

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE - SEMESTER-I & II (OLD) EXAMINATION – SUMMER-2019****Subject Code: 110008****Date: 06/06/2019****Subject Name: Maths - I****Time: 10:30 AM TO 01:30 PM****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** (i) Evaluate :  $\lim_{x \rightarrow 1} (1-x) \tan\left(\frac{\pi x}{2}\right)$  **02**
- (ii) Evaluate :  $\lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^{1-\cos x}$  **02**
- (iii) Find Jacobian  $\frac{\partial(u,v)}{\partial(x,y)}$  for functions  $u = x \sin y, v = y \sin x$ . **03**
- (b)** (i) Sketch the region and find the area bounded by ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$ . **03**
- (ii) Using Lagrange's mean value theorem, prove that **04**
- $$\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$$
- Q.2 (a)** (i) Verify Rolle's theorem for  $f(x) = x(x+3)e^{-\frac{x}{2}}$  in  $-3 \leq x \leq 0$ . **04**
- (ii) Find two non-negative numbers whose sum is 9 such that the product of one number and the square of the other is maximum. **03**
- (b)** (i) Prove that  $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \frac{1}{2}\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots\right)$ . **04**
- (ii) Find the absolute maximum and minimum values of  $f(x) = \frac{x^3}{x+2}$  in interval  $[-1, 1]$ . **03**
- Q.3 (a)** (i) Test the convergence of the series:  $\sum_{n=1}^{\infty} \frac{n^n x^n}{(n+1)^n}, x > 0$ . **04**
- (ii) Test the convergence of the series:  $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$  **03**
- (b)** (i) Test the convergence of the series:  $\frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \frac{4}{17} + \dots$  **04**
- (ii) Test the convergence of the series:  $\sum_{n=1}^{\infty} \frac{n!}{n^n}$  **03**
- Q.4 (a)** (i) Find extreme values of  $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$  **04**
- (ii) Find all first and first and second order partial derivatives for  $f(x, y) = x^2 \sin y + y^2 \cos x$ . Hence, verify mixed derivative Theorem. **03**
- (b)** (i) If  $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ ; show that  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u$ . **04**
- (ii) If  $u = f(x-y, y-z, z-x)$  then prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ . **03**

- Q.5 (a)** Sketch the region of integration and evaluate by reversing the order of Integration for integral  $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$  **07**
- (b)** (i) Evaluate the integral  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$  by changing into polar coordinates. **04**  
(ii) Evaluate  $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dx dy dz$ . **03**
- Q.6 (a)** (i) Find directional derivative of  $\phi = xy^2 + yz^2$  at point  $(2, -1, 1)$  in the direction of vector  $\hat{i} + 2\hat{j} + 2\hat{k}$ . **04**  
(ii) If  $\vec{r} = t^3\hat{i} + (2t^3 - \frac{1}{5t^2})\hat{j}$ , then show that  $\vec{r} \times \frac{d\vec{r}}{dt} = \hat{k}$ . **03**
- (b)** Evaluate  $\int_C \vec{F} \cdot d\vec{r}$ , where,  $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ . Where, C is the rectangle in XY-plane bounded by  $y = 0, x = a, y = b, x = 0$ . **07**
- Q.7 (a)** Verify Green's theorem for  $\oint_C [(x^2 - 2xy)dx + (x^2y + 3)dy]$ . Where, C is the boundary of the region bounded by  $y = x^2$  and the line  $y = x$ . **07**
- (b)** (i) Show that  $\vec{F} = (y^2 - z^2 + 3xy - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$  is both solenoidal and irrotational. **04**  
(ii) Evaluate  $\oint_C [e^x dx + 2ydy - dz]$  by Stoke's theorem. where, C is the curve  $x^2 + y^2 = 4, z = 2$ . **03**

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