CS-601

BACHELOR IN COMPUTER APPLICATIONS

Term-End Examination

June, 2007

CS-601 : DIFFERENTIAL AND INTEGRAL CALCULUS WITH APPLICATIONS

Time : 2 hours *Maximum Marks : 60*

Note : *Question number 1 is* ***compulsory****. Answer any* ***three*** *questions from the rest.*

1. (a) State whether the following statements are true or false. Give reasons for your answers. *5×2=10*

(i) $f(x) = |x|$ is differentiable in $[2, 6]$.

(ii) $\frac{d}{dx}\left[\int_3^{x^2} \frac{1}{t}\, dt\right] = \frac{1}{x^2} - \frac{1}{9}$

(iii) If $|x - 5| \leq 2$, then $x \in]3, 7[$.

(iv) A constant function is continuous.

(v) The equation of the tangent to $y = x(x - 1)$ at $(1, 0)$ is $y = x - 1$.

(b) If $y = (\sin^{-1} x)^2$ prove that
$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0$ for each positive integer 'n'. 5

(c) Prove that the function $x - \sin x + \cos x$ is increasing on $[0, \pi/4]$ and decreasing on $[\pi/4, \pi/2]$. 3

(d) Integrate :

$$\int \sin 4x \cos 3x \, dx$$

3

(e) Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. x. 4

(f) Evaluate $\int_0^6 2^x \, dx$ by dividing [0, 6] into 6 equal parts by Simpson's rule. Find the exact value of the integral and hence find the approximate value of *ln* 2. 5

2. (a) Find fog and gof, where f(x) = tan x and g(x) = 5. 3

(b) If $y = x + \sqrt{x^2 - 1}$, show that
$(x^2 - 1) y_2 + x y_1 - y = 0$ 3

(c) Find the area between the parabolas $y^2 = 8x$ and $x^2 = 8y$. 4

3. (a) Find the angle of intersection to the parabola $y^2 = 2x$ and the circle $x^2 + y^2 = 8$. *5*

(b) $\int \frac{t+1}{2t^2+5t+2}\,dt$ *5*

4. (a) Prove that $e^x > 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$ for all $x > 0$. *3*

(b) Determine the range of values of x at which $f(x) = 2x^3 - 15x^2 + 36x + 5$ is

(i) increasing,

(ii) decreasing. *4*

(c) Evaluate $\int_0^{\ln 2} \frac{e^x\,dx}{e^{2x}+3e^x+2}$ *3*

5. (a) Explain the geometrical properties : symmetry, maxima, minima, extent, tangents, asymptotes of the curve $y^2 = x^2(1 - x^2)$. *6*

(b) Find the volume of the solid generated by revolving the region between the curves, $y = x$ and $y = \sin x$ in $[0, \pi/2]$, about the x-axis. *4*