

**GUJARAT UNIVERSITY**  
**B.E. Sem VI (CE) Examination**  
**Theory of Computation**

Wednesday, 18th June, 2008]

[Time : 3 Hours  
 Max. Marks : 100

- Instructions :** (1) Attempt all questions.  
 (2) Answer to the two sections must be written in **separate** answer books.  
 (3) Figures to the right indicate **full** marks.  
 (4) Assume suitable data if necessary.  
 (5) Answer with neat sketches/block diagrams/flow charts, as required.

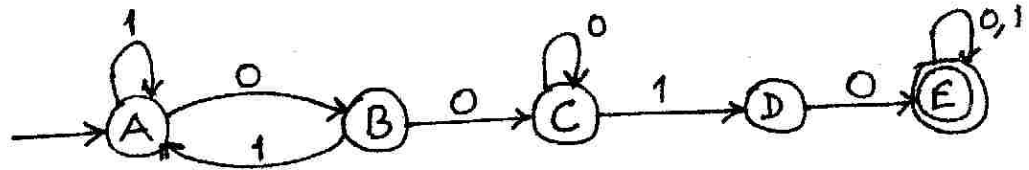
**SECTION I**

- 1 Do as directed (Any Six of the following Eight) (18)
- a. Describe following infinite set precisely using a formula that does not involve "...".  
 If you wish, you can use standard set notations such as N, R, Z, etc.  
 $\{1/2, 1/4, 3/4, 1/8, 3/8, 5/8, 7/8, 1/16, 3/16, 5/16, 7/16, 9/16, 11/16, 13/16, 15/16, \dots\}$
- b. Suppose A is a set having n elements.  
 i. How many relations are there on A?  
 ii. How many symmetric relations are there on A?
- c. Prove by using Mathematical Induction that for  $n \geq 0$ ,
- $$\sum_{i=1}^n i^2 = n(n+1)(2n+1)/6$$
- Hence deduce that 6 divides  $n(n+1)(2n+1)$  for any  $n \geq 0$ .
- d. Prove the formula  
 $1^* 1^* = 1^*$   
 and hence derive that  
 $1^* (1 + \wedge) = 1^*$
- e. Draw NFA -  $\wedge$  for the following regular expression over  $\{0,1\}$ :  
 $(0 + 1) (01)^* (011)^*$
- f. Find a string of minimum length in  $\{0,1\}^*$  NOT in the language corresponding to the following regular expression:  
 $(0^* + 1^*) (0^* + 1^*) (0^* + 1^*)$
- g. Consider the two regular expressions over  $\{0,1\}$ :  
 $r = 0^* + 1^*$                        $s = 01^* + 10^* + 1^*0 + (0^*1)^*$   
 Find a string corresponding to r but not to s.
- h. Prove that for every  $n \geq 1$  and every  $m \geq 1$ , the number of functions from  $\{1, 2, \dots, n\}$  to  $\{1, 2, \dots, m\}$  is  $m^n$ .
- 2 a. Find a regular expression corresponding to the following subset of  $\{0,1\}^*$ . (4)  
 "The language of all strings in which the number of 0's is even."

[P.T.O.]

# 186M108-2

- b. Write a regular expression that causes the following FA to be in each state: (4)



- c. For the following regular expression, draw an FA recognizing the corresponding language. (4)

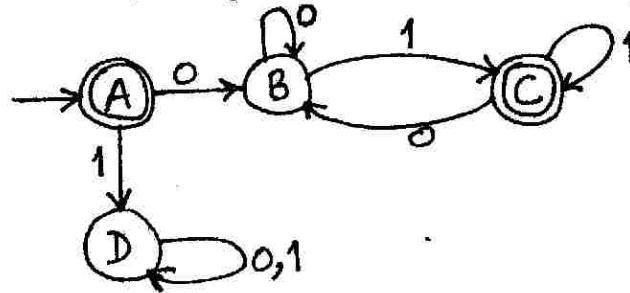
$$(0 + 1)^* (01 + 110)$$

- d. Suppose  $M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$  and  $M_2 = (Q_2, \Sigma, q_2, A_2, \delta_2)$  accept languages  $L_1$  and  $L_2$ , respectively. Define  $M = (Q, \Sigma, q_0, A, \delta)$ , which accepts the language  $L_1 \cup L_2$ . (4)

OR

- 2 a. Find a regular expression corresponding to the following subset of  $\{0,1\}^*$  "The language of all strings in which every 0 is followed immediately by 11." (4)

- b. Write a regular expression that causes the following FA to be in each state: (4)



- c. For the following regular expression, draw an FA recognizing the corresponding language: (4)

$$0 + 10^* + 01^*0$$

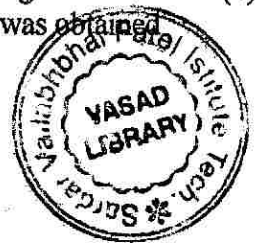
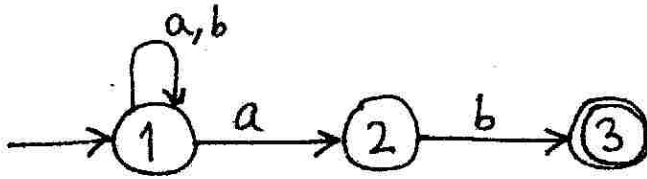
- d. Suppose  $M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$  and  $M_2 = (Q_2, \Sigma, q_2, A_2, \delta_2)$  accept language  $L_1$  and  $L_2$ , respectively. Define  $M = (Q, \Sigma, q_0, A, \delta)$  which accepts the language  $L_1 - L_2$  (4)

- 3 a. A transition table is given for an NFA -  $\wedge$  with seven states (4)

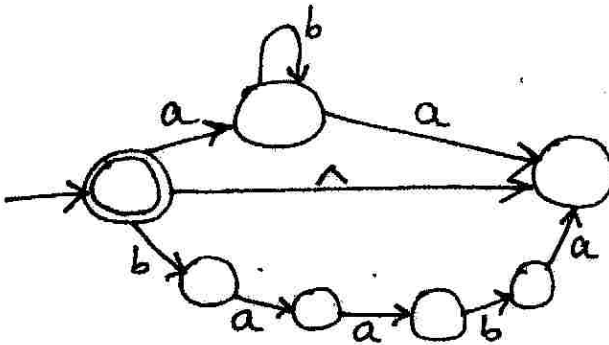
q	$\delta(q, a)$	$\delta(q, b)$	$\delta(q, \wedge)$
1	$\emptyset$	$\emptyset$	{2}
2	{3}	$\emptyset$	{5}
3	$\emptyset$	{4}	$\emptyset$
4	{4}	$\emptyset$	{1}
5	$\emptyset$	{6,7}	$\emptyset$
6	{5}	$\emptyset$	$\emptyset$
7	$\emptyset$	$\emptyset$	{1}

Find  $\delta^*(1, ab)$

- b. Using the subset construction, draw an FA accepting the same language as the following NFA. Label the final picture so as to make it clear how it was obtained from the subset construction. (4)



- c. For the following NFA -  $\wedge$ , find a regular expression corresponding to the language it recognizes: (4)



- d. For the following regular expression over  $\{0,1\}$ , draw an NFA -  $\wedge$  recognizing the corresponding language: (4)  
 $(0 + 1)^* (011 + 01010) (0 + 1)^*$

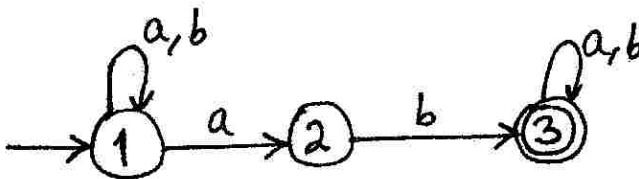
OR

- 3 a. A transition table is given for NFA -  $\wedge$  with seven states (4)

q	$\delta(q, a)$	$\delta(q, b)$	$\delta(q, \wedge)$
1	{5}	$\emptyset$	{4}
2	{1}	$\emptyset$	$\emptyset$
3	$\emptyset$	{2}	$\emptyset$
4	$\emptyset$	{7}	{3}
5	$\emptyset$	$\emptyset$	{1}
6	$\emptyset$	{5}	{4}
7	{6}	$\emptyset$	$\emptyset$

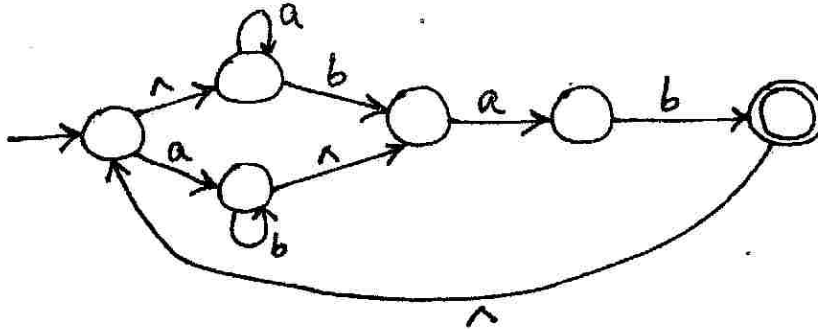
Calculate  $\delta^*(1, ba)$

- b. Using the subset construction, draw an FA accepting the same language as the following NFA. Label the final picture so as to make it clear how it was obtained from the subset construction (4)



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- c. For the following NFA -  $\wedge$ , find a regular expression corresponding to the language it recognizes (4)



- d. For the following regular expression over  $\{0,1\}$ , draw an NFA -  $\wedge$  recognizing the corresponding language (4)

$$010^* + 0(01 + 10)^* 11$$

### SECTION - II

- 4 a. What language is generated by the context-free grammar (CFG) with the following production: (4)

$$S \rightarrow aSa \mid bSb \mid aAb \mid bAa \\ A \rightarrow aAa \mid bAb \mid a \mid b \mid \wedge$$

- b. For the following context-free grammar (CFG)  $G$ , find a CFG  $G'$  in Chomsky normal form generating (4)

$$L(G) = \{\wedge\}.$$

$$G \text{ has productions } S \rightarrow SS \mid (S) \mid \wedge$$

- c. Show that the following grammar is ambiguous, and find an equivalent unambiguous grammar: (4)

$$S \rightarrow SS \mid a \mid b$$

- d. Find the CFG generating the following language: (4)

$$L = \{a^i b^j c^k \mid i = j + k\}$$

### OR

- 4 a. What language is generated by the CFG with the following production: (4)

$$S \rightarrow aT \mid bT \mid \wedge \\ T \rightarrow aS \mid bS$$

- b. For the following CFG  $G$ , find a CFG  $G'$  in Chomsky's normal form generating (4)

$$L(G) = \{\wedge\}.$$

$$G \text{ has productions } S(S) \mid \wedge$$

- c. Show that the following grammar is ambiguous, and find an equivalent unambiguous grammar: (4)

$$S \rightarrow aSb \mid aaSb \mid \wedge$$

- d. Find the CFG generating the following language: (4)

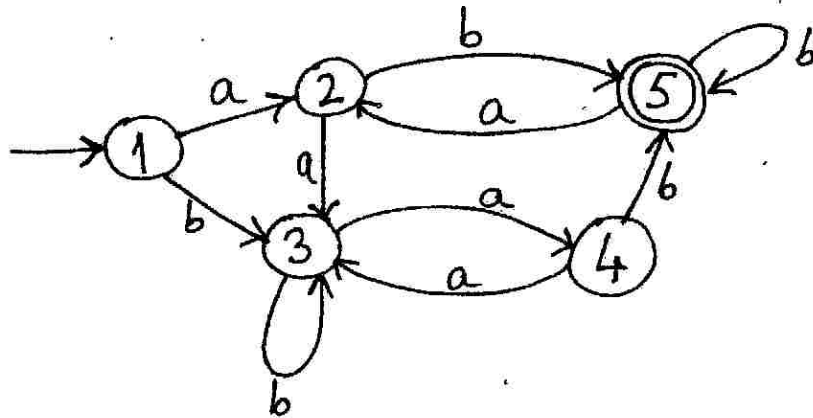
$$L = \{a^i b^j \mid i < 2j\}$$

- 5 a. Give transition tables for Push Down Automata (PDA) recognizing the following language (5)

$$L = \text{The language of all non-palindromes over } \{a, b\}.$$

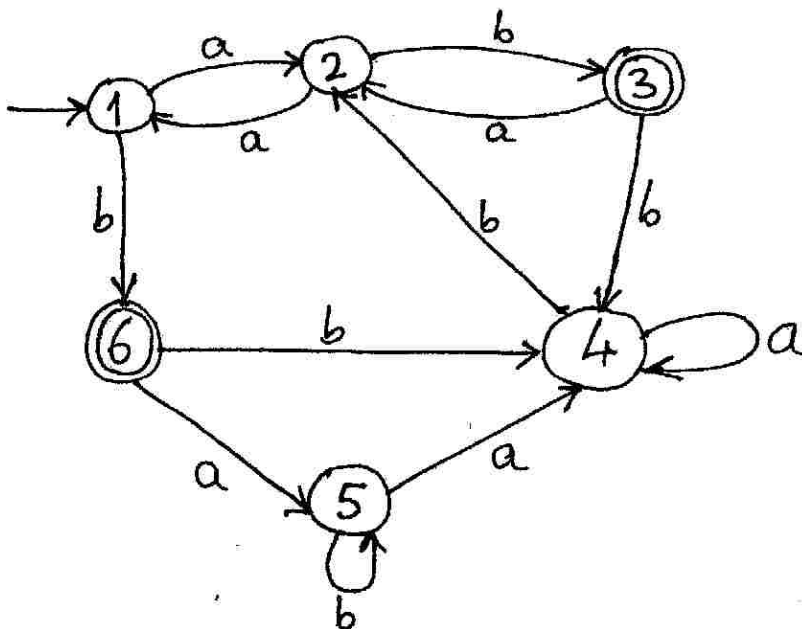


- b. Prove that  $L$  (as given below) is non-regular by showing that any two elements of the infinite set  $\{0^n \mid n \geq 0\}$  are distinguishable with respect to  $L = \{0^n 1 0^{2n} \mid n \geq 0\}$  (5)
- c. For the following FA, use the minimization algorithm to find a minimum-state FA recognizing the same language. (6)



OR

- 5 a. Give transition table for PDA recognizing the following language (5)  
 $L = \{x \in \{a, b, c\}^* \mid n_a(x) < n_b(x) \text{ or } n_a(x) < n_c(x)\}$
- b. Prove that  $L$  (as given below) is non-regular by showing that any two elements of the infinite set  $\{0^n \mid n \geq 0\}$  are distinguishable with respect to  $L = \{0^i 1^j \mid j = i \text{ or } j = 2i\}$  (5)
- c. For the following FA, use the minimization algorithm to find a minimum-state FA recognizing the same language (6)



[P.T.O.]

## 186M108-6

6 Write short notes on ANY SIX of the following:

(18)

- a. Universal Turing Machine
- b. Recursively Enumerable languages
- c. Definition of Primitive Recursive Function and its examples
- d. Chomsky's hierarchy of grammars (Type 0, 1, 2 and 3). Give salient comparative characteristics also.
- e. Halting Problem
- f. NP Complete Problems
- g. Decision Problems and Algorithms
- h. Parsing
- i. Pumping Lemma for Context-Free Language (CFL)