THIRD SEMESTER BTECH DEGREE EXAMINATION 2014

(SCHEME: 2013)

**13.303 DISCRETE STRUCTURES (FR)**

MODEL QUESTION PAPER

Time: 3 hours Maximum marks: 100

## PART-A

*Answer **all** questions. **Each** question carries 2 marks*

**1.** Write the given formula to an equivalent form and which contains the connectives $\neg$ and $\wedge$ only.

$$\neg\,(P \leftrightarrows (Q \rightarrow (R \vee P)))$$

**2.** Show that the following implication is a tautology without constructing the truth table

$$((P \vee \neg P) \rightarrow Q) \rightarrow ((P \vee \neg P) \rightarrow R) \Rightarrow (Q \rightarrow R)$$

**3.** Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ without constructing the truth table.

**4.** Differentiate between a partition and Covering of a Set with an example.

**5.** Give an example of an equivalence relation.

**6.** Let $(A,.)$ be a Group. Show that $(ab)^{-1} = b^{-1}a^{-1}$.

**7.** List out the properties of a ring.

**8.** Prove that the Zero element and Unit element of a Boolean algebra B are unique.

**9.** Define path in a Graph.

**10.** 51 numbers are chosen from the integers between 1 and 100 inclusively .Prove that 2 of the chosen integers are consecutive.

( 10 x 2 Marks = 20 Marks)

## PART-B

*Answer **one full** question from **each** module. **Each** question carries **20** marks.*

### MODULE - I

**11.** (a) Show that $(x)\,(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$ using Indirect method of Proof.

(10 Marks)

(b) Discuss Indirect method of Proof. Show that the following premises are inconsistent.

(i) If Jack misses many classes through illness, then he fails high school.

(ii) If Jack fails high School, then he is uneducated.

(iii) If Jack reads a lot of books, then he is not uneducated.

(iv) Jack misses many classes through illness and reads a lot of books. (10 marks)

**12.** (a) Show that

(i) $(\exists x)\ (F(x) \wedge S(x)) \rightarrow (y)\ (M(y) \rightarrow W(y))$

(ii) $(\exists y)\ (M(y) \wedge \urcorner W(y))$ the conclusion $(x)(F(x) \rightarrow \urcorner S(x))$ follows. (10 marks)

(b) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$ (5 marks)

(c) Show the following implication using rules of inferences

$P \rightarrow (Q \rightarrow R), Q \rightarrow (R \rightarrow S) \Rightarrow P \rightarrow (Q \rightarrow S)$ (5 marks)

## MODULE - II

**13.** (a) For any two sets A and B Show that $A - (A \cap B) = A - B$. (5 marks)

(b) ) Let R and S be two relations on a set of positive integers I.

$R = \{ \langle x, 2x \rangle / x \in I \}$ $S = \{ \langle x, 7x \rangle / x \in I \}$

Find RoS, RoR, RoRoR, RoSoR. (5 marks)

(c) Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$. $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$ Prove that $a_n \leq 3^n$

(5 marks)

**14.** (a) Construct a formula for the sum of first n positive odd numbers. Prove the same using mathematical Induction. (5 marks)

(b) Draw the Hasse Diagram of $(P(A), \leq)$ where $\leq$ represents $A \subseteq B$ and $A = \{a, b, c\}$ (5 marks)

(c) Five friends run a race everyday for 4 months (excluding Feb). If no race ends in a tie, show that there are at least 2 races with identical outcomes. (5 marks)

(d) What are Piano Axioms? Explain. (5 marks)

## MODULE - III

**15.** (a) If $(a+b)^2 = a^2 + 2ab + b^2$, $\forall\ a, b \in R$, prove that R is a commutative ring and conversely. (5 marks)

(b) Show that any subgroup of a cyclic group is cyclic. (5 marks)

(c) State and prove Lagrange's Theorem. (10 marks)

**16.** (a) Let $(A,*)$ be a Group. Show **that** $(A,*)$ is an abelian Group if and only if

$a^2 * b^2 = (a*b)^2$ for all a and b in A. (5 marks)

(b) Let $(H, .)$ be a subgroup of a Group $(G, .)$ . Let N =$\{x / x \in G, xHx-1 = H\}$. Show that $(N, .)$ is a subgroup of $(G, .)$. (10 marks)

(c) Prove that Every Field is an Integral Domain. (5 marks)

**MODULE - IV**

**17.** (a) Prove that in a distributive Lattice, if $b \wedge c = 0$, then $b \leq c$. (5 marks)

(b) Show that $a \vee b$ is the least upper bound of a and b in $(A, \leq)$. Show that $a \wedge b$ is the greatest lower bound of a and b in $(A, \leq)$. (5 marks)

(c) Differentiate between Connected ,Disconnected and Strongly Connected Graphs using examples (10 marks)

**18.** (a) State and prove any four basic properties of algebraic systems defined by Lattices.

(10 marks)

(b) Differentiate between a Boolean function & Boolean expression. (5 marks)

(c) Simplify the following Boolean expression

$(a \wedge b) \vee c) \wedge (a \vee b) \wedge c)$ (5 marks)