Reg. No. : | | | | | | | |

Question Paper Code : 20141

M.E./M.Tech. DEGREE EXAMINATION, JANUARY 2011.

First Semester

Communication Systems

(Common to Computer and Communication)

281109 — APPLIED MATHEMATICS FOR COMMUNICATION ENGINEERS

(Regulation 2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

2. Prove that $J_n(x) = 0$ has no repeated root except at $x = 0$.

3. What are the necessary conditions for Cholesky decomposition of a matrix?

4. If A is a nonsingular matrix, then what is A^+?

5. If X is a continuous RV with pdf $f(x) = 2x$, $0 < x < 1$ then find the pdf of the RV $Y = X^3$.

6. If the mean of a Poisson RV is 2, then what is its standard deviation?

7. If (X, Y) is a continuous two dimensional RV with joint pdf $f(x, y) = k(xy^2 + yx^2)$, $0 < x < 1$, $0 < y < 2$. Find the value of k.

8. Prove that correlation coefficient is the geometric mean of regression coefficients.

9. What are the characteristics of a queuing system?

10. What do you mean by transient state and steady state queuing system?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $e^{\frac{1}{2}x\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$. (8)

(ii) Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right]$. (8)

Or

(b) (i) Show that $x^n J_n(x)$ is a solution of the equation $xy'' + (1-2n)y' + xy = 0$. (8)

(ii) Show that $J_5(x) = \left(\frac{384}{x^4} - \frac{72}{x^2} - 1\right) J_1(x) + \left(\frac{12}{x} - \frac{192}{x^3}\right) J_0(x)$. (8)

12. (a) Construct a QR decomposition for the matrix $A = \begin{bmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{bmatrix}$. (16)

Or

(b) Construct a singular value decomposition for the matrix $A = \begin{bmatrix} -3 & 1 \\ -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$. (16)

13. (a) (i) Find the moment generating function of geometric distribution and hence find its mean and variance. (8)

(ii) Buses arrive at a specified stop at 15 minutes intervals starting at 7 a.m. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7.30 am, find the probability that he waits

(1) less than 5 minutes for a bus and

(2) atleast 12 minutes for a bus. (8)

Or

(b) (i) The number of monthly breakdowns of a computer is a RV having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month

(1) without a breakdown

(2) with only one breakdown

(3) with atleast one breakdown. (8)

(ii) Suppose the life of automobile batteries is exponentially distributed with parameter $\lambda = 0.0001$ days,

(1) What is the probability that a battery will last more than 1200 days?

(2) What is the probability that a battery will last more than 1200 days given that it has already served 1000 days? (8)

14. (a) (i) For the following bivariate distribution of X and Y, find

(1) $P(X \leq 2, Y = 2)$

(2) $F_X(2)$

(3) $P(Y = 3)$

(4) $P(X < 3, Y \leq 4)$

(5) $F_Y(3)$. (8)

X \ Y	1	2	3	4
1	0.1	0	0.2	0.1
2	0.05	0.12	0.08	0.01
3	0.1	0.05	0.1	0.09

(ii) Let (X,Y) be a two dimensional continuous RV with joint pdf $f(x,y) = k\,xye^{-(x^2+y^2)};\quad x \geq 0,\ y \geq 0$.

(1) Find the value of k

(2) Are X and Y independent? (8)

Or

(b) (i) If X and Y are independent standard normal variates, find the probability distribution of $U = \frac{X}{Y}$. (8)

(ii) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible.

Variance of $X = 9$, Regression equation: $8x - 10y + 66 = 0$; $40x - 18y = 214$. Find

(1) The mean values of X and Y

(2) The standard deviation of Y

(3) The correlation coefficient between X and Y. (8)

15. (a) Suppose people arrive to purchase tickets for a basket ball game at the average rate of 4 minutes. It takes an average of 10 seconds to purchase a tickets. If a sports fan arrives 2 minutes before the game starts and if it takes $1\frac{1}{2}$ minutes to reach the correct seat after the fan purchase a ticket, then

(i) Can the sports fan expect to be seated for the start of the game?

(ii) What is the probability that the sports fan will be seated for the start of the game?

(iii) How early must the fan arrive in order to be 99% sure of being seated for the start of the game? (Assume Poisson arrival rate and exponential service time) (16)

Or

(b) A supermarket has two girls serving at the counters. The customers arrive in a Poisson fashion at the rate of 12 per hour. The service time for each customer is exponential with mean 6 minutes. Find

(i) The probability that an arriving customer has to wait for service

(ii) The average number of customers in the system and

(iii) The average time spent by a customer in the supermarket. (16)
