205 : Operations Research

Assignment No. 1
Answer All Questions: 5x5 =25 Marks

1. A company makes two kinds of leather belts. Belt-A is a high quality belt and Belt-B is of lower quality. The respective profits are Rs.40/- and Rs.30/- per belt. Each belt of type-A requires twice the time as that of belt of type-B and if all belts are of type-B, the company can make1000 per day. The supply of leather is sufficient for 800 belts per day (both A and B combined). Belt A requires a fancy buckle and only 400 per day are available and for belt B, 700 buckles per day are available. Determine the product mix, while solving it using simplex method.

2. A company has factories at four different places (1,2,3 and 4) which supply items to warehouses A, B, C, D and E. Monthly factory capacities are 200, 175, 150 and 325 respectively. Monthly warehouse requirements are 110, 90, 120, 230 and 160 respectively. Unit shipping costs (in rupees are given in the following table:

		To				
		A	B	C	D	E
From	1	13	-	31	8	20
	2	14	9	17	26	10
	3	25	11	12	17	15
	4	10	16	13	-	17

Shipments from 1 to B and from 4 to D are not possible. Determine the optimum distribution plan to minimize the shipping cost.

3.   Consider the following linear programming problem:

Minimise Z $= 24X_1 + 30X_2$

Subject to $2X_1 + 3X_2 \geq 10$

$4X_1 + 9X_2 \geq 15$

$6X_1 + 6X_2 \geq 20$ $X_1, X_2 \geq 0$

(a)   Formulate the dual L.P. Model.
(b)   Solve the dual problem and hence find the solution to the primal problem

4.   Vehicles are passing through a toll gate at the rate of 70 per hour. The average time to pass through the gate is 45 seconds. The arrival rate and service rate follow Poisson distribution. There is a complaint that the vehicles wait for long duration. The authorities are willing to install one more gate to reduce the average time to pass through the toll gate to 35 seconds if the idle time of the toll gate is less that 9% and the average queue length at the gate is more than 8 vehicles. Check whether the installation of the second gate is justified.

5.   The flight timings between two cities, X and Y are as given in the following two tables. The minimum layover time of any crew in either of the cities is 3 hours. Determine the base city for each crew so that the sum of the layover times of all the crews in non-base cities is minimized.

Timings of Flights from City X to City Y

Flight number	Departure time (from City X)	Arrival time (to City Y)
101	6 a.m	8.00 a.m
102	10 a.m	12.00 noon
103	3 p.m	5.00 p.m
104	8.p.m	10.00 p.m.

Timings of Flights from City Y to City X

Flight number	Departure time (from City X)	Arrival time (to City Y)
201	5.30 a.m	7.00 a.m
202	9.00 a.m	10.30 a.m
203	4.00 p.m	5.30 p.m
204	10.00 p.m	11.30 p.m.

Answer all Questions:

1.   Solve the following integer linear programming problem optimally using branch-and-bound technique.

        Minimise $Z = 6X_1 + 8X_2$

        Subject to $4X_1 + 5X_2 \leq 22$

                   $5X_1 + 8X_2 \leq 30$

                   $X_1 , X_2 \geq 0$ and integers

2.   Consider the data of a project as shown in the following table :

Activity	Normal Time (Weeks)	Normal Cost (Rs.)	Crash Time (Weeks)	Cash Cost (Rs.)
1 – 2	8	800	5	950
1 – 3	5	500	3	700
1 – 4	9	600	6	1,050
2 – 4	10	900	8	1,300
3 – 4	5	700	3	1,100
3 – 6	6	1,200	5	1,500
4 – 6	7	1,300	5	1,400
5 – 7	2	400	1	500
6 – 7	4	500	2	900

If the indirect cost per week is Rs. 300, find the optimal crashed project completion time

3.   Players A and B play a game in which each player has three coins (25p, 50p and 100p (one rupee)). Each of them selects a coin without the knowledge of the other person. If the sum of the values of the coins is an even number, A wins B's coin. If that sum is an odd number, B wins A's coin.

    (a)   Develop a payoff matrix with respect to Player A.
    (b)   Find the optimal strategies for the players.

4.   An organization is planning to diversify its business with a maximum out lay of Rs. 4 crores. It has identified three different locations to install plans. The organization can invest in one or more these plants subject to the availability of the fund. The different possible alternatives and their investment (in crores of rupees) and present worth of returns during the useful life (in crores of rupees) of each of these plants are

summarized in the following table.   The first row of the table has zero cost and zero return for all the plants.   Hence, it is known as do-nothing alternative.   Find the optimal allocation of the capital to different plants which will maximize the corresponding sum of the present worth of returns.

Alternatives	Plant 1		Plant 2		Plant 3	
	Cost	Return	Cost	Return	Cost	Return
1	0	0	0	0	0	0
2	1	12	2	16	2	9
3	2	15	3	20	3	12
4	3	19	4	25	-	-

5.   A production manager is faced with the problem of job allocation between his two production teams.   The production rate of Team X is 6 units per hour, while the production rate of Team Y is 10 units per hour.   The normal working hours for each of the teams are 50 hours per week.   The production manager has prioritized the following goals for the coming week :

P_1 : Avoid underachievement of the desired production level of 825 units.
P_2 : Any overtime operation of Team X beyond 10 hours should be avoided.
P_3 : Any overtime operation of Team Y beyond 15 hours should be avoided.
P_4 : Any underutilization of regular working hours should be avoided. Again, assign differential weights according to the relative productivity of the two teams.

Formulate this problem as a goal programming model and solve it by simplex method.