

R 3419

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2007.

Third Semester

(Regulation 2004)

Civil Engineering

MA 1201 — MATHEMATICS — III

(Common to all branches of B.E./B.Tech. Except Bio-Medical Engineering)
(Common to B.E. (Part-Time) Second Semester Regulation 2005)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Form a partial differential equation by eliminating the arbitrary constants a and b from the equation $(x - a)^2 + (y - b)^2 = z^2 \cot^2 \alpha$.
2. Find the complete solution of the partial differential equation $p^2 + q^2 - 4pq = 0$.
3. If

$$f(x) = \begin{cases} \cos x & \text{if } 0 < x < \pi, \\ 50, & \text{if } \pi < x < 2\pi, \end{cases}$$

and $f(x) = f(x + 2\pi)$ for all x , find the sum of the Fourier series of $f(x)$ at $x = \pi$.

4. Find the coefficient b_5 of $\cos 5x$ in the Fourier cosine series of the function $f(x) = \sin 5x$ in the interval $(0, 2\pi)$.
5. Solve the equation $3 \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0$, given that $u(x, 0) = 4e^{-x}$ by the method of separation of variables.

6. Write the one dimensional wave equation with initial and boundary conditions in which the initial position of the string is $f(x)$ and the initial velocity imparted at each point x is $g(x)$.
7. Write the Fourier transform pair.
8. Find the Fourier sine transform of $f(x) = e^{-ax}$ ($a > 0$).
9. Express $Z\{f(n+1)\}$ in terms of $\bar{f}(z)$.
10. Find the value of $Z\{f(n)\}$ when $f(n) = na^n$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the equation (8)

$$(x^2 - y^2 - z^2)p + 2xyq = 2zx.$$

- (ii) Solve (8)

$$(D^2 - DD' - 2(D')^2)z = 2x + 3y + e^{3x+4y}.$$

Or

- (b) (i) Solve (8)

$$z^2(p^2 + q^2) = x^2 + y^2.$$

- (ii) Solve (8)

$$(D^2 + 3DD' - 4(D')^2)z = x + \sin y.$$

12. (a) (i) Find the Fourier series of $f(x) = x^2$ in $0 < x < 2\pi$ and $f(x) = f(x + 2\pi)$ for all x . (8)

- (ii) By finding the Fourier cosine series for $f(x) = x$ in $0 < x < \pi$, show that $\frac{\pi^4}{96} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^4}$. (8)

Or

- (b) (i) Obtain the Fourier series of $f(x)$ of period $2l$ given
- $$f(x) = \begin{cases} l-x, & 0 \leq x < l \\ 0, & l \leq x < 2l. \end{cases} \quad (8)$$

- (ii) Find the complex form of the Fourier series of the function $f(x) = e^x$ when $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$. (8)

13. (a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $\lambda x(l-x)$, find the displacement $y(x,t)$ of the string at any time t and at a distance x from the zero end. (16)

Or

- (b) A rectangular plate is bounded by the lines $x=0, y=0, x=a, y=b$. Its surfaces are insulated. The temperatures along $x=0$ and $y=0$ are kept at 0°C and the others at 100°C . Find the steady state temperature at any point of the plate. (16)

14. (a) (i) Find the Fourier transform of $f(x)$ defined by

$$f(x) = \begin{cases} 1, & |x| < a, \\ 0, & |x| > a, \end{cases}$$

and hence find the value of $\int_0^{\infty} \frac{\sin x}{x} dx$. (8)

- (ii) By finding the Fourier cosine transform of $f(x) = e^{-ax}$ ($a > 0$) and using Parseval's identity for cosine transform evaluate. (8)

$$\int_0^{\infty} \frac{dx}{(a^2 + x^2)^2}$$

Or

- (b) (i) Find the Fourier transform of $f(x)$ defined by (10)

$$f(x) = \begin{cases} 1 - |x|, & \text{if } |x| < 1, \\ 0, & \text{if } |x| > 1, \end{cases}$$

and hence prove that

$$\int_0^{\infty} \left(\frac{\sin x}{x} \right)^4 dx = \frac{\pi}{3}.$$

- (ii) If $F_c(f(x)) = F_c(s)$, prove that $F_c(F_c(x)) = f(s)$. (6)

(In the above F_c stands for the Fourier cosine transform)

15. (a) (i) Find the Z-transform of (8)

$$f(n) = \frac{2n + 3}{(n + 1)(n + 2)}.$$

- (ii) Using convolution theorem find (8)

$$Z^{-1} \left(\frac{z^2}{(z + a)^2} \right)$$

Or

- (b) (i) Solve the difference equation

$$y(n + 3) - 3y(n + 1) + 2y(n) = 0$$

given that $y(0) = 4$, $y(1) = 0$ and $y(2) = 8$, by the method of Z-transform. (8)

- (ii) Find

$$Z^{-1} \left(\frac{z^2}{(z + 2)(z^2 + 4)} \right),$$

by the method of partial fractions. (8)