Reg. No. | | | | | | | | | | | |

# Question Paper Code: **E3120**

**B.E./B.Tech. DEGREE EXAMINATIONS, MAY/JUNE 2010**
**Regulations 2008**

Second Semester

Common to all branches

**MA2161 Mathematics II**

Time: Three Hours | Maximum: 100 Marks

Answer ALL Questions

Part A - (10 x 2 = 20 Marks)

1. Transform the equation $x^2y'' + xy' = x$ into a linear differential equation with constant coefficients.
2. Find the particular integral of $(D^2 + 1)\, y = \sin x$.
3. Is the position vector $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ irrotational? Justify.
4. State Gauss divergence theorem.
5. Verify whether the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic.
6. Find the constants $a, b, c$ if $f(z) = x + ay + i(bx + cy)$ is analytic.
7. What is the value of the integral $\int_C \left(\frac{3z^2 + 7z + 1}{z + 1}\right) dz$ where $C$ is $|z| = \frac{1}{2}$?
8. If $f(z) = \frac{-1}{z-1} - 2\left[1 + (z-1) + (z-1)^2 + \cdots\right]$, find the residue of $f(z)$ at $z = 1$.
9. Find the Laplace transform of unit step function.
10. Find $L^{-1}\left\{\cot^{-1}(s)\right\}$.

Part B - (5 x 16 = 80 Marks)

11. (a) (i) Solve the equation $(D^2 + 4\,D + 3)\,y = e^{-x}\,\sin\,x$. (8)

(ii) Solve the equation $(D^2+1)\,y = x\,\sin\,x$ by the method of variation of parameters. (8)

OR

11. (b) (i) Solve $(x^2D^2 - 2xD - 4)\,y = x^2 + 2\log x$. (8)

(ii) Solve : (8)

$$\frac{dx}{dt} + 2x + 3y = 2e^{2t},$$
$$\frac{dy}{dt} + 3x + 2y = 0.$$

12. (a) (i) Prove that $\vec{F} = (6xy + z^3)\,\vec{i} + (3x^2 - z)\,\vec{j} + (3xz^2 - y)\,\vec{k}$ is irrotational vector and find the scalar potential such that $\vec{F} = \nabla\varphi$. (8)

(ii) Verify Green's theorem for $\int_C (3x^2-8y^2)\,dx+(4y-6xy)\,dy$ where $C$ is the boundary of the region defined by $x = y^2$, $y = x^2$. (8)

OR

12. (b) Verify Gauss-divergence theorem for the vector function $\vec{f} = (x^3 - yz)\hat{i} - 2x^2y\hat{j} + 2\hat{k}$ over the cube bounded by $x = 0,\ y = 0,\ z = 0$ and $x = a,\ y = a,\ z = a$. (16)

13. (a) (i) Prove that every analytic function $w = u + iv$ can be expressed as a function of $z$ alone, not as a function of $\bar{z}$. (8)

(ii) Find the bilinear transformation which maps the points $z = 0, 1, \infty$ into $w = i, 1, -i$ respectively. (8)

OR

13. (b) (i) Find the image of the hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$. (8)

(ii) Prove that the transformation $w = \frac{z}{1-z}$ maps the upper half of $z$-plane on to the upper half of $w$-plane. What is the image of $|z| = 1$ under this transformation? (8)

14. (a) (i) Find the Laurent's series of $f(z) = \frac{7z-2}{z(z+1)(z+2)}$ in $1 < |z+1| < 3$. (8)

(ii) Using Cauchy's integral formula, evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)}\,dz$, where '$C$' is the circle $|z| = \frac{3}{2}$. (8)

OR

14. (b) (i) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\, dx$ using contour integration. (8)

(ii) Evaluate $\int_{0}^{2\pi} \frac{d\theta}{2 + \cos\theta}$ using contour integration. (8)

15. (a) (i) Apply convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$. (8)

(ii) Find the Laplace transform of the following triangular wave function given by

$$f(t) = \begin{cases} t, & 0 \le t \le \pi \\ 2\pi - t, & \pi \le t \le 2\pi \end{cases}$$

and $f(t + 2\pi) = f(t)$. (8)

OR

15. (b) (i) Verify initial and final value theorems for the function $f(t) = 1 + e^{-t}(\sin t + \cos t)$. (8)

(ii) Using Laplace transform solve the differential equation $y'' - 3y' - 4y = 2e^{-t}$ with $y(0) = 1 = y'(0)$. (8)

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