 | 0.0002 |
| | 5 | 0.9999 | 0.9936 | 0.9803 | 0.9527 | 0.8338 | 0.6230 | 0.3669 | 0.1503 | 0.0328 | 0.0016 |
| | 6 | 1.0000 | 0.9991 | 0.9965 | 0.9894 | 0.9452 | 0.8281 | 0.6177 | 0.3504 | 0.1209 | 0.0128 |
| | 7 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9877 | 0.9453 | 0.8327 | 0.6172 | 0.3222 | 0.0702 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9983 | 0.9893 | 0.9536 | 0.8507 | 0.6242 | 0.2639 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9990 | 0.9940 | 0.9718 | 0.8926 | 0.6513 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 15 | 0 | 0.2059 | 0.0352 | 0.0134 | 0.0047 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | 1 | 0.5490 | 0.1671 | 0.0802 | 0.0353 | 0.0052 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | 2 | 0.8159 | 0.3980 | 0.2361 | 0.1268 | 0.0271 | 0.0037 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
| | 3 | 0.9444 | 0.6482 | 0.4613 | 0.2969 | 0.0905 | 0.0176 | 0.0019 | 0.0001 | 0.0000 | 0.0000 |
| | 4 | 0.9873 | 0.8358 | 0.6865 | 0.5155 | 0.2173 | 0.0592 | 0.0094 | 0.0007 | 0.0000 | 0.0000 |
| | 5 | 0.9978 | 0.9389 | 0.8516 | 0.7216 | 0.4032 | 0.1509 | 0.0338 | 0.0037 | 0.0001 | 0.0000 |
| | 6 | 0.9997 | 0.9819 | 0.9434 | 0.8689 | 0.6098 | 0.3036 | 0.0951 | 0.0152 | 0.0008 | 0.0000 |
| | 7 | 1.0000 | 0.9958 | 0.9827 | 0.9500 | 0.7869 | 0.5000 | 0.2131 | 0.0500 | 0.0042 | 0.0000 |
| | 8 | 1.0000 | 0.9992 | 0.9958 | 0.9848 | 0.9050 | 0.6964 | 0.3902 | 0.1311 | 0.0181 | 0.0003 |
| | 9 | 1.0000 | 0.9999 | 0.9992 | 0.9963 | 0.9662 | 0.8491 | 0.5968 | 0.2784 | 0.0611 | 0.0023 |
| | 10 | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9907 | 0.9408 | 0.7827 | 0.4845 | 0.1642 | 0.0127 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9981 | 0.9824 | 0.9095 | 0.7031 | 0.3518 | 0.0556 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9963 | 0.9729 | 0.8732 | 0.6020 | 0.1841 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9948 | 0.9647 | 0.8329 | 0.4510 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9953 | 0.9648 | 0.7941 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 20 | 0 | 0.1216 | 0.0115 | 0.0032 | 0.0008 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | 1 | 0.3917 | 0.0692 | 0.0243 | 0.0076 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | 2 | 0.6769 | 0.2061 | 0.0913 | 0.0355 | 0.0036 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | 3 | 0.8670 | 0.4114 | 0.2252 | 0.1071 | 0.0160 | 0.0013 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| | 4 | 0.9568 | 0.6296 | 0.4148 | 0.2375 | 0.0510 | 0.0059 | 0.0003 | 0.0000 | 0.0000 | 0.0000 |
| | 5 | 0.9887 | 0.8042 | 0.6172 | 0.4164 | 0.1256 | 0.0207 | 0.0016 | 0.0000 | 0.0000 | 0.0000 |
| | 6 | 0.9976 | 0.9133 | 0.7858 | 0.6080 | 0.2500 | 0.0577 | 0.0065 | 0.0003 | 0.0000 | 0.0000 |
| | 7 | 0.9996 | 0.9679 | 0.8982 | 0.7723 | 0.4159 | 0.1316 | 0.0210 | 0.0013 | 0.0000 | 0.0000 |
| | 8 | 0.9999 | 0.9900 | 0.9591 | 0.8867 | 0.5956 | 0.2517 | 0.0565 | 0.0051 | 0.0001 | 0.0000 |
| | 9 | 1.0000 | 0.9974 | 0.9861 | 0.9520 | 0.7553 | 0.4119 | 0.1275 | 0.0171 | 0.0006 | 0.0000 |
| | 10 | 1.0000 | 0.9994 | 0.9961 | 0.9829 | 0.8725 | 0.5881 | 0.2447 | 0.0480 | 0.0026 | 0.0000 |
| | 11 | 1.0000 | 0.9999 | 0.9991 | 0.9949 | 0.9435 | 0.7483 | 0.4044 | 0.1133 | 0.0100 | 0.0001 |
| | 12 | 1.0000 | 1.0000 | 0.9998 | 0.9987 | 0.9790 | 0.8684 | 0.5841 | 0.2277 | 0.0321 | 0.0004 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9935 | 0.9423 | 0.7500 | 0.3920 | 0.0867 | 0.0024 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9984 | 0.9793 | 0.8744 | 0.5836 | 0.1958 | 0.0113 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9941 | 0.9490 | 0.7625 | 0.3704 | 0.0432 |
| | 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9987 | 0.9840 | 0.8929 | 0.5886 | 0.1330 |
| | 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9964 | 0.9645 | 0.7939 | 0.3231 |
| | 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9924 | 0.9308 | 0.6083 |
| | 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9992 | 0.9885 | 0.8784 |
| | 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

**THE POISSON DISTRIBUTION**

(The cumulative values are shown in parentheses)

| $c$ \ $np'$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|
| 0 | 0.905 (0.905) | 0.819 (0.819) | 0.741 (0.741) | 0.670 (0.670) | 0.607 (0.607) |
| 1 | 0.091 (0.996) | 0.164 (0.983) | 0.222 (0.963) | 0.268 (0.938) | 0.303 (0.910) |
| 2 | 0.004 (1.000) | 0.016 (0.999) | 0.033 (0.996) | 0.054 (0.992) | 0.076 (0.986) |
| 3 | | 0.010 (1.000) | 0.004 (1.000) | 0.007 (0.999) | 0.013 (0.999) |
| 4 | | | | 0.001 (1.000) | 0.001 (1.000) |

| $c$ \ $np'$ | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|
| 0 | 0.549 (0.549) | 0.497 (0.497) | 0.449 (0.449) | 0.406 (0.406) | 0.368 (0.368) |
| 1 | 0.329 (0.878) | 0.349 (0.845) | 0.359 (0.808) | 0.366 (0.772) | 0.368 (0.736) |
| 2 | 0.099 (0.977) | 0.122 (0.967) | 0.144 (0.952) | 0.166 (0.938) | 0.184 (0.920) |
| 3 | 0.020 (0.997) | 0.028 (0.995) | 0.039 (0.991) | 0.049 (0.987) | 0.061 (0.981) |
| 4 | 0.003 (1.000) | 0.005 (1.000) | 0.008 (0.999) | 0.011 (0.998) | 0.016 (0.997) |
| 5 | | | 0.001 (1.000) | 0.002 (1.000) | 0.003 (1.000) |

| $c$ \ $np'$ | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
|---|---|---|---|---|---|
| 0 | 0.333 (0.333) | 0.301 (0.301) | 0.273 (0.273) | 0.247 (0.247) | 0.223 (0.223) |
| 1 | 0.366 (0.699) | 0.361 (0.662) | 0.354 (0.627) | 0.345 (0.592) | 0.335 (0.558) |
| 2 | 0.201 (0.900) | 0.217 (0.879) | 0.230 (0.857) | 0.242 (0.834) | 0.251 (0.809) |
| 3 | 0.074 (0.974) | 0.087 (0.966) | 0.100 (0.957) | 0.113 (0.947) | 0.126 (0.935) |
| 4 | 0.021 (0.995) | 0.026 (0.992) | 0.032 (0.989) | 0.039 (0.986) | 0.047 (0.982) |
| 5 | 0.004 (0.999) | 0.007 (0.999) | 0.009 (0.998) | 0.011 (0.997) | 0.014 (0.996) |
| 6 | 0.001 (1.000) | 0.001 (1.000) | 0.002 (1.000) | 0.003 (1.000) | 0.004 (1.000) |

| $c$ \ $np'$ | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
|---|---|---|---|---|---|
| 0 | 0.202 (0.202) | 0.183 (0.183) | 0.165 (0.165) | 0.150 (0.150) | 0.135 (0.135) |
| 1 | 0.323 (0.525) | 0.311 (0.494) | 0.298 (0.463) | 0.284 (0.434) | 0.271 (0.406) |
| 2 | 0.258 (0.783) | 0.264 (0.758) | 0.268 (0.731) | 0.270 (0.704) | 0.271 (0.677) |
| 3 | 0.138 (0.921) | 0.149 (0.907) | 0.161 (0.892) | 0.171 (0.875) | 0.180 (0.857) |
| 4 | 0.055 (0.976) | 0.064 (0.971) | 0.072 (0.964) | 0.081 (0.956) | 0.090 (0.947) |
| 5 | 0.018 (0.994) | 0.022 (0.993) | 0.026 (0.990) | 0.031 (0.987) | 0.036 (0.983) |
| 6 | 0.005 (0.999) | 0.006 (0.999) | 0.008 (0.998) | 0.010 (0.997) | 0.012 (0.995) |
| 7 | 0.001 (1.000) | 0.001 (1.000) | 0.002 (1.000) | 0.003 (1.000) | 0.004 (0.999) |
| 8 | | | | | 0.001 (1.000) |

**THE POISSON DISTRIBUTION**

(The cumulative values are shown in parentheses)

| $np'$ / $c$ | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 |
|---|---|---|---|---|---|
| 0 | 0.123 (0.123) | 0.111 (0.111) | 0.100 (0.100) | 0.091 (0.091) | 0.082 (0.082) |
| 1 | 0.257 (0.380) | 0.244 (0.355) | 0.231 (0.331) | 0.218 (0.309) | 0.205 (0.287) |
| 2 | 0.270 (0.650) | 0.268 (0.623) | 0.265 (0.596) | 0.261 (0.570) | 0.256 (0.543) |
| 3 | 0.189 (0.839) | 0.197 (0.820) | 0.203 (0.799) | 0.209 (0.779) | 0.214 (0.757) |
| 4 | 0.099 (0.938) | 0.108 (0.928) | 0.117 (0.916) | 0.125 (0.904) | 0.134 (0.891) |
| 5 | 0.042 (0.980) | 0.048 (0.976) | 0.054 (0.970) | 0.060 (0.964) | 0.067 (0.958) |
| 6 | 0.015 (0.995) | 0.017 (0.993) | 0.021 (0.991) | 0.024 (0.988) | 0.028 (0.986) |
| 7 | 0.004 (0.999) | 0.005 (0.998) | 0.007 (0.998) | 0.008 (0.996) | 0.010 (0.996) |
| 8 | 0.001 (1.000) | 0.002 (1.000) | 0.002 (1.000) | 0.003 (0.999) | 0.003 (0.999) |
| 9 | | | | 0.001 (1.000) | 0.001 (1.000) |

| $np'$ / $c$ | 2.6 | 2.7 | 2.8 | 2.9 | 3.0 |
|---|---|---|---|---|---|
| 0 | 0.074 (0.074) | 0.067 (0.067) | 0.061 (0.061) | 0.055 (0.055) | 0.050 (0.050) |
| 1 | 0.193 (0.267) | 0.182 (0.249) | 0.170 (0.231) | 0.160 (0.215) | 0.149 (0.199) |
| 2 | 0.251 (0.518) | 0.245 (0.494) | 0.238 (0.469) | 0.231 (0.446) | 0.224 (0.423) |
| 3 | 0.218 (0.736) | 0.221 (0.715) | 0.223 (0.692) | 0.224 (0.670) | 0.224 (0.647) |
| 4 | 0.141 (0.877) | 0.149 (0.864) | 0.156 (0.848) | 0.162 (0.832) | 0.168 (0.815) |
| 5 | 0.074 (0.951) | 0.080 (0.944) | 0.087 (0.935) | 0.094 (0.926) | 0.101 (0.916) |
| 6 | 0.032 (0.983) | 0.036 (0.980) | 0.041 (0.976) | 0.045 (0.971) | 0.050 (0.966) |
| 7 | 0.012 (0.995) | 0.014 (0.994) | 0.016 (0.992) | 0.019 (0.990) | 0.022 (0.988) |
| 8 | 0.004 (0.999) | 0.005 (0.999) | 0.006 (0.998) | 0.007 (0.997) | 0.008 (0.996) |
| 9 | 0.001 (1.000) | 0.001 (1.000) | 0.002 (1.000) | 0.002 (0.999) | 0.003 (0.999) |
| 10 | | | | 0.001 (1.000) | 0.001 (1.000) |

| $np'$ / $c$ | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 |
|---|---|---|---|---|---|
| 0 | 0.045 (0.045) | 0.041 (0.041) | 0.037 (0.037) | 0.033 (0.033) | 0.030 (0.030) |
| 1 | 0.140 (0.185) | 0.130 (0.171) | 0.122 (0.159) | 0.113 (0.146) | 0.106 (0.136) |
| 2 | 0.216 (0.401) | 0.209 (0.380) | 0.201 (0.360) | 0.193 (0.339) | 0.185 (0.321) |
| 3 | 0.224 (0.625) | 0.223 (0.603) | 0.222 (0.582) | 0.219 (0.558) | 0.216 (0.537) |
| 4 | 0.173 (0.798) | 0.178 (0.781) | 0.182 (0.764) | 0.186 (0.744) | 0.189 (0.726) |
| 5 | 0.107 (0.905) | 0.114 (0.895) | 0.120 (0.884) | 0.126 (0.870) | 0.132 (0.858) |
| 6 | 0.056 (0.961) | 0.061 (0.956) | 0.066 (0.950) | 0.071 (0.941) | 0.077 (0.935) |
| 7 | 0.025 (0.986) | 0.028 (0.984) | 0.031 (0.981) | 0.035 (0.976) | 0.038 (0.973) |
| 8 | 0.010 (0.996) | 0.011 (0.995) | 0.012 (0.993) | 0.015 (0.991) | 0.017 (0.990) |
| 9 | 0.003 (0.999) | 0.004 (0.999) | 0.005 (0.998) | 0.006 (0.997) | 0.007 (0.997) |
| 10 | 0.001 (1.000) | 0.001 (1.000) | 0.002 (1.000) | 0.002 (0.999) | 0.002 (0.999) |
| 11 | | | | 0.001 (1.000) | 0.001 (1.000) |

**THE POISSON DISTRIBUTION**

(The cumulative values are shown in parentheses)

| $np'$ / $c$ | 3.6 | 3.7 | 3.8 | 3.9 | 4.0 |
|---|---|---|---|---|---|
| 0 | 0.027 (0.027) | 0.025 (0.025) | 0.022 (0.022) | 0.020 (0.020) | 0.018 (0.018) |
| 1 | 0.098 (0.125) | 0.091 (0.116) | 0.085 (0.107) | 0.079 (0.099) | 0.073 (0.091) |
| 2 | 0.177 (0.302) | 0.169 (0.285) | 0.161 (0.268) | 0.154 (0.253) | 0.147 (0.238) |
| 3 | 0.213 (0.515) | 0.209 (0.494) | 0.205 (0.473) | 0.200 (0.453) | 0.195 (0.433) |
| 4 | 0.191 (0.706) | 0.193 (0.687) | 0.194 (0.667) | 0.195 (0.648) | 0.195 (0.628) |
| 5 | 0.138 (0.844) | 0.143 (0.830) | 0.148 (0.815) | 0.152 (0.800) | 0.157 (0.785) |
| 6 | 0.083 (0.927) | 0.088 (0.918) | 0.094 (0.909) | 0.099 (0.899) | 0.104 (0.889) |
| 7 | 0.042 (0.969) | 0.047 (0.965) | 0.051 (0.960) | 0.055 (0.954) | 0.060 (0.949) |
| 8 | 0.019 (0.988) | 0.022 (0.987) | 0.024 (0.984) | 0.027 (0.981) | 0.030 (0.979) |
| 9 | 0.008 (0.996) | 0.009 (0.996) | 0.010 (0.994) | 0.012 (0.993) | 0.013 (0.992) |
| 10 | 0.003 (0.999) | 0.003 (0.999) | 0.004 (0.998) | 0.004 (0.997) | 0.005 (0.997) |
| 11 | 0.001 (1.000) | 0.001 (1.000) | 0.001 (0.999) | 0.002 (0.999) | 0.002 (0.999) |
| 12 | | | 0.001 (1.000) | 0.001 (1.000) | 0.001 (1.000) |

| $np'$ / $c$ | 4.1 | 4.2 | 4.3 | 4.4 | 4.5 |
|---|---|---|---|---|---|
| 0 | 0.017 (0.017) | 0.015 (0.015) | 0.014 (0.014) | 0.012 (0.012) | 0.011 (0.011) |
| 1 | 0.068 (0.085) | 0.063 (0.078) | 0.058 (0.072) | 0.054 (0.066) | 0.050 (0.061) |
| 2 | 0.139 (0.224) | 0.132 (0.210) | 0.126 (0.198) | 0.119 (0.185) | 0.113 (0.174) |
| 3 | 0.190 (0.414) | 0.185 (0.395) | 0.180 (0.378) | 0.174 (0.359) | 0.169 (0.343) |
| 4 | 0.195 (0.609) | 0.195 (0.590) | 0.193 (0.571) | 0.192 (0.551) | 0.190 (0.533) |
| 5 | 0.160 (0.769) | 0.163 (0.753) | 0.166 (0.737) | 0.169 (0.720) | 0.171 (0.704) |
| 6 | 0.110 (0.879) | 0.114 (0.867) | 0.119 (0.856) | 0.124 (0.844) | 0.128 (0.832) |
| 7 | 0.064 (0.943) | 0.069 (0.936) | 0.073 (0.929) | 0.078 (0.922) | 0.082 (0.914) |
| 8 | 0.033 (0.976) | 0.036 (0.972) | 0.040 (0.969) | 0.043 (0.965) | 0.046 (0.960) |
| 9 | 0.015 (0.991) | 0.017 (0.989) | 0.019 (0.988) | 0.021 (0.986) | 0.023 (0.983) |
| 10 | 0.006 (0.997) | 0.007 (0.996) | 0.008 (0.996) | 0.009 (0.995) | 0.011 (0.994) |
| 11 | 0.002 (0.999) | 0.003 (0.999) | 0.003 (0.999) | 0.004 (0.999) | 0.004 (0.998) |
| 12 | 0.001 (1.000) | 0.001 (1.000) | 0.001 (1.000) | 0.001 (1.000) | 0.001 (0.999) |
| 13 | | | | | 0.001 (1.000) |

**THE POISSON DISTRIBUTION**

(The cumulative values are shown in parentheses)

| $np'$ / $c$ | 4.6 | 4.7 | 4.8 | 4.9 | 5.0 |
|---|---|---|---|---|---|
| 0 | 0.010 (0.010) | 0.009 (0.009) | 0.008 (0.008) | 0.008 (0.008) | 0.007 (0.007) |
| 1 | 0.046 (0.056) | 0.043 (0.052) | 0.039 (0.047) | 0.037 (0.045) | 0.034 (0.041) |
| 2 | 0.106 (0.162) | 0.101 (0.153) | 0.095 (0.142) | 0.090 (0.135) | 0.084 (0.125) |
| 3 | 0.163 (0.325) | 0.157 (0.310) | 0.152 (0.294) | 0.146 (0.281) | 0.140 (0.265) |
| 4 | 0.188 (0.513) | 0.185 (0.495) | 0.182 (0.476) | 0.179 (0.460) | 0.176 (0.441) |
| 5 | 0.172 (0.685) | 0.174 (0.669) | 0.175 (0.651) | 0.175 (0.635) | 0.176 (0.617) |
| 6 | 0.132 (0.817) | 0.136 (0.805) | 0.140 (0.791) | 0.143 (0.778) | 0.146 (0.763) |
| 7 | 0.087 (0.904) | 0.091 (0.896) | 0.096 (0.887) | 0.100 (0.878) | 0.105 (0.868) |
| 8 | 0.050 (0.954) | 0.054 (0.950) | 0.058 (0.945) | 0.061 (0.939) | 0.065 (0.933) |
| 9 | 0.026 (0.980) | 0.028 (0.978) | 0.031 (0.976) | 0.034 (0.973) | 0.036 (0.969) |
| 10 | 0.012 (0.992) | 0.013 (0.991) | 0.015 (0.991) | 0.016 (0.989) | 0.018 (0.987) |
| 11 | 0.005 (0.997) | 0.006 (0.997) | 0.006 (0.997) | 0.007 (0.996) | 0.008 (0.995) |
| 12 | 0.002 (0.999) | 0.002 (0.999) | 0.002 (0.999) | 0.003 (0.999) | 0.003 (0.998) |
| 13 | 0.001 (1.000) | 0.001 (1.000) | 0.001 (1.000) | 0.001 (1.000) | 0.001 (0.999) |
| 14 | | | | | 0.001 (1.000) |

| $np'$ / $c$ | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 |
|---|---|---|---|---|---|
| 0 | 0.002 (0.002) | 0.001 (0.001) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| 1 | 0.015 (0.017) | 0.006 (0.007) | 0.003 (0.003) | 0.001 (0.001) | 0.000 (0.000) |
| 2 | 0.045 (0.062) | 0.022 (0.029) | 0.011 (0.014) | 0.005 (0.006) | 0.002 (0.002) |
| 3 | 0.089 (0.151) | 0.052 (0.081) | 0.029 (0.043) | 0.015 (0.021) | 0.007 (0.009) |
| 4 | 0.134 (0.285) | 0.091 (0.172) | 0.057 (0.100) | 0.034 (0.055) | 0.019 (0.028) |
| 5 | 0.161 (0.446) | 0.128 (0.300) | 0.092 (0.192) | 0.061 (0.116) | 0.038 (0.066) |
| 6 | 0.161 (0.607) | 0.149 (0.449) | 0.122 (0.314) | 0.091 (0.207) | 0.063 (0.129) |
| 7 | 0.138 (0.745) | 0.149 (0.598) | 0.140 (0.454) | 0.117 (0.324) | 0.090 (0.219) |
| 8 | 0.103 (0.848) | 0.131 (0.729) | 0.140 (0.594) | 0.132 (0.456) | 0.113 (0.332) |
| 9 | 0.069 (0.917) | 0.102 (0.831) | 0.124 (0.718) | 0.132 (0.588) | 0.125 (0.457) |
| 10 | 0.041 (0.958) | 0.071 (0.902) | 0.099 (0.817) | 0.119 (0.707) | 0.125 (0.582) |
| 11 | 0.023 (0.981) | 0.045 (0.947) | 0.072 (0.889) | 0.097 (0.804) | 0.114 (0.696) |
| 12 | 0.011 (0.992) | 0.026 (0.973) | 0.048 (0.937) | 0.073 (0.877) | 0.095 (0.791) |
| 13 | 0.005 (0.997) | 0.014 (0.987) | 0.030 (0.967) | 0.050 (0.927) | 0.073 (0.864) |
| 14 | 0.002 (0.999) | 0.007 (0.994) | 0.017 (0.984 | 0.032 (0.959) | 0.052 (0.916) |
| 15 | 0.001 (1.000) | 0.003 (0.997) | 0.009 (0.993) | 0.019 (0.978) | 0.035 (0.951) |
| 16 | | 0.002 (0.999) | 0.004 (0.997) | 0.011 (0.989) | 0.022 (0.973) |
| 17 | | 0.001 (1.000) | 0.002 (0.999) | 0.006 (0.995) | 0.013 (0.986) |
| 18 | | | 0.001 (1.000) | 0.003 (0.998) | 0.007 (0.993) |
| 19 | | | | 0.001 (0.999) | 0.004 (0.997) |
| 20 | | | | 0.001 (1.000) | 0.002 (0.999) |
| 21 | | | | | 0.001 (1.000) |

**THE POISSON DISTRIBUTION**

(The cumulative values are shown in parentheses)

| $c$ \ $np'$ | 11.0 | 12.0 | 13.0 | 14.0 | 15.0 |
|---|---|---|---|---|---|
| 0 | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| 1 | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| 2 | 0.001 (0.001) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) | 0.000 (0.000) |
| 3 | 0.004 (0.005) | 0.002 (0.002) | 0.001 (0.001) | 0.000 (0.000) | 0.000 (0.000) |
| 4 | 0.010 (0.015) | 0.005 (0.007) | 0.003 (0.004) | 0.001 (0.001) | 0.001 (0.001) |
| 5 | 0.022 (0.037) | 0.013 (0.020) | 0.007 (0.011) | 0.004 (0.005) | 0.002 (0.003) |
| 6 | 0.041 (0.078) | 0.025 (0.045) | 0.015 (0.026) | 0.009 (0.014) | 0.005 (0.008) |
| 7 | 0.065 (0.143) | 0.044 (0.089) | 0.028 (0.054) | 0.017 (0.031) | 0.010 (0.018) |
| 8 | 0.089 (0.232) | 0.066 (0.155) | 0.046 (0.100) | 0.031 (0.062) | 0.019 (0.037) |
| 9 | 0.109 (0.341) | 0.087 (0.242) | 0.066 (0.166) | 0.047 (0.109) | 0.032 (0.069) |
| 10 | 0.119 (0.460) | 0.105 (0.347) | 0.086 (0.252) | 0.066 (0.175) | 0.049 (0.118) |
| 11 | 0.119 (0.579) | 0.114 (0.461) | 0.101 (0.353) | 0.084 (0.259) | 0.066 (0.184) |
| 12 | 0.109 (0.688) | 0.114 (0.575) | 0.110 (0.463) | 0.099 (0.358) | 0.083 (0.267) |
| 13 | 0.093 (0.781) | 0.106 (0.681) | 0.110 (0.573) | 0.106 (0.464) | 0.096 (0.363) |
| 14 | 0.073 (0.854) | 0.091 (0.772) | 0.102 (0.675) | 0.106 (0.570) | 0.102 (0.465) |
| 15 | 0.053 (0.907) | 0.072 (0.844) | 0.088 (0.763) | 0.099 (0.669) | 0.102 (0.567) |
| 16 | 0.037 (0.944) | 0.054 (0.898) | 0.072 (0.835) | 0.087 (0.756) | 0.096 (0.663) |
| 17 | 0.024 (0.968) | 0.038 (0.936) | 0.055 (0.890) | 0.071 (0.827) | 0.085 (0.748) |
| 18 | 0.015 (0.983) | 0.026 (0.962) | 0.040 (0.930) | 0.056 (0.883) | 0.071 (0.819) |
| 19 | 0.008 (0.991) | 0.016 (0.978) | 0.027 (0.957) | 0.041 (0.924) | 0.056 (0.875) |
| 20 | 0.005 (0.996) | 0.010 (0.988) | 0.018 (0.975) | 0.029 (0.953) | 0.042 (0.917) |
| 21 | 0.002 (0.998) | 0.006 (0.994) | 0.011 (0.986) | 0.019 (0.972) | 0.030 (0.947) |
| 22 | 0.001 (0.999) | 0.003 (0.997) | 0.006 (0.992) | 0.012 (0.984) | 0.020 (0.967) |
| 23 | 0.001 (1.000) | 0.002 (0.999) | 0.004 (0.996) | 0.007 (0.991) | 0.013 (0.980) |
| 24 | | 0.001 (1.000) | 0.002 (0.998) | 0.004 (0.995) | 0.008 (0.988) |
| 25 | | | 0.001 (0.999) | 0.003 (0.998) | 0.005 (0.993) |
| 26 | | | 0.001 (1.000) | 0.001 (0.999) | 0.003 (0.996) |
| 27 | | | | 0.001 (1.000) | 0.002 (0.998) |
| 28 | | | | | 0.001 (0.999) |
| 29 | | | | | 0.001 (1.000) |

**AREAS UNDER THE NORMAL CURVE**

| *z* | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| – 3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| – 3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| – 3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| – 3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| – 3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| – 2.9 | .0019 | .0018 | .0017 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| – 2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| – 2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| – 2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| – 2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| – 2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| – 2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| – 2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| – 2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| – 2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| – 1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| – 1.8 | .0359 | .0352 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| – 1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| – 1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| – 1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| – 1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0722 | .0708 | .0694 | .0681 |
| – 1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| – 1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| – 1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| – 1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| – 0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| – 0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| – 0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| – 0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| – 0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| – 0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| – 0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| – 0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| – 0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| – 0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

**AREAS UNDER THE NORMAL CURVE**

| *z* | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9278 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

**CRITICAL VALUES OF THE '*t*' DISTRIBUTION**

| | α | | | | |
|---|---|---|---|---|---|
| ν | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| inf. | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

**CRITICAL VALUES OF THE CHI-SQUARE DISTRIBUTION**

| | α | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ν | .995 | .99 | .98 | .975 | .95 | .90 | .80 | .75 | .70 | .50 |
| 1 | $.0^4393$ | $.0^3157$ | $.0^3628$ | $.0^3982$ | .00393 | .0158 | .0642 | .102 | .148 | .455 |
| 2 | .0100 | .0201 | .0404 | .0506 | .103 | .211 | .446 | .575 | .713 | 1.386 |
| 3 | .0717 | .115 | .185 | .216 | .352 | .584 | 1.005 | 1.213 | 1.424 | 2.366 |
| 4 | .207 | .297 | .429 | .484 | .711 | 1.064 | 1.649 | 1.923 | 2.195 | 3.357 |
| 5 | .412 | .554 | .752 | .831 | 1.145 | 1.610 | 2.343 | 2.675 | 3.000 | 4.351 |
| 6 | .676 | .872 | 1.134 | 1.237 | 1.635 | 2.204 | 3.070 | 3.455 | 3.828 | 5.348 |
| 7 | .989 | 1.239 | 1.564 | 1.690 | 2.167 | 2.833 | 3.822 | 4.255 | 4.671 | 6.346 |
| 8 | 1.344 | 1.646 | 2.032 | 2.180 | 2.733 | 3.490 | 4.594 | 5.071 | 5.527 | 7.344 |
| 9 | 1.735 | 2.088 | 2.532 | 2.700 | 3.325 | 4.168 | 5.380 | 5.899 | 6.393 | 8.343 |
| 10 | 2.156 | 2.558 | 3.059 | 3.247 | 3.940 | 4.865 | 6.179 | 6.737 | 7.267 | 9.342 |
| 11 | 2.603 | 3.053 | 3.609 | 3.816 | 4.575 | 5.578 | 6.989 | 7.584 | 8.148 | 10.341 |
| 12 | 3.074 | 3.571 | 4.178 | 4.404 | 5.226 | 6.304 | 7.807 | 8.438 | 9.034 | 11.340 |
| 13 | 3.565 | 4.107 | 4.765 | 5.009 | 5.892 | 7.042 | 8.634 | 9.299 | 9.926 | 12.340 |
| 14 | 4.075 | 4.660 | 5.368 | 5.629 | 6.571 | 7.790 | 9.467 | 10.165 | 10.821 | 13.339 |
| 15 | 4.601 | 5.229 | 5.985 | 6.262 | 7.261 | 8.547 | 10.307 | 11.036 | 11.721 | 14.339 |
| 16 | 5.142 | 5.812 | 6.614 | 6.908 | 7.962 | 9.312 | 11.152 | 11.912 | 12.624 | 15.338 |
| 17 | 5.697 | 6.408 | 7.255 | 7.564 | 8.672 | 10.085 | 12.002 | 12.792 | 13.531 | 16.338 |
| 18 | 6.265 | 7.015 | 7.906 | 8.231 | 9.390 | 10.865 | 12.857 | 13.675 | 14.440 | 17.338 |
| 19 | 6.844 | 7.633 | 8.567 | 8.907 | 10.117 | 11.651 | 13.716 | 14.562 | 15.352 | 18.338 |
| 20 | 7.434 | 8.260 | 9.237 | 9.591 | 10.851 | 12.443 | 14.578 | 15.452 | 16.266 | 19.337 |
| 21 | 8.034 | 8.897 | 9.915 | 10.283 | 11.591 | 13.240 | 15.445 | 16.344 | 17.182 | 20.337 |
| 22 | 8.643 | 9.542 | 10.600 | 10.982 | 12.338 | 14.041 | 16.314 | 17.240 | 18.101 | 21.337 |
| 23 | 9.260 | 10.196 | 11.293 | 11.689 | 13.091 | 14.848 | 17.187 | 18.137 | 19.021 | 22.337 |
| 24 | 9.886 | 10.856 | 11.992 | 12.401 | 13.848 | 15.659 | 18.062 | 19.037 | 19.943 | 23.337 |
| 25 | 10.520 | 11.524 | 12.697 | 13.120 | 14.611 | 16.473 | 18.940 | 19.939 | 20.867 | 24.337 |
| 26 | 11.160 | 12.198 | 13.409 | 13.844 | 15.379 | 17.292 | 19.820 | 20.843 | 21.792 | 25.336 |
| 27 | 11.808 | 12.879 | 14.125 | 14.573 | 16.151 | 18.114 | 20.703 | 21.749 | 22.719 | 26.336 |
| 28 | 12.461 | 13.565 | 14.847 | 15.308 | 16.928 | 18.939 | 21.588 | 22.657 | 23.647 | 27.336 |
| 29 | 13.121 | 14.256 | 15.574 | 16.047 | 17.708 | 19.768 | 22.475 | 23.567 | 24.577 | 28.336 |
| 30 | 13.787 | 14.953 | 16.306 | 16.791 | 18.493 | 20.599 | 23.364 | 24.478 | 25.508 | 29.336 |

**CRITICAL VALUES OF THE CHI-SQUARE DISTRIBUTION**

| | α | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ν | .30 | .25 | .20 | .10 | .05 | .025 | .02 | .01 | .005 | .001 |
| 1 | 1.074 | 1.323 | 1.642 | 2.706 | 3.841 | 5.024 | 5.412 | 6.635 | 7.879 | 10.827 |
| 2 | 2.408 | 2.773 | 3.219 | 4.605 | 5.991 | 7.378 | 7.824 | 9.210 | 10.597 | 13.815 |
| 3 | 3.665 | 4.108 | 4.642 | 6.251 | 7.815 | 9.348 | 9.837 | 11.345 | 12.838 | 16.268 |
| 4 | 4.878 | 5.385 | 5.989 | 7.779 | 9.488 | 11.143 | 11.668 | 13.277 | 14.860 | 18.465 |
| 5 | 6.064 | 6.626 | 7.289 | 9.236 | 11.070 | 12.832 | 13.388 | 15.086 | 16.750 | 20.517 |
| 6 | 7.231 | 7.841 | 8.558 | 10.645 | 12.592 | 14.449 | 15.033 | 16.812 | 18.548 | 22.457 |
| 7 | 8.383 | 9.037 | 9.803 | 12.017 | 14.067 | 16.013 | 16.622 | 18.475 | 20.278 | 24.322 |
| 8 | 9.524 | 10.219 | 11.030 | 13.362 | 15.507 | 17.535 | 18.168 | 20.090 | 21.955 | 26.125 |
| 9 | 10.656 | 11.389 | 12.242 | 14.684 | 16.919 | 19.023 | 19.679 | 21.666 | 23.589 | 27.877 |
| 10 | 11.781 | 12.549 | 13.442 | 15.987 | 18.307 | 20.483 | 21.161 | 23.209 | 25.188 | 29.588 |
| 11 | 12.899 | 13.701 | 14.631 | 17.275 | 19.675 | 21.920 | 22.618 | 24.725 | 26.757 | 31.264 |
| 12 | 14.011 | 14.845 | 15.812 | 18.549 | 21.026 | 23.337 | 24.054 | 26.217 | 28.300 | 32.909 |
| 13 | 15.119 | 15.984 | 16.985 | 19.812 | 22.362 | 24.736 | 25.472 | 27.688 | 29.819 | 34.528 |
| 14 | 16.222 | 17.117 | 18.151 | 21.064 | 23.685 | 26.119 | 26.873 | 29.141 | 31.319 | 36.123 |
| 15 | 17.322 | 18.245 | 19.311 | 22.307 | 24.996 | 27.488 | 28.259 | 30.578 | 32.801 | 37.697 |
| 16 | 18.418 | 19.369 | 20.465 | 23.542 | 26.296 | 28.845 | 29.633 | 32.000 | 34.267 | 39.252 |
| 17 | 19.511 | 20.489 | 21.615 | 24.769 | 27.587 | 30.191 | 30.995 | 33.409 | 35.718 | 40.790 |
| 18 | 20.601 | 21.605 | 22.760 | 25.989 | 28.869 | 31.526 | 32.346 | 34.805 | 37.156 | 42.312 |
| 19 | 21.689 | 22.718 | 23.900 | 27.204 | 30.144 | 32.852 | 33.687 | 36.191 | 38.582 | 43.820 |
| 20 | 22.725 | 23.828 | 25.038 | 28.412 | 31.410 | 34.170 | 35.020 | 37.566 | 39.997 | 45.315 |
| 21 | 23.858 | 24.935 | 26.171 | 29.615 | 32.671 | 35.479 | 36.343 | 38.932 | 41.401 | 46.797 |
| 22 | 24.939 | 26.039 | 27.301 | 30.813 | 33.924 | 36.781 | 37.659 | 40.289 | 42.796 | 48.268 |
| 23 | 26.018 | 27.141 | 28.429 | 32.007 | 35.172 | 38.076 | 38.968 | 41.638 | 44.181 | 49.728 |
| 24 | 27.096 | 28.241 | 29.553 | 33.196 | 36.415 | 39.364 | 40.270 | 42.980 | 45.558 | 51.179 |
| 25 | 28.172 | 29.339 | 30.675 | 34.382 | 37.652 | 40.646 | 41.566 | 44.314 | 46.928 | 52.620 |
| 26 | 29.246 | 30.434 | 31.795 | 35.563 | 38.885 | 41.923 | 42.856 | 48.642 | 48.290 | 54.052 |
| 27 | 30.319 | 31.528 | 32.912 | 36.741 | 40.113 | 43.194 | 44.140 | 46.963 | 49.645 | 55.476 |
| 28 | 31.391 | 32.620 | 34.027 | 37.916 | 41.337 | 44.461 | 45.419 | 48.278 | 50.993 | 56.893 |
| 29 | 32.461 | 33.711 | 35.139 | 39.087 | 42.557 | 45.722 | 46.693 | 49.588 | 52.336 | 58.302 |
| 30 | 33.530 | 34.800 | 36.250 | 40.256 | 43.773 | 46.979 | 47.962 | 50.892 | 53.672 | 59.703 |

**CRITICAL VALUES OF THE 'F' DISTRIBUTION**

$f_{0.05}(v_1, v_2)$

| | $v_1$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $v_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 161.4 | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.17 | 2.09 | 2.02 | 1.96 |
| ∞ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 |

**CRITICAL VALUES OF THE 'F' DISTRIBUTION**

$f_{0.05}(v_1, v_2)$

| | $v_1$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $v_2$ | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
| 1 | 241.9 | 243.9 | 245.9 | 248.0 | 249.1 | 250.1 | 251.1 | 252.2 | 253.3 | 254.3 |
| 2 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.50 |
| 3 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.59 | 8.57 | 8.55 | 8.53 |
| 4 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.69 | 5.66 | 5.63 |
| 5 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.46 | 4.43 | 4.40 | 4.36 |
| 6 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 | 3.74 | 3.70 | 3.67 |
| 7 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 | 3.30 | 3.27 | 3.23 |
| 8 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 | 3.01 | 2.97 | 2.93 |
| 9 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 | 2.79 | 2.75 | 2.71 |
| 10 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 | 2.66 | 2.62 | 2.58 | 2.54 |
| 11 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 | 2.53 | 2.49 | 2.45 | 2.40 |
| 12 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.41 | 2.38 | 2.34 | 2.30 |
| 13 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 | 2.34 | 2.30 | 2.25 | 2.21 |
| 14 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 | 2.27 | 2.22 | 2.18 | 2.13 |
| 15 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.20 | 2.16 | 2.11 | 2.07 |
| 16 | 2.49 | 2.42 | 2.35 | 2.28 | 2.24 | 2.19 | 2.15 | 2.11 | 2.06 | 2.01 |
| 17 | 2.45 | 2.38 | 2.31 | 2.23 | 2.19 | 2.15 | 2.10 | 2.06 | 2.01 | 1.96 |
| 18 | 2.41 | 2.34 | 2.27 | 2.19 | 2.15 | 2.11 | 2.06 | 2.02 | 1.97 | 1.92 |
| 19 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 1.98 | 1.93 | 1.88 |
| 20 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.99 | 1.95 | 1.90 | 1.84 |
| 21 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.92 | 1.87 | 1.81 |
| 22 | 2.30 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.78 |
| 23 | 2.27 | 2.20 | 2.13 | 2.05 | 2.01 | 1.96 | 1.91 | 1.86 | 1.81 | 1.76 |
| 24 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.79 | 1.73 |
| 25 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.82 | 1.77 | 1.71 |
| 26 | 2.22 | 2.15 | 2.07 | 1.99 | 1.95 | 1.90 | 1.85 | 1.80 | 1.75 | 1.69 |
| 27 | 2.20 | 2.13 | 2.06 | 1.97 | 1.93 | 1.88 | 1.84 | 1.79 | 1.73 | 1.67 |
| 28 | 2.19 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.77 | 1.71 | 1.65 |
| 29 | 2.18 | 2.10 | 2.03 | 1.94 | 1.90 | 1.85 | 1.81 | 1.75 | 1.70 | 1.64 |
| 30 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.79 | 1.74 | 1.68 | 1.62 |
| 40 | 2.08 | 2.00 | 1.92 | 1.84 | 1.79 | 1.74 | 1.69 | 1.64 | 1.58 | 1.51 |
| 60 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.59 | 1.53 | 1.47 | 1.39 |
| 120 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.50 | 1.43 | 1.35 | 1.25 |
| ∞ | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.39 | 1.32 | 1.22 | 1.00 |

**CRITICAL VALUES OF THE 'F' DISTRIBUTION**

$f_{0.01}(v_1, v_2)$

| | $v_1$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $v_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 4052 | 4999.5 | 5403 | 5625 | 5764 | 5859 | 5928 | 5981 | 6022 |
| 2 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 |
| 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 |
| 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 |
| 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 |
| 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 |
| ∞ | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 |

**CRITICAL VALUES OF THE 'F' DISTRIBUTION**

$f_{0.01}(v_1, v_2)$

| | $v_1$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $v_2$ | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
| 1 | 6056 | 6106 | 6157 | 6209 | 6235 | 6261 | 6287 | 6313 | 6339 | 6366 |
| 2 | 99.40 | 99.42 | 99.43 | 99.45 | 99.46 | 99.47 | 99.47 | 99.48 | 99.49 | 99.50 |
| 3 | 27.23 | 27.05 | 26.87 | 26.69 | 26.60 | 26.50 | 26.41 | 26.32 | 26.22 | 26.13 |
| 4 | 14.55 | 14.37 | 14.20 | 14.02 | 13.93 | 13.84 | 13.75 | 13.65 | 13.56 | 13.46 |
| 5 | 10.05 | 9.89 | 9.72 | 9.55 | 9.47 | 9.38 | 9.29 | 9.20 | 9.11 | 9.02 |
| 6 | 7.87 | 7.72 | 7.56 | 7.40 | 7.31 | 7.23 | 7.14 | 7.06 | 6.97 | 6.88 |
| 7 | 6.62 | 6.47 | 6.31 | 6.16 | 6.07 | 5.99 | 5.91 | 5.82 | 5.74 | 5.65 |
| 8 | 5.81 | 5.67 | 5.52 | 5.36 | 5.28 | 5.20 | 5.12 | 5.03 | 4.95 | 4.86 |
| 9 | 5.26 | 5.11 | 4.96 | 4.81 | 4.73 | 4.65 | 4.57 | 4.48 | 4.40 | 4.31 |
| 10 | 4.85 | 4.71 | 4.56 | 4.41 | 4.33 | 4.25 | 4.17 | 4.08 | 4.00 | 3.91 |
| 11 | 4.54 | 4.40 | 4.25 | 4.10 | 4.02 | 3.94 | 3.86 | 3.78 | 3.69 | 3.60 |
| 12 | 4.30 | 4.16 | 4.01 | 3.86 | 3.78 | 3.70 | 3.62 | 3.54 | 3.45 | 3.36 |
| 13 | 4.10 | 3.96 | 3.82 | 3.66 | 3.59 | 3.51 | 3.43 | 3.34 | 3.25 | 3.17 |
| 14 | 3.94 | 3.80 | 3.66 | 3.51 | 3.43 | 3.35 | 3.27 | 3.18 | 3.09 | 3.00 |
| 15 | 3.80 | 3.67 | 3.52 | 3.37 | 3.29 | 3.21 | 3.13 | 3.05 | 2.96 | 2.87 |
| 16 | 3.69 | 3.55 | 3.41 | 3.26 | 3.18 | 3.10 | 3.02 | 2.93 | 2.84 | 2.75 |
| 17 | 3.59 | 3.46 | 3.31 | 3.16 | 3.08 | 3.00 | 2.92 | 2.83 | 2.75 | 2.65 |
| 18 | 3.51 | 3.37 | 3.23 | 3.08 | 3.00 | 2.92 | 2.84 | 2.75 | 2.66 | 2.57 |
| 19 | 3.43 | 3.30 | 3.15 | 3.00 | 2.92 | 2.84 | 2.76 | 2.67 | 2.58 | 2.49 |
| 20 | 3.37 | 3.23 | 3.09 | 2.94 | 2.86 | 2.78 | 2.69 | 2.61 | 2.52 | 2.42 |
| 21 | 3.31 | 3.17 | 3.03 | 2.88 | 2.80 | 2.72 | 2.64 | 2.55 | 2.46 | 2.36 |
| 22 | 3.26 | 3.12 | 2.98 | 2.83 | 2.75 | 2.67 | 2.58 | 2.50 | 2.40 | 2.31 |
| 23 | 3.21 | 3.07 | 2.93 | 2.78 | 2.70 | 2.62 | 2.54 | 2.45 | 2.35 | 2.26 |
| 24 | 3.17 | 3.03 | 2.89 | 2.74 | 2.66 | 2.58 | 2.49 | 2.40 | 2.31 | 2.21 |
| 25 | 3.13 | 2.99 | 2.85 | 2.70 | 2.62 | 2.54 | 2.45 | 2.36 | 2.27 | 2.17 |
| 26 | 3.09 | 2.96 | 2.81 | 2.66 | 2.58 | 2.50 | 2.42 | 2.33 | 2.23 | 2.13 |
| 27 | 3.06 | 2.93 | 2.78 | 2.63 | 2.55 | 2.47 | 2.38 | 2.29 | 2.20 | 2.10 |
| 28 | 3.03 | 2.90 | 2.75 | 2.60 | 2.52 | 2.44 | 2.35 | 2.26 | 2.17 | 2.06 |
| 29 | 3.00 | 2.87 | 2.73 | 2.57 | 2.49 | 2.41 | 2.33 | 2.23 | 2.14 | 2.03 |
| 30 | 2.98 | 2.84 | 2.70 | 2.55 | 2.47 | 2.39 | 2.30 | 2.21 | 2.11 | 2.01 |
| 40 | 2.80 | 2.66 | 2.52 | 2.37 | 2.29 | 2.20 | 2.11 | 2.02 | 1.92 | 1.80 |
| 60 | 2.63 | 2.50 | 2.35 | 2.20 | 2.12 | 2.03 | 1.94 | 1.84 | 1.73 | 1.60 |
| 120 | 2.47 | 2.34 | 2.19 | 2.03 | 1.95 | 1.86 | 1.76 | 1.66 | 1.53 | 1.38 |
| ∞ | 2.32 | 2.18 | 2.04 | 1.88 | 1.79 | 1.70 | 1.59 | 1.47 | 1.32 | 1.00 |

**RANDOM NUMBERS**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 9069 | 7629 | 5756 | 2237 | 3069 | 6004 | 3792 | 2530 |
| 4321 | 5890 | 0822 | 5994 | 9996 | 8961 | 1262 | 5870 |
| 4195 | 5124 | 9161 | 6899 | 6857 | 6455 | 7662 | 7035 |
| 8589 | 4464 | 0905 | 8676 | 4514 | 8790 | 7186 | 4591 |
| 1007 | 3877 | 2592 | 8860 | 5753 | 8661 | 7694 | 5013 |
| 7047 | 2263 | 8242 | 9363 | 0458 | 5459 | 2369 | 3815 |
| 6974 | 5289 | 7527 | 6283 | 3635 | 1209 | 3791 | 1709 |
| 6203 | 5675 | 0586 | 8541 | 7337 | 3896 | 3060 | 1726 |
| 3888 | 0533 | 6091 | 6066 | 2169 | 4146 | 1047 | 3999 |
| 9860 | 9589 | 0814 | 1976 | 8775 | 8710 | 0231 | 8630 |
| 3845 | 7559 | 3167 | 1845 | 5491 | 4805 | 7966 | 9334 |
| 5732 | 0238 | 6134 | 5642 | 7306 | 2351 | 3150 | 2848 |
| 9534 | 6145 | 1823 | 0269 | 6577 | 4545 | 2181 | 9347 |
| 3574 | 9563 | 8359 | 4776 | 0111 | 9110 | 6160 | 8471 |
| 6574 | 1550 | 9890 | 5275 | 3005 | 3922 | 7048 | 1569 |
| 3756 | 6594 | 6634 | 9824 | 1318 | 6586 | 4075 | 5091 |
| 5569 | 2958 | 8823 | 3073 | 2471 | 1512 | 1015 | 9361 |
| 9109 | 2166 | 2146 | 9374 | 9483 | 2111 | 7095 | 8421 |
| 1165 | 2712 | 2021 | 6154 | 5522 | 9017 | 0354 | 0754 |
| 8078 | 2347 | 6410 | 2480 | 7247 | 1283 | 1307 | 6651 |
| 0179 | 4334 | 7117 | 2530 | 2504 | 4703 | 1756 | 0688 |
| 1125 | 2677 | 9553 | 7596 | 1407 | 3062 | 4701 | 9624 |
| 9936 | 2780 | 0687 | 7901 | 4265 | 5741 | 3310 | 2535 |
| 2827 | 1781 | 7272 | 4947 | 8892 | 7557 | 3134 | 8504 |
| 5389 | 9850 | 5081 | 5267 | 5164 | 1340 | 0605 | 5451 |
| 2166 | 6647 | 7554 | 4773 | 9682 | 3348 | 8503 | 8358 |
| 3760 | 1243 | 7458 | 6177 | 8038 | 2223 | 2679 | 4284 |
| 7522 | 6494 | 8298 | 7868 | 0822 | 8806 | 9255 | 3581 |
| 3111 | 6280 | 3705 | 0257 | 0298 | 6587 | 8677 | 8291 |
| 6589 | 0555 | 8479 | 4523 | 0150 | 4309 | 2756 | 9037 |
| 3879 | 9015 | 1218 | 3420 | 1552 | 8760 | 2758 | 3897 |
| 4607 | 5549 | 8957 | 1643 | 7731 | 6421 | 4639 | 0839 |
| 6202 | 0118 | 0479 | 4969 | 5067 | 3423 | 2718 | 1440 |
| 6226 | 1693 | 7411 | 0887 | 8890 | 0987 | 6252 | 8683 |
| 8490 | 3667 | 9016 | 6370 | 3826 | 4061 | 4548 | 6521 |
| 0267 | 5886 | 8597 | 3128 | 1833 | 7218 | 2997 | 4017 |
| 4977 | 9118 | 3327 | 7049 | 0913 | 0947 | 9262 | 8071 |
| 3846 | 7549 | 8036 | 7688 | 4659 | 9984 | 4752 | 7859 |
| 4786 | 4360 | 7316 | 7631 | 4046 | 0174 | 8035 | 4080 |
| 1680 | 4395 | 6313 | 9927 | 0274 | 1499 | 7072 | 4169 |

Factors for Computing 3σ Control Limits

**CHART FOR AVERAGES**

| Number of Observations in Sample, $n$ | Factors for Control Limits | | |
|---|---|---|---|
| | $A$ | $A_1$ | $A_2$ |
| 2 | 2.121 | 3.760 | 1.880 |
| 3 | 1.732 | 2.394 | 1.023 |
| 4 | 1.500 | 1.880 | 0.729 |
| 5 | 1.342 | 1.596 | 0.577 |
| 6 | 1.225 | 1.410 | 0.483 |
| 7 | 1.134 | 1.277 | 0.419 |
| 8 | 1.061 | 1.175 | 0.373 |
| 9 | 1.000 | 1.094 | 0.337 |
| 10 | 0.949 | 1.028 | 0.308 |
| 11 | 0.905 | 0.973 | 0.285 |
| 12 | 0.866 | 0.925 | 0.266 |
| 13 | 0.832 | 0.884 | 0.249 |
| 14 | 0.802 | 0.848 | 0.235 |
| 15 | 0.775 | 0.816 | 0.223 |
| 16 | 0.750 | 0.788 | 0.212 |
| 17 | 0.728 | 0.762 | 0.203 |
| 18 | 0.707 | 0.738 | 0.194 |
| 19 | 0.688 | 0.717 | 0.187 |
| 20 | 0.671 | 0.697 | 0.180 |
| 21 | 0.655 | 0.679 | 0.173 |
| 22 | 0.640 | 0.662 | 0.167 |
| 23 | 0.626 | 0.647 | 0.162 |
| 24 | 0.612 | 0.632 | 0.157 |
| 25 | 0.600 | 0.619 | 0.153 |

Factors for Computing 3σ Control Limits

**CHART FOR STANDARD DEVIATIONS**

| Number of Observations in Sample, $n$ | Factors for Control Line $C_2$ | Factors for Control Limits | | | |
|---|---|---|---|---|---|
| | | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
| 2 | 0.5642 | 0 | 1.843 | 0 | 3.267 |
| 3 | 0.7236 | 0 | 1.858 | 0 | 2.568 |
| 4 | 0.7979 | 0 | 1.808 | 0 | 2.266 |
| 5 | 0.8407 | 0 | 1.756 | 0 | 2.089 |
| 6 | 0.8686 | 0.026 | 1.711 | 0.030 | 1.970 |
| 7 | 0.8882 | 0.105 | 1.672 | 0.118 | 1.882 |
| 8 | 0.9027 | 0.167 | 1.638 | 0.185 | 1.815 |
| 9 | 0.9139 | 0.219 | 1.609 | 0.239 | 1.761 |
| 10 | 0.9227 | 0.262 | 1.584 | 0.284 | 1.716 |
| 11 | 0.9300 | 0.299 | 1.561 | 0.321 | 1.679 |
| 12 | 0.9359 | 0.331 | 1.541 | 0.354 | 1.646 |
| 13 | 0.9410 | 0.359 | 1.523 | 0.382 | 1.618 |
| 14 | 0.9453 | 0.384 | 1.507 | 0.406 | 1.594 |
| 15 | 0.9490 | 0.406 | 1.492 | 0.428 | 1.572 |
| 16 | 0.9523 | 0.427 | 1.478 | 0.448 | 1.552 |
| 17 | 0.9551 | 0.445 | 1.465 | 0.466 | 1.534 |
| 18 | 0.9576 | 0.461 | 1.454 | 0.482 | 1.518 |
| 19 | 0.9599 | 0.477 | 1.443 | 0.497 | 1.503 |
| 20 | 0.9619 | 0.491 | 1.433 | 0.510 | 1.490 |
| 21 | 0.9638 | 0.504 | 1.424 | 0.523 | 1.477 |
| 22 | 0.9655 | 0.516 | 1.415 | 0.534 | 1.466 |
| 23 | 0.9670 | 0.527 | 1.407 | 0.545 | 1.455 |
| 24 | 0.9684 | 0.538 | 1.399 | 0.555 | 1.445 |
| 25 | 0.9696 | 0.548 | 1.392 | 0.565 | 1.435 |

Factors for Computing 3σ Control Limits

**CHART FOR RANGES**

| Number of Observations in Sample, $n$ | Factors for Central Line $d_2$ | Factors for Control Limits | | | | |
|---|---|---|---|---|---|---|
| | | $d_3$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
| 2 | 1.128 | 0.853 | 0 | 3.686 | 0 | 3.267 |
| 3 | 1.693 | 0.888 | 0 | 4.358 | 0 | 2.575 |
| 4 | 2.059 | 0.880 | 0 | 4.698 | 0 | 2.282 |
| 5 | 2.326 | 0.864 | 0 | 4.918 | 0 | 2.115 |
| 6 | 2.534 | 0.848 | 0 | 5.078 | 0 | 2.004 |
| 7 | 2.704 | 0.833 | 0.205 | 5.203 | 0.076 | 1.924 |
| 8 | 2.847 | 0.820 | 0.387 | 5.307 | 0.136 | 1.864 |
| 9 | 2.970 | 0.808 | 0.546 | 5.394 | 0.184 | 1.816 |
| 10 | 3.078 | 0.797 | 0.687 | 5.469 | 0.223 | 1.777 |
| 11 | 3.173 | 0.787 | 0.812 | 5.534 | 0.256 | 1.744 |
| 12 | 3.258 | 0.778 | 0.924 | 5.592 | 0.284 | 1.716 |
| 13 | 3.336 | 0.770 | 1.026 | 5.646 | 0.308 | 1.692 |
| 14 | 3.407 | 0.762 | 1.121 | 5.693 | 0.329 | 1.671 |
| 15 | 3.472 | 0.755 | 1.207 | 5.737 | 0.348 | 1.652 |
| 16 | 3.532 | 0.749 | 1.285 | 5.779 | 0.364 | 1.636 |
| 17 | 3.588 | 0.743 | 1.359 | 5.817 | 0.379 | 1.621 |
| 18 | 3.640 | 0.738 | 1.426 | 5.854 | 0.392 | 1.608 |
| 19 | 3.689 | 0.733 | 1.490 | 5.888 | 0.404 | 1.596 |
| 20 | 3.735 | 0.729 | 1.548 | 5.922 | 0.414 | 1.586 |
| 21 | 3.778 | 0.724 | 1.606 | 5.950 | 0.425 | 1.575 |
| 22 | 3.819 | 0.720 | 1.659 | 5.979 | 0.434 | 1.566 |
| 23 | 3.858 | 0.716 | 1.710 | 6.006 | 0.443 | 1.557 |
| 24 | 3.895 | 0.712 | 1.759 | 6.031 | 0.452 | 1.548 |
| 25 | 3.931 | 0.709 | 1.804 | 6.058 | 0.459 | 1.541 |