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F. Y. B. A. Examination, October – 2012

1394 – MATHEMATICS

(Applied Mathematics)

AMG - I : Calculus

(General - I)

(2008 Pattern)

Time : Three Hours   **Total Marks : 80**

Note : *(i) All questions are compulsory.*

(ii) Figures to the right indicate full marks.

Q. 1. Attempt **all** following sub questions : **[16]**

(a) Find the rational number between $\sqrt{6}$ and $\sqrt{7}$.

(b) Discuss the convergence of the series

$$1-\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)^3+\ ....$$

(c) If $y = e^{\tan^{-1}x}$ then show that $(Hx^2)y_2+(2+-1)\,y_1=0$

(d) Define contractive sequence.

(e) Find left hand and right hand limits of the functio f (x) at x = 0

where $f(x)=\frac{|x|}{x}$.

(f) If the function $f: IR \to IR$ is defined as

$f(x) = -1,$ if $x < c$

$= 1,$ if $x \geq c$

then show that $|f|$ is continous at $x = c$.

(g) Evaluate : $\lim\limits_{x \to 0} \dfrac{a^x - 1}{b^x - 1}$

(h) Test whether Rolle's theorem is applicable for the function $f(x) = |x|$ on $[-1, 1]$.

Q. 2. Attempt **any four** of the following : **[16]**

(a) State completeness axion for IR and prove that for any number y, $\exists\, n \in N$ such that $nx > y$, where x is any positive real number.

(b) If $\{x_n\}_{n-1}^{\infty}$, $\{y_n\}_{n=1}^{\infty}$, $\{z_n\}_{n=1}^{\infty}$ are three sequences such that $z_n \le y_n \le z_n, \forall\, n \in N$ and $\{x_n\}_{n=1}^{\infty}, \{z_n\}_{n=1}^{\infty}$ both converge to l then prove that sequence $\{y_n\}_{n=1}^{\infty}$ converges to l.

(c) Solve $|3x + 4| < |x + 2|$.

(d) Show that sequence $\left\{1 - \dfrac{(-1)^n}{n}\right\}_{n=1}^{\infty}$ is convergent.

(e) Discuss the convergence of the series $\sum\limits_{n=1}^{\infty} \dfrac{3n+1}{n(n+1)(n+2)}$

(f) Evaluate : $\lim\limits_{x \to 0} \dfrac{e^{y_x}}{1 + e^{y_x}}$, if it exists.

Q. 3. Answer **any two** of the following : **[16]**

(a) Show that the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent.

where $x_n = 1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \ldots\ldots + \dfrac{1}{n!}$

State the interval in which the limit lies.

(b) (i) Define Cauchy sequence. Prove that every Cauchy sequence of real numbers is bounded.

(ii) Using definition, show that the sequence $\{x_n\}_{n=1}^{\infty}$ converges to $\frac{2}{3}$, where $x_n = \frac{2n+3}{3n+5}$.

(c) (i) Prove that $\sum\limits_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for $p > 1$.

(ii) Discuss the convergence of the series $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$.

(d) (i) State the field axioms for the set of real numbers.

(ii) Discuss the continuity of the function f(x) in $(0,\infty)$ where

$$f(x) = \begin{cases} \frac{x^2}{4} - 4, & \text{for } 0 < x < 4 \\ 2, & \text{for } x = 4 \\ 4 - \frac{64}{x^2}, & \text{for } x > 4 \end{cases}$$

Q. 4. Attempt **any four** of the following : **[16]**

(a) State and prove Cauchy's mean value theorem.

(b) Separate the intervals in which the polynomial function $x^3 - 3x - 4$ is increasing or decreasing.

(c) Evaluate : $\lim\limits_{x \to 0} x^x$

(d) Verify Rolle's mean value theorem for the function

$f(x) = e^x(\sin x - \cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

(e) Expand sin x in ascending powers of $\left(x-\frac{\pi}{2}\right)$

(f) Prove that, if f(x) is differentiable at point x = a, then it is continuous at x = a. Is the converse true? Justify your answer.

Q. 5. Attempt **any two** of the following : **[16]**

(a) State and prove Leibnitz's theorem and hence find y_n, if $y = x^2e^x$.

(b) (i) Find y_n, if y = sin (bx + c)

(ii) Prove that every continous function defined on closed and bounded interval is bounded.

(c) (i) Using $\in-\delta$ detinition, show that absolute value function $f(x) = |x|$, is continous at every point in IR.

(ii) Evaluate $\lim\limits_{x\to1} x^{\left(\frac{1}{1-x}\right)}$.

(d) (i) Assuming validity of expansion, expand $\log\sqrt{\frac{1+x}{1-x}}$.

(ii) Prove that

$$\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}<\frac{b-a}{1+a^2},\quad \text{if } 0<a<b.$$

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