

**M.Sc. (MATHEMATICS WITH APPLICATIONS  
IN COMPUTER SCIENCE)**

**M.Sc. (MACS)**

**Term-End Examination**

00792

June, 2017

**MMTE-005 : CODING THEORY**

*Time : 2 hours*

*Maximum Marks : 50*

*(Weightage : 50%)*

---

**Note :** Answer any *four* questions from questions no. 1 to 5. Question no. 6 is *compulsory*. Calculators are *not allowed*.

---

---

1. (a) Give an example, with justification, of a non-linear binary code of length 4 and cardinality 4, with minimum distance 2. 4
- (b) Give an example of a perfect code, and justify your answer. 3
- (c) Construct a field with 9 elements. 3

2. (a) Prove that the dimension of a self-dual is half of its length. 3

(b) Let  $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$  be

the parity check matrix of a code C. Then :

- (i) construct the tanner graph of C;
- (ii) determine a generator matrix for C;
- (iii) obtain the code word if the message is 101. 5
- (c) Give an example, with justification, of a convolutional code having code rate  $\frac{1}{2}$ . 2

3. (a) Let C be the binary [15, 7] narrow-sense BCH code of designed distance  $\delta = 5$ , which has defining set  $T = \{1, 2, 3, 4, 6, 8, 9, 12\}$ . Let  $\alpha$  be the primitive 15<sup>th</sup> root of unity such that  $\alpha^4 = \alpha + 1$ . If  $g(x) = 1 + x^4 + x^6 + x^7 + x^8$  is the generator polynomial of C and  $y(x) = x + x^3 + x^5 + x^8$  is the received word, find the transmitted code word. 5

The following table may be of use to you :

0000	0	1000	$\alpha^3$	1011	$\alpha^7$	1110	$\alpha^{11}$
0001	1	0011	$\alpha^4$	0101	$\alpha^8$	1111	$\alpha^{12}$
0010	$\alpha$	0110	$\alpha^5$	1010	$\alpha^9$	1101	$\alpha^{13}$
0100	$\alpha^2$	1100	$\alpha^6$	0111	$\alpha^{10}$	1001	$\alpha^{14}$

- (b) Prove that a self-orthogonal binary cyclic code is doubly even. 5

4. (a) Let C be the [5, 2] binary code generated by

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

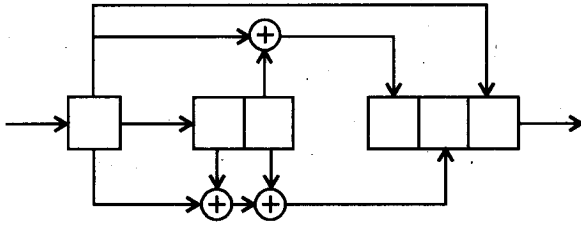
- (i) Find the weight distribution of C.  
 (ii) Using the Mac-Williams identity, find the weight distribution of the dual of C. 4

- (b) (i) Define the Gray map.  
 (ii) Find the Gray image of the code C generated by the matrix

$$G = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}.$$
 4

- (c) Define quadratic residue code and give an example. 2

5. (a) Consider the following convolutional encoder shown below :



- (i) Draw the corresponding Trellis diagram.
- (ii) Decode the received codeword 010000100001 using the Viterbi algorithm. 6
- (b) Find 8 values of  $n$ ,  $2 < n < 200$ , for which there is a self-dual extended cyclic binary code of length  $n + 1$ . 4
6. State whether the following statements are *True* or *False*. Give reasons for your answers. 10
- (a) A Hamming code has a double error correction capability.
- (b)  $\mathbf{Z}_3 \times \mathbf{Z}_3$  is a field.
- (c) Every Reed-Solomon code is a BCH code.
- (d) 15 is a square in  $\mathbf{Z}_{13}$ .
- (e) The generator matrix of a Turbo code is the identity matrix.