[4161] – 107

Seat No.	

F.E. (Semester – II) Examination, 2012
ENGINEERING MATHEMATICS – II
(2008 Pattern)

Time : 3 Hours Max. Marks : 100

Instructions : 1) *In Section* **I**, *solve Q. No.* **1** *or Q. No.* **2**, *Q. No.* **3** *or Q. No.* **4**, *Q. No.* **5** *or Q. No.* **6**. *In Section –* **II**, *solve Q. No.* **7**. *or Q. No.* **8**, *Q. No.* **9** *or Q. No.* **10**, *Q. No.* **11** *or Q. No.* **12**.

2) *Answers to the* **two** *Sections should be written in* **separate** *answer books.*

3) *Black figures to the* **right** *indicate* **full** *marks.*

4) **Use** *of electronic pocket calculator is* **allowed**.

5) *Assume suitable data, if* **necessary**.

SECTION – I

1. A) Form the differential equation whose general solution is $y = ae^{-2x} + be^{-3x}$, a and b are arbitrary constants. 6

B) Solve the following (**any two**) : 10

i) $(\cos x \cos y - \sin x \sin y)\, dy = dx$

ii) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

iii) $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$

OR

P.T.O.

2. A) Form the differential equation whose general solution is $y = A\cos\left(\frac{4x}{3}\right) + B\sin\left(\frac{4x}{3}\right)$, where A and B are arbitrary constants. 6

B) Solve the following (**any two**) : 10

i) $(2x^2y + e^x)\,dx = (e^x + y^3)dy$

ii) $2xydy = (3y^2 + x^2)dx$

iii) $\frac{dy}{dx} + (1+2x)y = e^{-x^2}$

3. Solve **any three** : 18

i) Water at temperature 100°C cools in 10 minutes to 88°C in a room of temperature 25°C. Find the temperature of water after 20 minutes.

ii) A particle is moving in a straight line with an acceleration $k\left[x + \frac{a^4}{x^3}\right]$ directed towards origin. If it starts from rest at a distance 'a' from the origin, prove that it will arrive at origin at the end of time $\frac{\pi}{4\sqrt{k}}$.

iii) The equation of an L-R circuit is given by $L\frac{dI}{dt} + RI = 10\cos t$. If $I = 0$ at $t = 0$, express I as a function of t, if L = 5 henries and R = 12 ohms.

iv) Find the orthogonal trajectories of the family of curves $r^2 = a^2\cos 2\theta$

OR

4. Solve **any three** : 18

i) A steam pipe 20 cm in diameter is protected with a covering 6 cm thick for which the coefficient of thermal conductivity is k = 0.0003 cal/cm deg. sec. in steady state. Find the heat lost per hour through a meter length of the pipe, if the inner surface of the pipe is at 200°C and the outer surface of the covering is at 30°C.

ii) A circuit consists of resistance 'R' ohms and a condenser of 'C' Farads connected to a constant e.m.f. E. If $\frac{q}{C}$ is the voltage of the condenser at time 't' after closing the circuit show that the voltage at time 't' is $E(1 - e^{-t/CR})$.

iii) A tank contains 10,000 litres of brine in which 200 kg salt are dissolved. Fresh water runs into the tank at the rate of 100 litres per minute, and the mixture kept uniform by stirring, runs out at the same rate. How long will it be before only 20 kg of salt is left in the tank ?

iv) If 30% of a radio active substance disappeared in 10 days, how long will it take for 90% of it to disappear ?

5. A) Obtain Fourier series expansion for function $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 \le x \le 2\pi$ and $f(x + 2\pi) = f(x)$.

Hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}$. 7

B) Prove that $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}\,dx = \frac{1}{a^n b^m} B(m,n)$. 4

C) If $I_n = \int_0^{\pi/2} x\cos^n x\,dx$, then obtain the relation between I_n and I_{n-2}. 5

OR

6. A) Find the harmonics a_0, a_1, a_2, b_1, b_2 of the Fourier series of the following data : 8

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$
y	1.0	1.4	1.9	1.7	1.5	1.2

B) Evaluate : $\int_0^\infty \sqrt{y}\, e^{-y^3}\, dy$. 4

C) If $I_n = \int_0^{\pi/4} \cos^{2n}x\, dx$, then

Prove that $I_n = \frac{1}{n2^{n+1}} + \frac{2n-1}{2n} I_{n-1}$. 4

SECTION – II

7. a) Trace the following curves (**any two**) : 8

i) $y^2 = (x - 1)(x - 2)(x - 3)$.

ii) $x^3 + y^3 = 3axy$

iii) $r = \frac{a}{2}(1 + \cos\theta)$

b) Prove that $\int_0^{\infty} \frac{e^{-\alpha x} \sin x}{x} dx = \cot^{-1} \alpha$. 5

c) Show that the arc length s of the curve $y = c \cosh\left(\frac{x}{c}\right)$, measured from its vertex to any point (x, y) is given by $s^2 = y^2 - c^2$. 4

OR

8. a) Trace the following curves (**any two**) : 8

i) $y^2 (x^2 - 1) = x$

ii) $r = a \sin 2\theta$

iii) $x = a (t - \sin t)$, $y = a (1 - \cos t)$.

b) Evaluate $\int xyds$, along the arc s of the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, in the first quadrant. 5

c) Show that $\int_0^{\infty} e^{-(x+a)^2} dx = \frac{\sqrt{\pi}}{2} [1 - \text{erf}(a)]$. 4

9. a) Find the equation of the sphere, whose centre is in the positive octant, and touching the three co-ordinate/planes and the plane $2x + y + 2z = 6$. 6

b) Find the equation of the right circular cone with vertex at (2, 1, – 3), semi vertical angle 30° and direction ratios of axis are 3, 4, – 1. 5

c) Determine the radius of the right circular cylinder and hence obtain its equation if the cylinder passes through the point (1, 2, 3) and has $\frac{x}{3} = \frac{y}{2} = \frac{z}{1}$ as the axis. 6

OR

10. a) Find the equation of the sphere having its centre on the plane $8x - 10y - 4z = 9$, and passing through the circle : 5

$x^2 + y^2 + z^2 - 2x - 3y + 4z - 8 = 0, x - 2y - z = 8$

b) Find the equation of the right circular cone generated by straight lines drawn from origin to cut the circle passing through the points (1, 2, 2), (2, 1, – 2), (2, – 2, 1). 6

c) Find the equation of the right circular cylinder of radius 2 and whose axis is $\frac{x-1}{2} = y = 3 - z$. 6

11. Solve **any two** : 16

a) Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$

b) Find the area of the region inside the circle $x^2 + y^2 = 2ax$ but lying outside the circle $x^2 + y^2 = a^2$.

c) ABCD is a square plate of side a and O is the midpoint of AB. If the density varies as the square of the distance from O, show that the C.G. of the plate is at a distance $\frac{7a}{10}$ from AB.

OR

12. Solve **any two** : 16

a) Express the following as a single integral and hence evaluate

$$\int_0^{\frac{a}{2}} \int_0^{x} \frac{dx\,dy}{\sqrt{a^2 - x^2 - y^2}} + \int_{\frac{a}{2}}^{a} \int_0^{\sqrt{ax - x^2}} \frac{dx\,dy}{\sqrt{a^2 - x^2 - y^2}}$$

b) Evaluate $\iiint_V \frac{z^2 dx\,dy\,dz}{(x^2 + y^2 + z^2)}$, where V is the volume of the sphere $x^2 + y^2 + z^2 = 2$.

c) Find the moment of inertia of one loop of the curve $r^2 = a^2 \cos 2\theta$, about the initial line.