Code No: RR10102 **Set No. 1**

I B.Tech Supplimentary Examinations, Aug/Sep 2007
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering, Bio-Technology and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test for convergence of the series $\sum_{\infty}^{1}\left[\sqrt{n^4+1}\ -\ \sqrt{n^4-1}\right]$ [5]

   (b) State and prove Cauchy's Mean value theorem. [5]

   (c) If a < b prove that $\frac{b-a}{(1+b^2)} < tan^{-1}b - tan^{-1}a < \frac{b-a}{(1+a^2)}$ using Lagrange's Mean value theorem. Deduce the following [6]

      i. $\frac{\Pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\Pi}{4} + \frac{1}{6}$

      ii. $\frac{5\Pi+4}{20} < \tan^{-1}2 < \frac{\Pi+2}{4}$

2. (a) If z=log (e^x+e^y)
   show that rt-s^2 = 0 where $r = \frac{\partial^2 z}{\partial x^2}$, $t = \frac{\partial^2 z}{\partial y^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$

   (b) Determine the center of curvature to the curve in parametric form x = $3t^2$, y = 3t - t^3. [8+8]

3. (a) Trace the Folium of Decartes : $x^3 + y^3 = 3axy$.

   (b) Determine the volume of the solid generated by revolving the limacon r = a + b cosθ (a>b) about the initial line. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant xy = x log x − x + c. [3]

   (b) Solve the differential equation:
   (2y sinx + cosy) dx = (x sin y + 2 cos x + tan y) dy [7]

   (c) Radium decomposes at a rate proportional to the amount present at that time. If a fraction p of the original amount disappears in 1 year how much Radium will remain at the end of 21 years. [6]

5. (a) Solve the differential equation: $y'' + 4y' + 4y = 4\cos x + 3\sin x$, y(0) = 1, y′(0) = 0.

(b) Solve the differential equation: $\left((2x+3)^2 D^2 - (2x+3)\,D - 12\right)\, y = 6x$.
[8+8]

6. (a) Find $L^{-1}\left[\frac{s^2}{(s^4+4)(s^2+9)}\right]$ using convolution theorem.

(b) Show that $\int\limits_0^{4a} \int\limits_{y^2/4x}^{y} \frac{\left(x^2-y^2\right)\, dxdy}{(x^2+y^2)} = 8\,a^2\left(\frac{\pi}{2} - \frac{5}{3}\right)$ [8+8]

7. (a) Prove that $\nabla(\mathbf{A}.\mathbf{B})=(\mathbf{B}.\nabla)\mathbf{A}+(\mathbf{A}.\nabla)\mathbf{B}+\mathbf{B}\times(\nabla\times\mathbf{A})+\mathbf{A}\times(\nabla\times\mathbf{B})$.

(b) If $\phi = 2xy^2z + x^2y$, evaluate $\int\limits_C \phi$ d**r** where C is the curve $x = t, y = t^2, z = t^3$ from t=0 to t=1. [8+8]

8. Verify Stoke's theorem for **F** = (y–z+2)**i**+(yz+4) **j** - xz**k** where S is the surface of the cube x=0, y=0, z=0, x=2, y=2, z=2 above the xy-plane. [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the series
$1 + \left[\frac{1}{2}\right]^2 + \left[\frac{1.3}{2.4}\right]^3 + \left[\frac{1.3.5}{2.4.6}\right]^4 + \ldots.$ [5]
   (b) Examine whether the following series is absolutely convergent or conditionally convergent
$\frac{1}{5\sqrt{2}} - \frac{1}{5\sqrt{3}} + \frac{1}{5\sqrt{4}} - \ldots\ldots + (-1)^n \frac{1}{5\sqrt{n}} + \ldots.$ [5]
   (c) Verify Cauchy's mean value theorem for sin$x$ cos$x$ in (a,b) [6]

2. (a) Find the stationary points of the following function 'u' and find the maximum or the minimum
u = $x^2 + 2xy + 2y^2 + 2x + y$
   (b) Considering the evolute of a curve as the envelope of its normals, find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [8+8]

3. (a) Trace the curve r = a + b cos θ. (a > b).
   (b) Find the surface area got by rotating the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the minor axis. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant 'c':
$y = 1 + x^2 + c\sqrt{1 + x^2}$. [3]
   (b) Solve the differential equation:
$\frac{dy}{dx} + (y - 1)\cos x = e^{-\sin x}\cos^2 x.$ [7]
   (c) An object whose temperature is 75^0C cools in an atmosphere of constant temperature 25 ^{0}C at the rate kθ , θ being the excess temperature of the body over the atmosphere. If after 10 minutes the temperature of the objects falls to 65 ^{0}C . Find its temperature after 20 minutes. Find the time required to cool down to 55 ^{0}C. [6]

5. (a) Solve the differential equation: $(D^3 - 4D^2 - D + 4)y = e^{3x}$ cos 2x.

(b) Solve the differential equation: $(D^2+1)y =$ cosec x by variation of parameters method. [8+8]

6. (a) Prove that L $\left[\left[\frac{1}{t} f(t)\right]\right. = \int\limits_0^\infty \overline{f}(s)\, ds$ where L [f(t)] = $\overline{f}\,(s)$ [5]

   (b) Find the inverse Laplace Transformation of $\frac{3(s^2-2)^2}{2\,s^5}$ [6]

   (c) Evaluate $\int\int$ $(x^2 + y^2)$ dxdy over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [5]

7. Prove that $\mathbf{F}=(y^2 \cos x + z^3)i + (2y \sin x - 4)j + (3xz^2 + 2)k$ is a conservative force field. Find the work done in moving an object in this field from (0, 1, –1) to (π/2, –1, 2). [16]

8. Verify divergence theorem for $\mathbf{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ taken over the surface of the sphere $x^2+y^2+z^2= a^2$. [16]

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Code No: RR10102 **Set No. 3**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\sum_{n=1}^{\infty} n!\, \frac{2^n}{n^n}$ [5]
   (b) Show that the series $\frac{\sin x}{1} - \frac{\sin 2x}{2^2} + \frac{\sin 3x}{3^3} + \ldots..\infty$ converges absolutely. [5]
   (c) Show that $\log\,(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \ldots..$ and hence deduce that $\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \ldots..$ [6]

2. (a) If $u = \tan^{-1}(\mathrm{y}^2/\mathrm{x})$ show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin 2u \sin^2 u}{u}$
   (b) Find the shortest distance from origin to the surface $\mathrm{xyz}^2 = 2$. [8+8]

3. Trace the curve : r = a (1 + cos θ). Show that the volume of revolution of the curve about the initial line is $8\pi \mathrm{a}^3/\ 3$. [16]

4. (a) Form the differential equation by eliminating the arbitrary constant $\sin^{-1}(\mathrm{xy}) + 4\mathrm{x} = \mathrm{c}$. [3]
   (b) Solve the differential equation: (x+1) $\frac{dy}{dx}$ - xy = (x+1). [7]
   (c) Obtain the orthogonal trajectories of the semi cubical parabolas $\mathrm{ay}^2 = \mathrm{x}^3$. [6]

5. (a) Solve the differential equation: $y'' + 4y' + 20y = 23 \sin t - 15 \cos t$, $y(0) = 0$, $y'(0) = -1$
   (b) Solve the differential equation: $(2x+5)^2\,\frac{d^2y}{dx^2} + 6\,(2x+5)\,\frac{dy}{dx} + 8y = 4(2x+5)$ [8+8]

6. (a) Prove that $L\left[\int_0^t f(u)\,du = \frac{\overline{f}(s)}{s}\right]$, where L{f(t)}= $\overline{f}$ (s). [5]
   (b) Find $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$ [6]
   (c) Evaluate $\int\int$ r sinθ drdθ over the cardioid r = a(1 - cosθ) above the initial line. [5]

7. (a) Evaluate $\nabla.[\mathrm{r}\nabla(1/\mathrm{r}^3)]$ where $r = \sqrt{x^2+y^2+z^2}$

(b) Evaluate $\iint_s \mathbf{A}.\mathbf{n}$ ds where $\mathbf{A}$=18zi-12j+3yk and s is that part of the plane 2x+3y +6z=12 which is located in the first octant. [8+8]

8. State Green's theorem and verify Green's theorem for $\oint_C [(xy + y^2)dx + x^2dy]$, where C is bounded by y = x and y = x^2. [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series
$\frac{x}{1} + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{x^5}{5} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{x^7}{7} + \ldots\ldots$ (x>0). [5]

   (b) Examine whether the following series is absolutely convergent or conditionally convergent $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots (x > 0)$ [5]

   (c) Write the Maclaurin's series with Lagrange's form of remainder for f(x) = cos$x$. [6]

2. (a) If u = f(r,s,t) where r=x/y, s=y/z and t=z/x
show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

   (b) State and prove the necessary and sufficient conditions for extrema of a function 'f' of two variables. [8+8]

3. Trace the lemniscate of Bernouli : $r^2 = a^2 \cos 2\theta$. Prove that the volume of revolution about the initial line is $\frac{\pi a^3}{6\sqrt{2}} \left[3 \ \log\left(\sqrt{2} + 1\right) - \sqrt{2}\right]$ [16]

4. (a) Form the differential equation by eliminating the arbitrary constant : log y/x = cx. [3]

   (b) Solve the differential equation: dr + (2r cotθ + sin2θ) dθ = 0. [6]

   (c) The number N of bacteria in a culture groups at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after 1 ½ hour. [6]

5. (a) Solve the differential equation: $(D^2 - 6D + 13)$y = $8e^{3x}$ sin 2x.

   (b) Solve the differential equation: $(x^2D^2 + 2xD - 2)y = (x + 1)^2$. [8+8]

6. (a) Find the Laplace transformation of $e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t$. [5]

   (b) Find $L^{-1}\left[\frac{s^3 - 3s^2 + 6s - 4}{(s^2 - 2s + 2)^2}\right]$ [6]

(c) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$ [5]

7. (a) If ω is constant vector, evaluate curl **V** where $\mathbf{V}=\omega\times\mathbf{r}$.
   (b) Evaluate $\int_c \mathbf{F}.d\mathbf{r}$ where $\mathbf{F}=(x-3y)\mathbf{i}+(y-2x)\mathbf{j}$ and c is the closed curve in the xy-plane, x=2cost, y=3sint from t=0 to t=2π. [8+8]

8. Verify divergence theorem for $\mathbf{F} = 2xz\mathbf{i} + yz\mathbf{j} + z^2\,\mathbf{k}$ over upper half of the sphere $x^2+y^2+z^2=a^2$. [16]

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