

(b) Derive Gauss equations of surface theory.

14. (a) Solve the equation

$$xy'' - (2x-1)y' + (x-1)y = e^x.$$

(b) Find the series solution of the equation.

$$2x^2y'' + x(2x+1)y' - y = 0.$$

15. (a) Define gamma function. Show that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

(b) Obtain Rodrigue's formula which gives an expression for  $P_n(x)$ .

Register Number :

Name of the Candidate :

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**M.Sc. DEGREE EXAMINATION, 2007**

( MATHEMATICS )

( FIRST YEAR )

( PAPER - III )

**130. DIFFERENTIAL GEOMETRY AND  
DIFFERENTIAL EQUATIONS**

( *Revised Regulations* )

May ]

[ Time : 3 Hours

Maximum : 100 Marks

**SECTION – A** (8 × 5 = 40)

*Answer any EIGHT questions.*

*All questions carry equal marks.*

1. Define the curvature and torsion of the curve.

Find the curvature and torsion of the cubic curve

$$\vec{r} = (u, u^2, u^3).$$

**Turn over**

2. If the radius of spherical curvature is constant, prove that the curve either lies on a sphere or has constant curvature.

3. Prove that the curves of the family  $\frac{v^3}{u^2} = \text{constant}$  are geodesics on a surface with metric

$$v^2 du^2 - 2 u v du dv + 2u^2 dv^2; (u > 0, v > 0).$$

4. State and prove the normal property of geodesics.

5. Find the Gaussian curvature of the surface

$$x = u + v, \quad y = u - v, \quad z = uv.$$

6. If there is a surface of minimum area passing through closed space curve, show that it is necessarily a minimal surface.

7. Solve the equation  $y'' + 4y = 4\tan 2x$  using variation of parameters method.

8. Find the general solution of

$$2x^2 y'' + x(2x + 1)y' - y = 0.$$

9. Locate and classify the singular points of

$$(3x + 1)xy'' - (x + 1)y' + 2y = 0.$$

10. Prove the following :

$$(i) \quad \frac{d}{dx} (x^p J_p(x)) = x^p J_{p-1}(x)$$

$$(ii) \quad J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x.$$

**SECTION – B** (3 × 20 = 60)

*Answer any THREE questions.*

*All questions carry equal marks.*

11. (a) Find the arc length of the curve given as the intersection of the surfaces

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \quad x = a \cosh\left(\frac{z}{a}\right)$$

(b) State and prove the fundamental existence theorem for space curves.

12. (a) Obtain Liouville's formula for  $K_g$ .

(b) State and prove Tissot's theorem.

13. (a) Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature is zero.

**Turn over**