First Year B.Sc., Degree Examinations
September / October 2015

(Directorate of Distance Education)

Paper –I: DSA 230: MATHEMATICS

Time: 3hrs.]   *[Max. Marks: 90*

Instructions to candidates:

Answer any SIX full questions of the following choosing at least ONE from each Part.

PART – A

1. a) i) Find the least non negative remainder when 2^{151} is divided by 7.

   ii) If p is prime and p/ab then prove that p/a or p/b.   (2 + 2)

   b) Solve the simultaneous congruences $x \equiv 2 \pmod 5$ and $3x \equiv 1 \pmod 8$ (5)

   c) Prove that any two integers of which atleast one is nonzero have a unique GCD and it can be expressed in the form $ma + nb$ where $m, n \in z$.   (6)

2. a) i) Define equivalence relation. Give an example.

   ii) Let $f : R \to R^+$ and $g : R^+ \to R$ be defined by $f(x) = e^x$ and $g(x) = \log_e x$ then find $fog(x)$ and $gof(x)$ (2 + 2)

   b) Find the partition of the set Z of all integers defined by the equivalence relation aRb iff (a – b) is a multiple of 5.   (5)

   c) Prove that $(0,1)$ is uncountable.   (6)

PART – B

3. a) i) If a function $f(x)$ is differentiable at *a* then prove that it is continuous at *a*.

   ii) Find the nth derivative of $\log(ax + b)$ (2 + 2)

   b) Examine the differentiability of the function $f(x) = \begin{cases} x^2 & if & x \le 3 \\ 6x - 9 & if & x > 3 \end{cases}$ at $x = 3$ (5)

   c) If $y = \cos(m \sin^{-1} x)$ then show that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$ (6)

4. a) i) Find $\dfrac{ds}{d\theta}$ for the curve $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

      ii) Find the pedal equation of the curve $r = a(1 + \cos\theta)$ (2 + 2)

  b) Show that the pair of curves $r = a\sec^2\dfrac{\theta}{2}$, $r = b\operatorname{cosec}^2\dfrac{\theta}{2}$ intersect orthogonally.   (5)

  c) Find the evolute of the parabola $y^2 = 4ax$ (6)

PART – C

5. a) i) Find the numbers x and y such that $(3,\ x,\ y)$ lie on the line passing through $(2,\ 4,\ 4)$ and $(-1,\ 4,\ 1)$

      ii) Find the equation of the plane passing through the points $(2,\ 3,\ 1)$, $(4,\ 5,\ 0)$ and $(2,\ 1,\ 7)$.   (2 + 2)

  b) Find the equation of the plane passing through the point $(1,\ 3,\ 5)$ and the line $x = 3 + 3t,\ y = 1 + t,\ z = -1 - 4t$ (5)

  c) Find the mutual position of the lines $l_1\ \&\ l_2$ given by

$$l_1:\ x = 1 - t,\ \ y = 2 + t,\ \ z = 2t$$
$$l_2:\ x = 3 - 2s,\ \ y = 4 + 2s,\ z = 6 + 4s$$

                                          (6)

6. a) i) Find the centre and radius of the sphere whose equation is
$$4x^2 + 4y^2 + 4z^2 - 4y - 8z - 22 = 0$$

      ii) Find the asymptotes parallel to the co-ordinate axes for the curve $y^2(x^2 - a^2) = x$
                                                (2 + 2)

  b) Find the position and nature of double points of the curve $x^3 + y^3 = 3axy$ (5)

  c) Show that the volume generated by the revolution of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ about the major axis is $\dfrac{4}{3}\pi ab^2$.   (6)

PART – D

7. a) i) If A and B are symmetric matrices then prove that $AB + BA$ is symmetric and $AB - BA$ is skew symmetric.

      ii) Find the rank of the matrix   (2 + 2)

$$A = \begin{bmatrix} 1 & 2 & -3 & -4 \\ 1 & 3 & 1 & -2 \\ 2 & 5 & -2 & -6 \end{bmatrix}$$

b) Find the inverse of the matrix (5)

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

c) Solve completely the system of equations

$$x + 3y - 2z = 0$$
$$2x - y + 4z = 0 \qquad (6)$$
$$x - 11y + 14z = 0$$

8. a) i) Evaluate $\int \dfrac{dx}{1 - \sin x}$

   ii) Evaluate $\int e^x \left(\dfrac{x-1}{x^2} \right) dx$ (2 + 2)

b) Evaluate $\int \dfrac{dx}{(1+x^2)\sqrt{1-x^2}}$ (5)

c) Evaluate $\int_0^{\pi} x \sin^7 x \cos^2 x \ dx$ (6)

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