Reg. No. .... (2 pages) **K 6121**

Name ....

FIRST YEAR B.Sc. DEGREE EXAMINATION, APRIL/MAY 2005

Part III—Mathematics

Subsidiary Paper I—MATHEMATICS

(Common for all subsidiary subjects)

Time : Three Hours Maximum : 70 Marks

A maximum of 14 marks will be awarded from each unit.
All questions carry 5 marks each.

Unit I

1. Find the sum of infinity of the series $1 - \frac{3}{4} + \frac{3.5}{4.8} - \frac{3.5.7}{4.8.12} + \ldots$

2. If n is large show that $\left(1 + \frac{1}{n}\right)^n = e\left(1 - \frac{1}{2n} + \frac{11}{24n^2}\right)$ approximately.

3. If α, β, γ are the roots of the equation $x^3 + px + q = 0$, find the equation whose roots are $(\alpha - \beta)^2$, $(\beta - \gamma)^2$, $(\gamma - \alpha)^2$.

4. Solve the biquadratic equation $x^4 - 4x^3 + 7x^2 - 6x + 2 = 0$.

Unit II

5. If $\tan(x + iy) = u + iv$, prove that $\frac{u}{v} = \frac{\sin 2x}{\sinh 2y}$.

6. Sum to infinity the series $c \sin \alpha + \frac{c^2}{2!} \cos 2\alpha + \frac{c^3}{3!} \cos 3\alpha + \ldots$

7. Transform the equation $6x^2 + 24xy - y^2 = 0$ into another in which the xy term is absent.

8. The asymptotes of a hyperbola are parallel to the lines $2x + 3y = 0$ and $3x - 2y = 0$ and its centre is at (1, 2). Find its equation if it passes through the point (5, 3). Find also the equation to the conjugate hyperbola.

Unit III

9. Expand $\sin x$ as an infinite series.

10. Evaluate $\lim_{x \to \frac{\pi}{4}} (\tan x)^{\tan 2x}$.

11. Find the maxima and minima of the function $x^3 - 18x^2 + 96x + 4$.

12. Find the radius of curvature of the curve $x^{1/2} + y^{1/2} = 1$ at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$.

Turn over

Unit IV

13. Find the area lying above the x-axis and included between the circle $x^2 + y^2 - 2x = 0$ and the parabola $y^2 = x$.

14. Find the area of the surface formed by the rotation of the curve $y^2 = 8x$ about the x-axis from $x = 2$ to $x = 7$.

15. Evaluate $\iint (x^2 + y^2)\, dx\, dy$ over the region for which $x \geq 0$, $y \geq 0$ and $x + y \leq 1$.

16. Evaluate $\int_0^{\pi/2} \sqrt{\tan \theta}\, d\theta$.

Unit V

17. Show that $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal.

18. Test for consistency and solve :

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

19. Find the eigen vectors of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$.

20. If $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$, find A^4 using Cayley-Hamilton theorem.