Second Year B.Sc., Degree Examinations

September / October 2015

(Directorate of Distance Education)

Paper –II: DSB 230: MATHEMATICS

Time: 3hrs.] *[Max. Marks: 90*

Instructions to candidates:

Answer any SIX full questions of the following choosing at least ONE from each Part.

PART – A

1. a) i) Find the order and degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$

ii) Solve $\frac{d^4y}{dx^4}+8\frac{d^2y}{dx^2}+16y=0$ (2 + 2)

b) Solve $\frac{dy}{dx}=\frac{4x+6y+5}{2x+3y+4}$ (5)

c) Solve $x(y^2-x)dx+(3x^2y^2+x^2y-2x^3+y^2)dy=0$ (6)

2. a) i) Solve $xp^2+(y-x)p-y=0$

ii) Find the general and singular solution of $y=px-e^p$ (2 + 2)

b) Solve $4p^3+3xp=y$ (5)

c) Find orthogonal trajectories of the family of circles through the origin and the centre on the $x-$axis. (6)

PART – B

3. a) i) Solve $(D^2+9)y=\cos 2x+\sin 3x$, where $D=\frac{d}{dx}$

ii) Solve $(D^4+5D^3+6D^2-4D-8)y=0$, where $D=\frac{d}{dx}$ (2 + 2)

b) Solve $(D^2-4)y=\cosh(2x-1)$, where $D=\frac{d}{dx}$ (5)

c) Solve $(D^3+1)y=5e^x x^2$: $D=\frac{d}{dx}$ (6)

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4. a) i) Evaluate $\lim_{x\to 1}\left[\frac{\sqrt{x-1}+\sqrt{x-1}}{\sqrt{x^2-1}}\right]$

ii) Evaluate $\lim_{x\to \pi/2}(\sec x - \tan x)$ (2 + 2)

b) State and prove Lagrange's mean value theorem (5)

c) Prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots$ using Maclaurin's expansion (6)

PART – C

5. a) i) Show that in a group G, $(ab)^{-1} = b^{-1}a^{-1}$

ii) Find the distinct generators of the cyclic group of order 10 (2 + 2)

b) If a is a generator of a cyclic group G. then $0(a) = 0(G)$ (5)

c) State and prove Fermat's theorem. (6)

6. a) i) Solve $x-6 < 2x-5 \le x-3$

ii) If *f* be the permutation in 5 symbols defined by

$f(1)=3$ $f(2)=4$ $f(3)=2$ $f(4)=5$ $f(5)=1$ find f^{-1} (2 + 2)

b) Find the order of the permutation and find whether they are odd or even (5)

$$\phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 6 & 7 & 8 & 1 & 2 \end{pmatrix}$$

c) Find the envelope of the family of ellipses , where a & b are parameters connected by the relation $ab = c^2$, where 'C' is a known constant. (6)

PART – D

7. a) i) If $\lim_{n\to\infty} a_n = l$ then ST $\lim_{n\to\infty}|a_n| = |l|$

ii) Examine the nature of the sequence $a_n = \left\{\frac{n+1}{n-1}\right\}^n$ (2 + 2)

b) Discuss the convergence of $\left\{x^{1/n}\right\}$, $x > 0$ (5)

c) i) Prove that A monotonic decreasing sequence bounded below is convergent

ii) Define leibnitz's rule on alternating series. (4 + 2)

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8. a) i) Discuss the convergence of $\sum \frac{1}{n} \sin \frac{1}{n}$.

ii) Verify whether the series $1^3 + 2^3 + 3^3 + \ldots\ldots$ is converges or not (2 + 2)

b) Test the convergence of $1 + \frac{\alpha+1}{\beta+1} + \frac{(\alpha+1)(2\alpha+1)}{(\beta+1)(2\beta+1)} + \frac{(\alpha+1)(2\alpha+1)(3\alpha+1)}{(\beta+1)(2\beta+1)(3\beta+1)} + \ldots\ldots\ldots$ (5)

c) Sum to infinity the series $\frac{1}{24} - \frac{1.3}{24.32} + \frac{1.3.5}{24.32.40} - + \ldots\ldots\ldots$ (6)

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