T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE 2011

Paper II

PART I

1	2	3	4	5	6	7	8
C	**D**	**B**	**A**	**D**	**C**	**A**	**B**

9	10	11	12
C, D	**A, B, C, D**	**A, B, D**	**A, C, D**

13	14	15	16	17	18
8	**6**	**4**	**7**	**6**	**8**

19	20
A – r, s, t	**A – p, r, s**
B – p, s	**B – r, s**
C – r, s	**C – t**
D – q, r	**D – p, q, r, t**

Section I

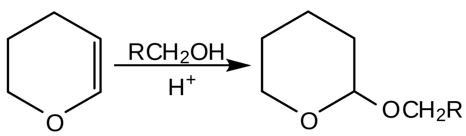
1. $[Co(NH_3)_6]Cl_3$, $Na_3[Co(ox)_3]$, $[K_2Pt(CN)_4]$ & $[Zn(H_2O)_6](NO_3)_2$ are diamagnetic.

2. $E_{cell} = E^0_{cell} + \frac{0.06}{4} \log \frac{P_{O_2} \times [H^+]^4}{[Fe^{2+}]^2}$

$= 1.67 + 0.015 \log \frac{0.1 \times (10^{-3})^4}{(10^{-3})^2}$

$= 1.67 - 0.015 \times 7$

$= 1.57$ V

3. 

4. CuS & HgS are insoluble in dilute mineral acids. Cu^{2+} & Hg^{2+} ions belong to group II of qualitative analysis.

5. Haematite is Fe_2O_3. Oxidation state of Fe is +3
Magnetite is Fe_3O_4. Oxidation state of Fe is +2 and +3.

6. Aromatic primary amines form diazonium salt with $NaNO_2$ and HCl at low temperature which couples with β–naphthol to form coloured azo dye.

7. $\Delta T_f = i \times K_f \times m$

$= 4 \times 1.86 \times \frac{0.1}{329} \times 10$

$= 0.023$

F.P $= -2.3 \times 10^{-2}$ °C

8. The structure given is that of β–D–glucose.

Section II

9. Cu^{+2} is reduced to Cu^{+1} by CN^- & SCN^-

10. In (B), (C) and (D), $X–(CH_2)_4–X$ is converted to a diamine which can form condensation polymer with adipic acid. In (A), $X–(CH_2)_4–X$ is converted to a diol which gives polyester.

11. $K = \frac{0.693}{t_{1/2}}$

K increases with increase of temperature and hence half life decreases.

$$t = \frac{t_{1/2}}{0.3}\log\frac{100}{0.4} = 8t_{1/2}$$

$R = R_0e^{-kt}$

12. $MnO_4^- \rightarrow Mn^{2+}$ (acid medium)
$MnO_4^- \rightarrow MnO_2$ (Neutral and aqueous mediums)

Section III

13. A truncated octahedrous has 8 hexagonal and 6 square faces. (36 edges and 24 vertices)

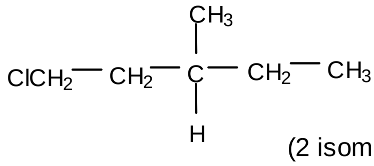
14. There are six C – H bonds that can involve in hyperconjugation.

15. $PCl_5 + SO_2 \rightarrow POCl_3 + SOCl_2$
$PCl_5 + H_2O \rightarrow POCl_3 + 2HCl$
$PCl_5 + H_2SO_4 \rightarrow SO_2Cl_2 + 2POCl_3 + 2HCl$
$6PCl_5 + P_4O_{10} \rightarrow 10POCl_3$

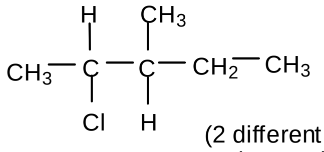
16. $[Cl^-]$ from CuCl = 10^{-3}
$K_{sp}(AgCl) = [Ag^+]\,[Cl^-]$
$[Ag^+] = \frac{1.6 \times 10^{-10}}{10^{-3}} = 1.6 \times 10^{-7}$

17. Millimoles of $Cl^- = 30 \times 0.01 \times 2$
$= 0.6$
Vol. of 0.1 M $AgNO_3 = \frac{0.6}{0.1} = 6$ mL

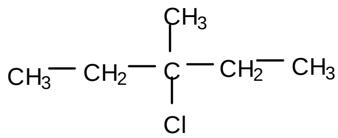
18.



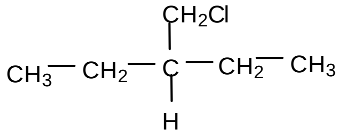
(2 isomers)



(2 different chiral carbon - 4 isomers)



(1 isomer)



(1 isomer)

Section IV

19. (A) is intramolecular aldol condensation
(B) involves Grignard reagent addition. To carbonyl compound.
(C) involves nucleophilic addition and dehydration.
(D) involves dehydration and intramolecular Friedel–Craft reaction.

20. (A) involves transition from solid to gas phase with the absorption of heat.
(B) is exothermic and a gaseous product is formed from a solid.
(C) involves association
(D) white phosphorous is converted to the polymeric red allotropic form as heating. Different solids are considered as different phases.

PART II

21	22	23	24	25	26	27	28
B	**B**	**C**	**A**	**C**	**D**	**C**	**A**

29	30	31	32
B, C	**A, C**	**C, D**	**A, B, D**

33	34	35	36	37	38
2	**7**	**5**	**5**	**4**	**4**

39	40
A – p, t	**A – p, r, t**
B – p, s	**B – p, r**
C – q, s	**C – q, s**
D – q, r	**D – r, t**

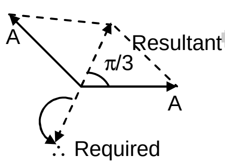
Section I

21. Escape KE = $\frac{GMm}{r}$

    and V = $\sqrt{\frac{GM}{r}}$

    Eliminate r

22. Phasor:



    A sin $\left(\omega t + \frac{4\pi}{3}\right)$

23. L.C = 0.01 mm
    diameter = 2.70 mm
    % error = $\frac{1}{2.70}$
    $\frac{\Delta\rho}{\rho} = \frac{3\Delta dia}{dia} + \frac{\Delta m}{m}$
    % error = $\frac{3}{2.70} + 2 = 3.1\%$

24. Take dr at r
    No. of turns = $\frac{N.dr}{(b-a)}$

    $-dB = \frac{\mu_0 N}{(b-a)} dr. \frac{i}{2r}$

    $B = \int dB = \frac{\mu_0 N.i}{2(b-a)} \ln\frac{b}{a}$

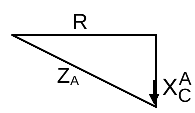
25. At θ_c, 100% reflected.
    (B or C) But C is correct.
    Θ At θ = 0, T + R = 100,
    R ≠ 0

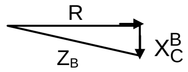
26. 0.01 V = 0.01 V' + 0.2 V''
    But time of fall = $\sqrt{\frac{2h}{g}} = 1$
    ∴ V' = 100, V'' = 20
    ∴ V = 500

27. If non-conservative induced electric field this pattern is possible.

28. T = $2\pi\sqrt{\frac{m}{K}}$ holds.
    At equilibrium position,
    $kx_0 = QE$
    ∴ Equilibrium shifted.

Section II

29. 

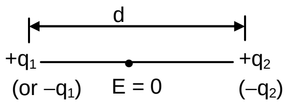


$X_C^B = \frac{1}{C\omega} < X_C^A$

$\Rightarrow Z_B < Z_A$

$\Rightarrow I_R^B > I_R^A$ and $V_C^A > V_C^B$

30. Linear momentum in the horizontal direction is zero before and after collision $\Rightarrow$ CM is stationary. for conservation of angular momentum, ω of ring increases after collision $\Rightarrow$ slip $\Rightarrow$ friction towards left.

31. 

$W_{agent} = \Delta U = q(V_2 - V_1)$

$= 1 \times (V_B - V_A)$

$= (V_B - V_A)$

32. $Vd_A + Vd_B = 2Vd_F = 0$

$\therefore\ d_A + d_B = 2d_F$

Obviously $d_A < d_F$ and $d_B > d_F$

Section III

33. $\frac{\frac{4}{3}}{v} - \frac{1}{-24} = \frac{\frac{7}{4}-1}{6} + \frac{\frac{4}{3}-\frac{7}{4}}{\infty}$

$\Rightarrow$ v = 16 from top surface of liquid

$\Rightarrow$ 2 cm from bottom surface of liquid

34. $h\upsilon = \frac{1242\ \text{eV nm}}{200\text{nm}} = 6.21$ eV

KE = 6.21 – 4.7 = 1.51 eV

$\frac{kq}{r} = 1.5\ \text{V}$ for stopping electrons.

$\therefore\ q = \frac{1.5 \times (10^{-2})}{9 \times 10^9} = Ne$

$\therefore\ N = 1 \times 10^7$

35. $T = \frac{2u \sin 60°}{g} = \frac{2 \times 10 \times \sqrt{3}}{10 \times 2} = \sqrt{3}$ s

$(u \cos\theta)\ T = \frac{1}{2} aT^2 + 1.15$

($u = 10$ m s^{-1})

$\Rightarrow a = 5$ m s^{-2}

36. 1st surface

$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$

$-\frac{1}{(-24)} + \frac{\left(\frac{7}{4}\right)}{V_1} = \frac{\left(\frac{7}{4} - 1\right)}{6}$

$\Rightarrow v_1 = 21$ cm

2nd surface

($\because R = \infty$ for 2nd surface)

$-\frac{\left(\frac{7}{4}\right)}{21} + \frac{4}{3 v_2} = 0$

$\Rightarrow v_2 = 16$ cm $\Rightarrow x = 18 - 16 = 2$ cm

37. $\frac{1}{2} mv^2 = \mu mg\ S + \frac{1}{2} kS^2$

m = 0.18 kg, μ = 0.1,

g = 10 m s^{-2}, S = 0.06 m

$\Rightarrow v^2 = 0.12 + 0.04 = 0.16$

$\Rightarrow v = \sqrt{0.16} = 0.4$ m s^{-1}

$= \frac{4}{10} \Rightarrow N = 4$

38. $Z^2 = R^2 + X_C^2$

$\Rightarrow X_C^2 = 1.25\ R^2 - R^2$

$= 0.25\ R^2$

$\Rightarrow X_C = 0.5\ R$

$\Rightarrow C = \frac{1}{X_C \omega} = \frac{1}{0.5 R \times 500} = \frac{1}{250 R}$

$\tau = RC = \frac{1}{250}$ s = 4 ms

Section IV

39. Knowledge based.

40. Knowledge based.

PART III

41	42	43	44	45	46	47	48
B	D	A	A	B	C	D	C

49	50	51	52
A, D	**C, D**	**A, B, D**	**A, B, C, D**

53	54	55	56	57	58
9	2	3	1	2	9

59	60
A – p, r, s	**A – q**
B – r, t	**B – p**
C – r	**C – s**
D – r	**D – s**

Section I

41. Let α denote the common root
$\alpha^2 + b\alpha - 1 = 0$
$\alpha^2 + \alpha + b = 0$
$b\alpha - 1 = \alpha + b$
$\alpha (b - 1) = b + 1$
$\alpha = \frac{b+1}{b-1}$
$(b + 1)^2 + b(b^2 - 1) - (b - 1)^2 = 0$
$4b + b(b^2 - 1) = 0$
$4 + b^2 = 0$
$b^2 = -3$
$b = \pm I\sqrt{3}$

42. Let the circle be
$x^2 + y^2 + 2gx + 2fy + c =$
Passed thro $(-1, 0) \longrightarrow -2g + c = -1 (1)$
Passes thro $(0, 2) \longrightarrow 4f + c = -4 \longrightarrow (2)$
$\sqrt{g^2 + f^2 - c} = g$
$f^2 - c = 0$
$f^2 = c$
$4f + f^2 = -4$
$(f + 2)^2 = 0 \Rightarrow f = -2$
$\Rightarrow c = -4 - 4f = 4$
$2g = 5, g = \frac{5}{2}$
Circles is $x^2 + y^2 + 5x - 4y + 4 = 0$
$(-4, 0)$ satisfies the equation

43. fogogo f(x)
$= fogog(x^2)$
$= fog(\sin(x)^2$
$= f(\sin(\sin(x^2)$
$= \sin^2(\sin(x^2)$
$gogof(x) = gog(x^2)$
$= g(\sin(x)^2$
$= \sin(\sin (x^2))$
$\therefore \sin^2(\sin(x^2) = \sin \sin (x^2)$
$\Rightarrow \sin(\sin(x^2) (\sin \sin(x^2) - 1) = 0$
$\Rightarrow \sin(\sin(x^2) = 0$ or $\sin (\sin(x^2) = 1$
$\Rightarrow \sin(x^2) = 0 \quad \sin (x^2)$
$\Rightarrow x^2 = n\pi$
$\therefore x = \pm \sqrt{n\pi} \quad n = \{0, 1, 2 \ldots\ldots\}$

44. for non-singular matrics, $\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \varpi^2 & \omega & 1 \end{vmatrix} \neq 0$
$\Rightarrow 1 - c\omega - a\omega - ac\omega^2 \neq 0$
$(1 - c\omega) (1 - c\omega) \neq 0$
$\Rightarrow a \neq \frac{1}{\omega} = \omega^2$ and $c \neq \frac{1}{\omega} = \omega^2$
$\therefore a = c = \omega$ and $b = \omega$ pr ω^2
Hence there are two such matrices.

45. Let the point be 'θ'
Normal at θ is
$\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$
Normal passes thro (9, 0)
$\frac{9a}{\sec\theta} = a^2 + b^2 — (1)$
$a \sec\theta = 6$
$\sec\theta = \frac{6}{a}$
(1) reduces

$9a \times \frac{a}{6} = a^2 + b^2$

$\frac{3a^2}{2} = a^2 + b^2$

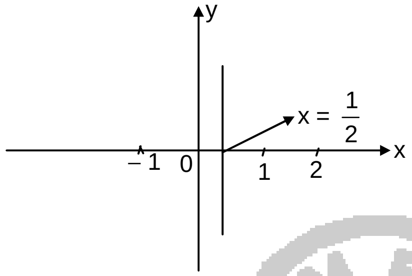
$= a^2 + a^2 (e^2 - 1)$

$\frac{3}{2} = 1 + e^2 - 1$

$= e^2$

$e = \sqrt{\frac{3}{2}}$

46.



f(x) = f(1 – x)

$\Rightarrow$ curve is symmetrical about $x = \frac{1}{2}$

$R_1 = \int_1^2 xf(x)\,dx$

$= \int_1^2 (-1+2-x)\,f(-1+2-x)\,dx$

$\int_1^2 (1-x)\,f(1-x)\,dx$

$= \int_{-1}^2 (1-x)\,f(x)\,dx$

$\int_{-1}^2 f(x)\,dx - \int_{-1}^2 xf(x)\,dx$

$= R_2 - R_1$

$2R_1 = R_2$

47. Which is of the form 1^α

$e^{\lim_{x\to 0}(f(x)-1)g(x)}$

$= e^{(1+x\log(1+b^2)-1)\frac{1}{x}}$

$= e^{\frac{x\log(1+b^2)}{x}}$

$= e^{\log(1+b^2)}$

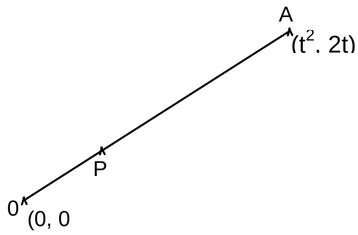
$\therefore 1 + b^2 = 2b \sin^2\theta$

$\sin^2\theta = \frac{1+b^2}{2b}$

Since b > 0

$\therefore \theta = \pm \frac{\pi}{2}$

48.



$\frac{OP}{PA} = \frac{1}{3}$

Let P be (x, y)

$X = \frac{t^2}{4}$, $Y = \frac{2t}{4}$

Lows of P is

$4X = 4Y^2$

$\Rightarrow Y^2 = X$

Section II

49. $P(E \cap \overline{F}) + P(F \cap \overline{E}) = \frac{11}{25}$

Also $P(\overline{E} \cap \overline{F}) = \frac{2}{25}$

But $P(\overline{E} \cap \overline{F}) = P(\overline{E} \cap \overline{F}) = P(\overline{E \cup F})$

$\therefore P(E \cap F) = 1 - \frac{2}{25} = \frac{23}{25}$

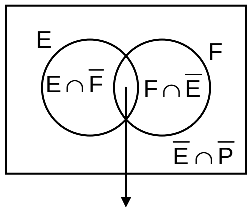
$P(E \cup F) = 1 - \frac{2}{25} = \frac{23}{25}$

$P(E \cup F) = P(E \cap \overline{F}) + P(F \cap \overline{E}) + P(E \cap F)$

(refer figure)

$\therefore \frac{23}{25} = \frac{11}{25} + P(E \cap F)$

$\therefore P(E \cap F) = \frac{12}{25} = P(E) \times P(F)$



From the options

$P(E) = \frac{3}{5}$ and $P(F) = \frac{4}{5}$

or

$P(E) = \frac{4}{5}$ and $P(F) = \frac{3}{5}$

50. Let $y = \frac{b-x}{1-bx}$

$\Rightarrow (1 - bx)\,y = b - x$

$\Rightarrow x(1 - by) = b - y$

$\Rightarrow x = \frac{b-y}{1-by}$

$f^{-1}(x) = f(x)$

$f'(x) = \frac{(1-bx)(-1)+(b-x)b}{(1-bx)^2}$

$f'(b) = \frac{-\left(1-b^2\right)}{\left(1-b^{2^2}\right)} = \frac{1}{\left(b^2-1\right)}$

$f'(0) = \frac{-1+b^2}{1} = \frac{1}{f'(b)}$

51. Normal at 't' is
$y + xt = 2t + t^3$
$6 + 9t = 2t + t^3$
$t^3 - 7t - 6 = 0$
$t = -1, t = -2, t = 3$
Normal are
$Y - x = -3$,
And $y + 3x = 33$
And $y - 2x = -12$

52. $f\left(-\frac{\pi-}{2}\right) = 0$
$f\left(\frac{-\pi+}{2}\right) = -\cos\left(\frac{-\pi}{2}\right) = 0$
(A) is true
f(x) is continuous at x = 0
$f'(0^-) = 0$
$f'(0^+) = 1$
(B) is true
F(x) is continuous at x = 1
$f'(1^-) = 1$
$f'(1^+) = 1$
f(x) is differentiable at x = 1.
(c) is true.
$\frac{22}{14} = \frac{11}{7} = 1$
$x = \frac{-3}{2}$ lies in $\left(-\infty, \frac{-\pi}{2}\right)$
$\Rightarrow$ f(x) is differentiable there, since f(x) is a linear function in that interval
(D) is true.

Section III

53. $M = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$

$M\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \Rightarrow a_2 = -1;\ b_2 = 2;\ c_2 = 3$

$M\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \Rightarrow a_1 + 1 = 1 \Rightarrow a_1 = 0$

$C_1 - 3 = -1 \Rightarrow c_1 = 2$

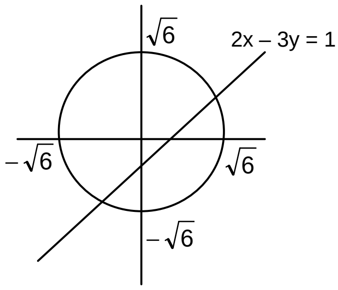
$M\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix} \Rightarrow 2 + 3 + c_3 = 12$

$\Rightarrow c_3 = 7.$

$A_1 + b_2 + c_3 = 0 + 2 + 7 = 9$

54. Origin lies on the side 2x – 3y - < 0

The smaller intersection Is on 2x – 3y – 1 > 0



All points ∈ S lies inside the circle, except $\left(\frac{5}{2}, \frac{3}{4}\right)$ and $\left(\frac{1}{8}, \frac{1}{4}\right)$ satisfies the inequality
$2x - 3y - 1 < 0$.
∴ The remaining 2 points lie on the smaller intersection.

55. Let $\overline{A} = ai + bj + ck$
$\overline{P}_1 = i + j + k \qquad |P_1|^2 = 0$
$\overline{P}_2 = i + wj + w^2 k \qquad |P_2|^2 = 0$
$\overline{P}_3 = i + w^2 j + wk \qquad |P_3|^2 = 0$
Them $x = \overline{A}, \overline{P}_1$, $y = \overline{A}, \overline{P}_2$ $z = \overline{A}, \overline{P}_3$
$\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2} = \frac{|A|^2\left(|P_1|^2 + |P_2|^2 + |P_3|^2\right)}{|A|^2}$
= 3 + 0 + 0 = 3

56. Let y(x) = n as g (x) = u
∴ The given equation becomes
du + udu = udu
$\Rightarrow \frac{dy}{du} + u = v \Rightarrow ue^u = e^u + c$
i.e $y(x)\, e^{g(x)} = e^{g(x)} + c$
$y(0) = 0$ as $g(0) = 0 \Rightarrow C = 0$
∴ $y(2)\, e^{g(2)} = e^{g(2)} + 0 \Rightarrow y(2) = 1$

57. $x^4 {}_{-4x}{}^3 + 12x^2 + x - 1 = 0$
$f(0) < 0, f(1) > 0$
∴ one root in (0, 1)
$f^1(x) = 4x^3 - 12x^2 + 24x + 1$
$f^{11}(x) = 12(x^2 - 2x + 2) > 0$
$\Rightarrow f^1(x)$ is increasing

∴ it has exactly one root
⇒ f(x) may have almost 2 distinct roots
Since real roots are even in number
f(x) has 2 distinct real roots

58. $(\bar{r}-\bar{c})\times\bar{b}=0$
$\bar{r}=\bar{c}+m\bar{b}$
$0=\bar{r}.\bar{a} = \bar{c}.\bar{a} + m\ \bar{b}.\bar{a}$
$= -4 + m, \quad m = 4$
$\bar{r}=\bar{c}+4\bar{b}$
$\bar{r}.\bar{b} = \bar{c}.\bar{b} + 4\bar{b}^2$
$= 1 + 8 = 9$
$\in (-\infty, -1]\cup[1, \infty)$

Section IV

59. (a). (a) $R_e\left(\frac{2iz}{1-z^2}\right)$

$R_e\left[\frac{2i}{\frac{1}{z}-z}\right]$

$= R_e\left(\frac{2i}{\bar{z}-z}\right)$

$= R_e\ \frac{1}{(-\operatorname{Im} z)}$

$= \frac{1}{-\operatorname{Im} z}$

(b) Put $y = 3^{x-1}$

$-1\dfrac{<\frac{8}{3}y}{1-y^2}\le 1$

$\Rightarrow \left|\frac{8}{3}xy\right|\le|1-xy^2|$

$\Rightarrow \frac{8}{3}y\le 1-y^2$ if $1-y^2\ge 0$

$3y^2+8y-3\le 0$ i.e $y\in[-1,1]$
$3y^2+a_y-y-3\le 0$
$(3y-1)(y+3)\le 0\ \ y\in\left[-3,\frac{1}{3}\right]$

$y\in\left[0,\frac{1}{3}\right] = 3^{-1}\le\frac{1}{3}$

$x\le 0$

$\frac{8}{3}y\le y^2-1$ if $1-y^2\le 0$

$3y^2-8y-3\ge 0$
$3y^2-9y+y-3\ge 0$
$(3y+1)(y-3)\ge 0 \quad y\ge 3$
$\Rightarrow 3^{x-1}\ge 3$
$x\ge 2$ (r, t)

∴ $x\in(-\infty, 0]\cup[z, \infty)$

(c) $\frac{1}{\cos^3\theta}\begin{vmatrix}\cos\theta & \sin\theta & \cos\theta\\ -\sin\theta & \cos\theta & \sin\theta`\\ -\cos\theta & -\sin\theta & \cos\theta\end{vmatrix}$

$=\frac{1}{\cos^3\theta}\begin{vmatrix}0 & 0 & 2\cos\theta\\ -\sin\theta & \cos\theta & \sin\theta\\ -\cos\theta & -\sin\theta & \cos\theta\end{vmatrix}$

$=\frac{2\cos\theta}{\cos^3\theta}=2\sec^2\theta\in[2,\infty)$

(d) $x^{\frac{3}{2}}(3x-10)$
$= 3x^{\frac{5}{2}}-10x^{\frac{3}{2}}$
$f'(x) = \frac{15}{2}x^{\frac{3}{2}}-15\sqrt{x}$
$= 15\sqrt{x}\left(\frac{x}{2}-1\right)\ge 0$
If $x\ge 2$.

60. (a) $\cos\theta = \frac{-1+3}{4}=\frac{1}{2}$
∴ $\theta = \frac{\pi}{3}$
∴ required angle is $\pi - \frac{\pi}{3}=\frac{2\pi}{3}$

(b) $\int_a^x (f(x)-3x)dx = a^2-x^2$
$f(x) - 3x = -2x \Rightarrow f(x) = x$
∴ $f\left(\frac{\pi}{6}\right)=\frac{\pi}{6}$

(c) $\frac{\pi}{\log 3}\log[\sec\pi x+\tan\pi x]_{7/6}^{5/6}=\pi$

(d) $\left|\operatorname{Arg}\left(\frac{1}{z-1}\right)\right| = |\operatorname{Arg}(z-1)|$
Maximum value = π.