DAY — 11 SEAT NUMBER

2008	III	12	1100	J - 109	(E)

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE) PAPER - II
(REVISED COURSE)

Time : 2 Hrs. (4 Pages) Max. Marks : 40

Note : (i) All questions are compulsory.

(ii) Figures to the right indicate full marks.

(iii) Answer to every question must be written on a new page.

Q. 1. (A) Attempt any TWO of the following : **[8]**

(i) Evaluate : $\lim_{x \to 2} (x-1)^{\frac{1}{3x-6}}$ (3)

(ii) Evaluate : $\lim_{x \to 1} \frac{4^{x-1} - 2^x + 1}{(x-1)^2}$ (3)

(iii) If a function 'f' is continuous at $x = 0$ where, (3)

$$f(x) = \frac{\sin 3x}{5x} + a, \quad \text{for } x < 0$$

$$= x + 4 - b, \quad \text{for } x \geq 0$$

find the value of $a + b$.

(B) Attempt any ONE of the following :

(i) Evaluate : $\int \frac{dx}{x + \sqrt{x}}$ (2)

(ii) Evaluate : $\int \frac{dx}{1 + \sin x}$ (2)

Q. 2. (A) Attempt any TWO of the following : **[8]**

(i) If $y = \tan^{-1}\left(\frac{5x+1}{3-x-6x^2}\right)$, show that (3)

$$\frac{dy}{dx} = \frac{3}{1+(3x+2)^2} + \frac{2}{1+(2x-1)^2}$$

(ii) If $2y = \sqrt{x+1} + \sqrt{x-1}$, show that (3)

$$4(x^2-1)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - y = 0$$

(iii) Find the approximate value of $\tan^{-1}(0.999)$ (3)

(B) Attempt any ONE of the following :

(i) Differentiate $(x^x + a^a)$ w. r. t. x. (2)

(ii) Find $\frac{dy}{dx}$, if $y = \tan(xe^x)$ (2)

Q. 3. (A) (a) Attempt any ONE of the following : **[8]**

(i) Evaluate : $\int \sin(\log x)\,dx$ (3)

(ii) Evaluate : $\int \frac{x\,dx}{(x-1)(x^2+1)}$ (3)

(b) Attempt any ONE of the following :

(i) Evaluate : $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{\tan x}}$ (3)

(ii) Show that : $\int_0^1 \frac{dx}{\sqrt{x^2-x+1}} = \log 3$ (3)

(B) Attempt any ONE of the following :

(i) Form the differential equation by eliminating the arbitrary constants a and b from the relation $y = ae^{2x} + be^{-2x}$. (2)

(ii) Solve the differential equation : (2)

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

Q. 4. (A)(a) Attempt any ONE of the following : **[8]**

(i) Prove that $\left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{E(e^x)}{\Delta^2(e^x)} = e^x$. (3)

(ii) Using the relation between Δ and E, estimate the missing term in the following table : (3)

x	0	1	2	3	4
$f(x)$	-5	-2	7	---	91

(b) Attempt any ONE of the following :

(i) Solve the differential equation : (3)

$$\frac{dy}{dx} = (9x + y + 2)^2, \text{ by using } 9x + y + 2 = u.$$

(ii) Find the particular solution of the differential equation : (3)

$$y(1 + \log x)\frac{dx}{dy} - x \log x = 0, \text{ when } x = e \text{ and } y = e^2.$$

(B) Attempt any ONE of the following :

(i) In a Boolean algebra, prove that, the zero element '0' and unit element '1' are unique. (2)

(ii) If B is a Boolean algebra, for $x \in B$ prove that (2)

(a) $x + x = x$

(b) $x \cdot x = x$

Q. 5. (A) (a) Attempt any ONE of the following : [8]

(i) If y is a differentiable function of u, and u is a differentiable function of x, then show that, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ (3)

(ii) If $y = f(x)$ is a differentiable function of x such that, the inverse function $x = f^{-1}(y)$ is defined.

Then prove that $\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$, where $\frac{dy}{dx} \neq 0$. (3)

(b) Attempt any ONE of the following :

(i) Prove that :

$$\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left(x + \sqrt{x^2 + a^2}\right) + c. \quad (3)$$

(ii) Prove that :

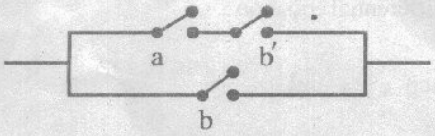
$$\int_{-a}^{a} f(x)\, dx = 2 \int_{0}^{a} f(x)\, dx, \quad \text{if } f(x) \text{ is an even function.}$$

$$= 0, \quad \text{if } f(x) \text{ is an odd function.} \quad (3)$$

(B) Attempt any ONE of the following :

(i) Draw the switching circuit of the Boolean expression $a + [b \cdot (c + a')]$ (2)

(ii) Simplify the switching circuit given below : (2)



◆◆◆