Third Year B.Sc., Degree Examinations

September / October 2015

(Directorate of Distance Education)

Paper –IV: DSC 231: MATHEMATICS

Time: 3hrs.] *[Max. Marks: 90*

Note: Answer any SIX of the following:

PART – A

1. a) i) Evaluate $\int_C (3x+y)dx+(2y-x)dy$ along the curve $y=x^2+1$ from $(0, 1)$ to $(3, 0)$.

ii) Evaluate $\int_0^1\int_0^2 (x+y)\,dy\,dx$. (2 + 2)

b) Evaluate $\int_C x^2y^2\,ds$ around the circle $x^2+y^2=1$ (5)

c) Evaluate $\iint_R xy\,dx\,dy$ where R is the quadrant of the circle $x^2+y^2=a^2$ and $x,\ y\ge 0$ (6)

2. a) i) Evaluate $\iint_R xye^x\,dy\,dx$ where R is $0\le x\le 1$ and $2\le y\le 3$

ii) Evaluate $\int_0^1\int_0^2\int_0^3 x^2yz\,dx\,dy\,dz$ (2 + 2)

b) Find the area of the surface $y^2+z^2=2x$ cut by the plane $x=1$ (5)

c) Find the volume of the sphere $x^2+y^2+z^2=a^2$ (6)

3. a) i) Define Gamma function. Find $\Gamma(1)$.

ii) Define Beta function and find $\int_0^\infty \frac{x^6(1-x^8)dx}{(1+x)^{22}}$. (2 + 2)

b) P.T $\int_0^{\pi/2} \sin^m\theta\cos^n\theta,\ d\theta = \frac{1}{2}\beta\left(\frac{m+1}{2},\ \frac{n+1}{2}\right)$ (5)

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c) P.T $\Gamma(n)\,\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{n-1}}\,\Gamma(2n)$ (6)

4. a) i) Define upper and lower Riemann sums.

ii) If $f(x)=2x+3$ by Riemann integration S.T. $\int_0^1 f(x)\,dx=4$ (2 + 2)

b) Compute $\int_a^b x^r dx$ where r is a positive integer. (5)

c) State and prove Darboux theorem. (6)

PART – B

5. a) i) Find the Wronstian w for equation $y''-2y'+y=e^x \log x$.

ii) Find a known part of complementary function for the equation

$(\sin x - x\cos x)\,y''-(x\sin x)\,y'+(\sin x)\,y=0$. (2 + 2)

b) Solve $\cos x\frac{d^2y}{dx^2}+\sin x\frac{dy}{dx}-2y\cos^3 x=2\cos^5 x$ by change of independent variable. (5)

c) Solve $\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}-(a^2+1)y=e^x\sec x$ by reducing into the normal form. (6)

6. a) i) Verify the condition of exactness of the equation $(1+x^2)\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=\sec^2 x$.

ii) Solve: $\frac{dx}{x^2+2y^2}=\frac{dy}{xy}=\frac{dz}{xz}$. (2 + 2)

b) Solve: $\frac{dx}{x^2-y^2-z^2}=\frac{dy}{2xy}=\frac{dz}{2xz}$. (5)

c) Solve $y_2+y=\operatorname{cosec} x$ by the method of variation of parameter. (6)

7. a) i) Verify the condition for integrability of the equation

$3x^2dx+3y^2dy-\left(x^3+y^3+e^{2z}\right)dz=0$

ii) Form the partial differential equation by eliminating the constants a and b in

$2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ (2 + 2)

b) Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$ (5)

c) Solve $zxp+yzq=xy$ (6)

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8. a) i) Find the Fourier Co-efficient a_0 for $f(x) = x^2$ in the interval $(-\pi, \pi)$.

ii) Find the Fourier Co-efficient a_n for $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$ (2 + 2)

b) Find the Fourier series generated by the periodic function $|x|$ of period 2π. (5)

c) Obtain the half range Fourier sine series of $f(x) = (x-1)^2$ in the interval $(0, 1)$. (6)

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