

- (b) Let V be a finite dimensional inner product space. Prove that V has an orthonormal set as a basis.
14. (a) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.
- (b) If $T, S \in A(V)$ and if S is regular, prove that T and $S T S^{-1}$ have the same minimal polynomial.
15. (a) If N is normal and $AN = NA$, prove that $AN^* = N^* A$, where A is any linear transformation on V .
- (b) Prove that any two finite fields having the same number of elements are isomorphic.

Register Number :

Name of the Candidate :

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M.Sc. DEGREE EXAMINATION, 2007

(MATHEMATICS)

(FIRST YEAR)

(PAPER - I)

110. ALGEBRA

(Revised Regulations)

May]

[Time : 3 Hours

Maximum : 100 Marks

PART – A (8 × 5 = 40)

Answer any EIGHT questions.

Each questions carries FIVE marks.

- Let H, K be two subgroups of a group G . Prove that $H K$ is a subgroup of G if and only if, $H K = K H$.
- Let G be a group. Let $A(G)$ be the set of all automorphisms of G . Prove that $A(G)$ is also a group.

Turn over

3. Prove that a finite integral domain is a field.
4. Prove that a Euclidean ring possesses a unit element.
5. If $v_1, v_2, \dots, v_n \in V$ are linearly independent, prove that every vector in their linear span has a unique representation in the form $\lambda_1 v_1 + \lambda_2 v_2 + \dots + \lambda_n v_n$ with the $\lambda_n \in F$.
6. State and prove Schwarz inequality.
7. Let F, K, L be fields. If L is an algebraic extension of K and if K is an algebraic extension of F , prove that L is an algebraic extension of F .
8. Let V be a finite dimensional vector space over a field F . Prove that if $T \in A(V)$ is singular if and only if \exists here exists a non-zero vector v in V such that $(v)T = 0$.
9. If $T \in A(V)$ is Hermitian, prove that all its characteristic roots are real.
10. If T is unitary and if λ is a characteristic root of T , prove that $|\lambda| = 1$.

PART – B (3 × 20 = 60)

Answer any THREE questions.

Each questions carries TWENTY marks.

11. (a) Let G, \bar{G} be groups Let ϕ be a homomorphism of G onto \bar{G} with Kernel K . Prove that $\frac{G}{K}$ is isomorphic to \bar{G} .
- (b) If p is a prime number and if p^α divides $O(G)$, prove that G has a subgroup of order p^α .
12. (a) Let R be a commutative ring with unit element. Let M be an ideal of R . Prove that M is a maximal ideal of R if and only if $\frac{R}{M}$ is a field.
- (b) If $f(x)$ and $g(x)$ are primitive polynomials, prove that $f(x)g(x)$ is a primitive polynomial.
13. (a) If V is a finite dimensional vector space over a field F , prove that any two bases of V have the same number of elements.

Turn over