(b) Show that if $T \in A(V)$ has all its characteristic roots in F, then there is a basis of V in which the matrix of T is triangular.

15. Let D be a division ring such that for every $a \in D$, there exists a positive integer $n(a) > 1$, depending on a such that $a^{n(a)} = a$. Show that D is a commutative field.

Register Number :

Name of the Candidate :

6 0 9 3

M.Sc. DEGREE EXAMINATION, 2012

(MATHEMATICS)

(FIRST YEAR)

(PAPER - I)

110. ALGEBRA

May] [Time : 3 Hours

Maximum : 100 Marks

SECTION - A ($8 \times 5 = 40$)

Answer any EIGHT questions.
ALL questions carry equal marks.

1. Show that the subgroup of N of G is a normal subgroup of G if and only if, every left coset of N in G is a right coset of N in G.

2. Show that S_n has as a normal subgroup of index 2 the alternating group A_n consiisting of even permutations.

3. Show that if F is a field, prove its only ideals are { 0 } and F itself.

4. If f(x), g(x) are non zero elements in F(x) then, show that

$$\deg f(x) \leq \deg [\, f(x)\, g(x) \,].$$

5. If S is a subset of V then, show that

$$L\,(L\,(S)) = L(S).$$

6. Show that if V is finite dimensional and $v \neq 0$ then, there is an element $f \in V^*$ such that $f(v) \neq 0$.

7. Prove that a subgroup of a solvable group is solvable.

8. Define fixed field of G. Show that the fixed field of G is a subfield of K.

9. If S and T are nil potent linear transformations with commute, then ST and S + T are nil potent linear transformations.

10. Show that if T is Hermitian and $vT^k = 0$ for $k \geq$ when $vT = 0$.

SECTION - B (3 × 20 = 60)

Answer any THREE questions.
ALL questions carry equal marks.

11. (a) Let ϕ be a homomorphism of G onto $\overline{G}$ with kernel K. Then, show that $\frac{G}{K}$ is isomorphic to $\overline{G}$

(b) Show that if p is a prime number and p/o (G) then, G has an element of order p.

12. (a) Show that the ideal $A = \langle a_0 \rangle$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R.

(b) Show that J[i] is Euclidean ring.

13. (a) Show that if V is finite dimensional over F, then any two basis of V have the same number of elements.

(b) Let V be a finite dimensional inner product space. Show that V has an orthonormal set as a basis.

14. (a) Show that if F is of characteristic O and a , b are algebraic over F, then there exist $C \in F(a, b)$ such that $F(a, b) = F(c)$.