

II Semester B.A./B.Sc. Examination, June 2008
(Semester Scheme)
Paper - II : MATHEMATICS

Time : 3 Hours

Max. Marks : 90

Instructions : i) Answer all questions.

ii) Answers should be written completely either in **English** or in **Kannada**.

I. Answer any 15 of the following :

(15×2=30)

- 1) If A is any square matrix and A' is its transpose then prove that A and A' have the same eigen values.
- 2) Find the value of λ for which the system $7x + 4y + 3z = 0$, $x + 2y + \lambda z = 0$, $x + 3y + 2z = 0$ has a non-trivial solution.
- 3) Show that 0 is the characteristic root of a matrix iff the matrix is singular.

4) Find the eigen values of $\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$.

5) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{pmatrix}$.

6) For the curve $r = a(1 + \cos \theta)$ at $\theta = \frac{\pi}{3}$ find the polar subtangent.

7) For the curve $x = a \cos t$, $y = b \sin t$, find $\frac{ds}{dt}$.

8) Find the angle between the radius vector and the tangent for the curve

$$r^2 = a^2 \cos 2\theta \text{ at } \theta = \frac{\pi}{2}.$$



- 9) For the curve $r = a\theta$, show that $p = \frac{r^2}{\sqrt{r^2 + a^2}}$.
- 10) Show that $y = e^x$, $x > 0$, is concave upwards.
- 11) Find the envelope of the family of curves $x^2 + y^2 - 2gx + g^2 - c^2 = 0$ where g is a parameter.
- 12) Find the asymptotes parallel to the coordinate axes for the curve $y^2(x^2 - a^2) = x$.
- 13) Define node and cusp of a given curve.
- 14) Find the area of the loop of the curve $3ay^2 = x(a - x^2)$.
- 15) Find the volume generated by an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about y-axis.
- 16) Solve $x \frac{dy}{dx} + y = x^2$.
- 17) Solve $(ax + hy + g) dx + (hx + by + f) dy = 0$.
- 18) Solve $p^2 - 5p + 6 = 0$ where $p = \frac{dy}{dx}$.
- 19) Solve $1 + \frac{dy}{dx} = e^{x+y}$.
- 20) Find the general solution of $y = px + \frac{ap}{\sqrt{1+p^2}}$.

II. Answer any three :

(3×5=15)

- 1) Find the inverse of matrix A by elementary transformations where

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$



2) Reduce the matrix $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$ to normal form and hence find the rank.

3) For what values of λ and μ the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have 1 no solution 2 a unique solution 3 infinite number of solutions.

4) State and prove Cayley Hamilton theorem.

5) Show that the following system of equations $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$ are consistent and solve.

III. Answer any two : (2×5=10)

1) With usual notation, prove that $\tan \phi = r \frac{d\theta}{dr}$.

2) Show that $r = a(1 + \sin \theta)$ and $r = b(1 - \sin \theta)$ intersect orthogonally.

3) Find the pedal equation of the curve $r^n = a^n \cos n\theta$.

4) Show that the evolute of parabola $y^2 = 4ax$ is $4(x - 2a)^3 = 27ay^2$.

IV. Answer any two : (2×5=10)

1) Find the asymptotes of the curve $4x^2(y - x) + y(y - 2)(x - y) - 4x - 4y + 7 = 0$.

2) Find the position and nature of the double points on the curve

$$x^3 + x^2 + y^2 - x - 4y + 3 = 0.$$

3) Find the envelope of family of lines $\frac{x}{a} + \frac{y}{b} = 1$ where $a \cdot b = c^2$.

4) Trace the curve $r = a(1 + \cos \theta)$.



V. Answer any two :

(2×5=10)

- 1) Find the perimeter of the cardioid $r = a(1 + \cos\theta)$.
- 2) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta - \sin\theta)$ $y = a(1 - \cos\theta)$ about the axis.
- 3) Find the area of the Astroid $x = a\cos^3\theta$ $y = a\sin^3\theta$.

VI. Answer any three :

(3×5=15)

- 1) Solve $(x^2 + y^2) dx = 2xydy$.
- 2) Solve $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$.
- 3) Find the orthogonal trajectories of $r^n = a^n \sin n\theta$.
- 4) Solve $(px - y)(py + x) = a^2p$ using transformation $x^2 = u$, $y^2 = v$.

ಕನ್ನಡ ರೂಪಾಂತರ

I. ಯಾವುದಾದರೂ 15 ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿ :

(15×2=30)

- 1) A ಒಂದು ಕೋಶವಾಗಿದ್ದು, A' ಅದರ ಟ್ರಾನ್ಸ್‌ಪೋಸ್ ಕೋಶವಾದರೆ A ಮತ್ತು A' ಕೋಶಗಳು ಒಂದೇ ಐಗನ್ ಬೆಲೆಗಳನ್ನು ಹೊಂದಿರುತ್ತವೆ ಎಂದು ಸಾಧಿಸಿ.
- 2) $7x + 4y + 3z = 0$, $x + 2y + \lambda z = 0$, $x + 3y + 2z = 0$ ಸಮೀಕರಣಗಳ ವ್ಯವಸ್ಥೆ ಅಕ್ಷದ್ವಯ ಪರಿಹಾರಗಳನ್ನು ಹೊಂದಿದ್ದಲ್ಲಿ λ ಬೆಲೆಯನ್ನು ಕಂಡುಹಿಡಿಯಿರಿ.
- 3) 'O' ಒಂದು ಕೋಶದ ಕ್ಯಾಂಪ್ಲೆಕ್ಸ್ ಪರಿಹಾರ ಆಗಿದ್ದರೆ ಕೋಶವು ಸಿಂಗ್ಯುಲರ್ ಮತ್ತು ಇದರ ವಿಲೋಮವನ್ನು ಸಾಧಿಸಿ.
- 4) $\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$ ಕೋಶದ ಐಗನ್ ಬೆಲೆಗಳನ್ನು ಕಂಡುಹಿಡಿಯಿರಿ.
- 5) $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{pmatrix}$ ಕೋಶದ ದರ್ಜೆಯನ್ನು ಕಂಡುಹಿಡಿಯಿರಿ.