Final Year M.Sc., Degree Examinations

September / October 2015

(Directorate of Distance Education)

MATHEMATICS

Paper – PM 10.05: DPB 510: COMPLEX ANALYSIS

Time: 3hrs.] *[Max. Marks: 70/80*

Instructions to candidates:

i) ***Students who have attended 30 marks IA scheme will have to answer for total 70 Marks.***

ii) ***Students who have attended 20 marks IA scheme will have to answer for total 80 Marks.***

iii) ***Answer any FIVE questions for both 70-80 marks scheme and Question No. (9) in Section – B is compulsory for 80 marks.***

SECTION – A

1. a) Prove that every power series represents an analytic function within the radius of convergence.

   b) Verify that the function $f(z) = z^2$ satisfies the Cauchy-Riemann equations and determine the derivative $f'(z)$.

   c) Let $u(x, y) = x^2 - y^2 + 2x$. Find the conjugate function $v(x, y)$ such that $f(z) = u(x, y) + i\,v(x, y)$ is an analytic function of z throughout the Z-Plane.

   (7 + 3 + 4)

2. a) Determine the image path in the w – plane corresponding to the circle $|z-3| = 2$ in the z – plane under the mapping $\omega = 1/z$.

   b) Find the images in w – plane of the lines $\mathrm{Re}(z) =$ constant and $\mathrm{Im}(z) =$ constant of the mapping $W = \dfrac{z-1}{z+1}$

   (7 + 7)

3. a) Find the bilinear transformation that maps the three points $z = 0, -i$ and -1 onto the three points $w = i, 1, 0$ respectively in the w – plane.

   b) If $f(z)$ is analytic in the domain D and $f'(z_0) \neq 0$ then show that $f(z)$ is conformal at $z = z_0$.

   c) Determine where the following complex mappings are conformal :
   $(i)\ f(z) = z^3 - 3z + 1 \quad (ii)\ f(z) = \cos z$

   (7 + 3 + 4)

4. a) State and prove Cauchy theorem for a triangle.

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b) Define an index of a closed curve. Prove that index of a closed curve is an integer. (8 + 6)

5. a) Prove that a bounded entire function is constant. Deduce the fundamental theorem of algebra.

b) Define an isolated singular point of the analytic function $f(z)$. Classify three different types of isolated singular points and illustrate each with example. (8 + 6)

6. a) Derive i) the Taylor series representation of $f(z)=\frac{1}{1-z}$ about the point $z_0=i$ and

ii) The Laurent series representation for $f(z)=\frac{e^z}{(z+1)^2}$ about $z_0=-1$

b) State and prove Cauchy residue theorem. Hence evaluate $\int_0^\infty \frac{x^2}{x^6+1}dx$. (7 + 7)

7. a) State and prove maximum modulus principle.

b) Given an entire function $f(z)=\sin z$ and if R denotes the rectangular region $0\le x\le\pi,\ 0\le y\le 1$, Show that $|f(z)|$ has a maximum value on the boundary of R and not in the interior.

c) Show that the roots of $z^7-5z^3+2=0$ lie between the circles $|z|=1$ and $|z|=2$. (6 + 5 + 3)

8. a) State and establish Poisson integral formula.

b) State and prove Weirstrass factorization theorem. (7 + 7)

SECTION – B

9. a) Obtain the Taylor series representations of (i) $f(z)=\frac{1}{z}$ and (ii) $f(z)=z^2e^{3z}$ about the point $z=3$ and state the region of convergence for each. (5)

b) Use residue theorem, evaluate

$$\int_0^{2\pi}\frac{d\theta}{(1+a\cos\theta)}\ ,\quad (-1<a<1)$$ (5)

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Final Year M.Sc., Degree Examinations

September / October 2015

(Distance Education)

MATHEMATICS

Paper – PM 10.06: DPB 520: TOPOLOGY

Time: 3hrs.] *[Max. Marks: 70/80*

Instructions to candidates:

i) ***Students who have attended 30 marks IA scheme will have to answer for total 70 Marks.***

ii) ***Students who have attended 20 marks IA scheme will have to answer for total 80 Marks.***

iii) ***Answer any FIVE questions for both 70-80 marks scheme and Question No. (9) in Section – B is compulsory for 80 marks.***

SECTION – A

1. a) Let X be a topological space, $A, B \subset X$. Show that

$(i)\ \overline{A \cup B} = \overline{A} \cup \overline{B} \quad (ii)\ \overline{A \cap B} \neq \overline{A} \cap \overline{B}$

b) Let A be a subset of a topological space X and let A^1 be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$.

c) Let B and B^1 be bases for the topologies τ and τ^1 respectively on X. Then show that τ^1 is finer than τ if and only if for each $x \in X$ and each basis element $B \in B$ containing x there is a basis element $B^1 \in B^1$ such that $x \in B^1 \subset B$. (5 + 4 + 5)

2. a) Let X and Y be spaces, $f : X \rightarrow Y$. Show that f is continuous on X if and only if $f(\overline{A}) \subset \overline{f(A)}$ for $A \subset X$.

b) Prove that the space R^{ω}, the countably infinite product of R in the box topology, is not metrizable. (7 + 7)

3. a) Define a connected space. For a subset A of X, prove or disprove: A is connected implies $\overline{A}$ is connected.

b) Let X_i be connected, $\bigcap X_i \neq \phi$. Show $\bigcup X_i$ is connected. (5 + 9)

4. a) Define a compact space. Show that a closed subspace of a compact space is compact and the image of a compact space under a continuous map is compact.

b) Define a first countable space. Let X be first countable and $A \subseteq X$, Prove that $x \in \overline{A}$ if and only if there is a sequence of points of A converging to x. (8 + 6)

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5. a) Let X be a topological space, $A \subset X$. Show that

   i) If X is Hausdorff and A is compact, then A is closed.

   ii) If X is compact and A is closed, then A is compact.

   b) Prove that the space R_l , the real line with the lower limit topology is Lindelof and separable but not second countable. (8 + 6)

6. a) Define regular space, prove that X is regular if and only if given a point x in X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subset U$.

   b) Prove that every metrizable space is normal. (7 + 7)

7. State and prove Urysohn's lemma. (14)

8. a) Show that

   i) a metric space is normal

   ii) a regular Lindeloff space is paracompact.

   b) Is R_e, i) Paracompact?

   ii) Metrizable? Justify (8 + 6)

SECTION – B

9. a) Define a locally connected space X. Prove that a space X is locally connected if and only if for every set U of X, each component of U is open in X.

   b) If $\{U_1, U_2, \ldots, U_n\}$ is a finite open covering of the normal space X, prove that there exists a partition of unity dominated by $\{U_i\}$. (5 + 5)

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Final Year M.Sc., Degree Examinations

September / October 2015

(Directorate of Distance Education)

MATHEMATICS

Paper – PM 10.07: DPB 530: MEASURE THEORY AND FUNCTIONAL ANALYSIS

Time: 3hrs.] *[Max. Marks: 70/80*

Instructions to candidates:

i) ***Students who have attended 30 marks IA scheme will have to answer for total 70 Marks.***

ii) ***Students who have attended 20 marks IA scheme will have to answer for total 80 Marks.***

iii) ***Answer any FIVE questions for both 70-80 marks scheme and Question No. (9) in Section – B is compulsory for 80 marks.***

SECTION – A

1. a) Define a Borel Set. For any singleton set $\{x\}$, prove that $m^*\{(x)\}=0$.

   b) Construct an uncountable set of measure zero.

   c) If $\{I_n\}$ is a finite covering of open intervals of $Q \cap [0,1]$, show that $\sum l(I_n) \geq 1$. Is this true if $\{I_n\}$ is infinite? (5 + 4 + 5)

2. a) State Littlewood's three principles and prove any one of them.

   b) State and prove Fatou's lemma. (8 + 6)

3. a) If $f \geq 0$ and measurable, show that $\exists$ a sequence $\{\phi_n\}$ of simple functions such that $\phi_n \uparrow f$.

   b) Let $f:[a, b] \to R'$ be increasing. Show f' exists a.e., and that $\int_a^b f' \leq f(b)-f(a)$.

   (7 + 7)

4. a) Find the Dini Derivatives of $f(x)=\begin{cases} 0 & x=0 \\ x\sin\frac{1}{x} & x \neq 0 \end{cases}$ at $x=0$.

   b) Define a function of bounded variation on $[a, b]$. With usual notations show that $T=N+P$ and $f(b)-f(a)=P-N$.

   c) Let *f* be absolutely continuous on $[a, b]$. Show that *f* is of bounded variation on $[a, b]$. (4 + 5 + 5)

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5. a) Define a complete metric space. Prove that $1_p, 1 \le p < \infty$ is a complete metric space. Do 1_∞ is complete? Justify.

   b) State and prove Banach Fixed point theorem. (8 + 6)

6. a) Show that there is no $f : R^1 \to R^1$ continuous only at rationals.

   b) State and prove Lebesgue Covering lemma. (6 + 8)

7. a) Prove that the set of all continuous linear operators of a normed linear space into a Banach space is itself a Banach space.

   b) Show that any two normed linear spaces with the same finite dimension are topologically isomorphic. (8 + 6)

8. a) State Hanh Banach theorem. Let M be a closed linear subspace of a normed linear space X and $x_0 \notin M$. If $d = d(x_0, M)$, then show that there exists a functional $f_0 \in X^*$ such that $f_0(M) = 0$ and $f_0(x_0) = 1$ and $\|f_0\| = \frac{1}{d}$.

   b) Prove that the space $l_p, 1 \le p < \infty$ is reflexive. Is the space l_1 reflexive? Justify. (8 + 6)

SECTION – B

9. If f is a real continuous function defined on a closed and bounded interval $[a, b]$ and if $\varepsilon > 0$, prove that there exists a polynomial p such that $|f(t) - p(t)| < \varepsilon$ for $a \le t < b$. (10)

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Final Year M.Sc., Degree Examinations

September / October 2015

(Directorate of Distance Education)

MATHEMATICS

Paper – PM 10.08: DPB 540: NUMERICAL ANALYSIS

Time: 3hrs.] *[Max. Marks: 70/80*

Instructions to candidates:

i) ***Students who have attended 30 marks IA scheme will have to answer for total 70 Marks.***

ii) ***Students who have attended 20 marks IA scheme will have to answer for total 80 Marks.***

iii) ***Answer any FIVE questions for both 70-80 marks scheme and Question No. (9) in Section – B is compulsory for 80 marks.***

SECTION – A

1. a) Derive the Newton-Raphson scheme to obtain the roots of an equation $f(x)=0$. use it to find an approximate root of the equation $x^3-5x+1=0$.

   b) Describe Bairstow's method to extract a quadratic factor of the form x^2+px+q from a polynomial of degree *n*. (7 + 7)

2. a) Find the solution of

$$\begin{aligned}83x+11y-4z&=95\\7x+52y+13z&=104\\3x+3y+29z&=71\end{aligned}$$

   by performing four iterations using any one of iteration matrix.

   b) Explain successive over relaxation method to solve the system $Ax=b$ (8 + 6)

3. a) Find all the eigen values and the corresponding eigen vectors of the matrix

$$A=\begin{pmatrix}1&\sqrt{2}&2\\\sqrt{2}&3&\sqrt{2}\\2&\sqrt{2}&1\end{pmatrix}.$$

   b) Find all the eigen values of the following matrix using Given's method

$$A=\begin{pmatrix}1&6&0\\6&2&1\\0&1&3\end{pmatrix}.$$

(8 + 6)

Contd...........2

4. a) Discuss the convergence criteria of Hermite interpolation polynomial of degree $\leq 2n+1$.

   b) Evaluate $I = \int_0^1 \frac{2x}{1+x^4}\,dx$ using Gauss Legendre and Gauss Chebyshev integration formula. (6 + 8)

5. a) Derive Lagrange's interpolation formula for the given data points $(x_i, y_i), i = 1, 2, \ldots n$

   b) Obtain the least square approximation polynomial of degree one and two for $f(x) = \sqrt{x}$ on $[0, 1]$. (7 + 7)

6. a) Determine the cubic spline $S(x)$ for the interval $[2, 3]$ for the following tabulated values of x and y.

X	1	2	3	4	5
Y	10	17	36	73	134

   b) Derive cubic spline interpolation polynomial. (7 + 7)

7. a) Derive Runge-Kutta 2nd order method to find the numerical solution to an IVP $y' = f(x, y), x_0, = y_0$

   b) Use Adams predictor-corrector method to find $y(0,8)$ and $y(1,0)$ for an IVP, $y' = x^2 + y,\ y(0) = 1,$ choose $h = 0.2$. (7 + 7)

8. a) Solve the Laplace equation $u_{xx} + u_{yy} = 0$ by employing five point formulae which satisfies the following boundary conditions.
   $u(0, y) = 0 \quad u(x, 0) = 0$
   $u(x, 1) = 100x,\ u(1, y) = 100y$ Choose $h = k = 1$.

   b) Derive Crank – Nicolson implicit formula for solving parabolic partial differential equation $\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta x^2}$. (7 + 7)

SECTION – B

9. a) Use Secant method to find the real roots of $2x^3 + 3x - 5 = 0$, perform four iterations.

   b) Evaluate an integral $\int_0^1 \frac{dx}{x^2 + 2x + 2}$ by dividing the given interval into equal 4, 6, 8 subintervals using Simpson's ⅓rd rule. (5 + 5)

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