PG – 622

Second Semester M.Sc. Degree Examination, July 2017
(R.N.S. – Repeaters) (2011-12 and Onwards)
MATHEMATICS
Paper – M-205 : Continuum Mechanics

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **any five** questions choosing atleast **one** from **each** Part.*
*2) **All** questions carry **equal** marks.*

PART – A

1. a) For an arbitrary vector with components b_i, if $a_{ij}b_j$ are components of a vector then show that a_{ij} are components of a second order tensor. Hence show that δ_{ij} are components of a second-order tensor.

b) Using the $\in - \delta$ identity prove the vector identity
$\left(\vec{a} \times \vec{b}\right).\left(\vec{c} \times \vec{d}\right) = \left(\vec{a}\,.\,\vec{c}\right)\left(\vec{b}\,.\,\vec{d}\right) - \left(\vec{a}\,.\,\vec{d}\right)\left(\vec{b}\,.\,\vec{c}\right)$.

c) If $\vec{A}$ is a skew tensor then prove that there exists a vector $\overrightarrow{W}$ of $\vec{A}$ such that $\vec{A}\,\vec{u} = \overrightarrow{W} \times \vec{u}$. **(6+4+6)**

2. a) Prove the following :

i) $\operatorname{div}\left(\nabla \vec{u}\right)^T = \nabla\left(\operatorname{div} \vec{u}\right)$

ii) $\operatorname{curl}\left(\nabla \vec{u}\right)^T = \nabla\left(\operatorname{curl} \vec{u}\right)$.

b) If $\underset{\sim}{W}$ is a dual vector of a skew tensor $\underset{\sim}{w}$, prove that $\operatorname{curl} \underset{\sim}{W} = \left(\operatorname{div} \underset{\sim}{W}\right) \underset{\sim}{I} - \nabla \underset{\sim}{W}$.

c) State and prove divergence theorem for a tensor field $\underset{\sim}{A}$. **(7+5+4)**

P.T.O.

PART – B

3. a) Explain briefly the following :

   i) Continuum hypothesis.

   ii) Deformation of arc, surface and volume elements.

   b) For the deformation defined by the equations

   $x_1 = x_1^0 + x_2^0, x_2 = x_1^0 - 2x_2^0, x_3 = x_1^0 + x_2^0 - x_3^0$. Find $\underset{\sim}{F}$, J and $\underset{\sim}{F}^{-1}$. Is the deformation isochoric ?

   c) Obtain an expression for Green strain tensor. **(8+5+3)**

4. a) Obtain a formula for the material derivative in the spatial form.

   b) Find the velocity and acceleration in both material and spatial forms for the motion defined by the equations.

   $x_1^0 = x_1 \cos\alpha\, t - x_2 \sin\alpha\, t$

   $x_2^0 = x_1 \sin\alpha\, t + x_2 \cos\alpha\, t$

   $x_3^0 = x_3 \ (\alpha \text{ in a constant})$.

   c) Show that a motion is circulation preserving if and only if $\frac{\partial \vec{w}}{\partial t} = \text{curl}\,(\vec{v} \times \vec{w})$, where $\vec{w} = \text{curl}\,\vec{v}$. **(3+8+5)**

5. a) Establish Cauchy's law in the form $\vec{S}(\hat{n}) = \underset{\sim}{T}^T \hat{n}$, where the quantities have their usual meaning. Further, prove that $\hat{n} . \vec{s}(\hat{n}') = \hat{n}' . \vec{s}(\hat{n})$ if and only if $\underset{\sim}{T}$ is symmetric.

   b) The stress matrix at a point in a material is $[\tau_{ij}] = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$

   Then find the normal stress and the shear stress on the octahedral plane element through the point. **(8+8)**

PART – C

6. a) Derive the equation of continuity in the Eulerian form from its Lagrangian form.

   b) Using the appropriate conservation law, show that the Cauchy's stress tensor is symmetric at every point of a material body and for all time t.

   c) Stating the assumptions made, prove that $\frac{D}{Dt}(K+E) = \int_s \left(T\vec{V} - \vec{E}\right).\hat{n}\, ds$.

(4+8+4)

7. a) For a linear, isotropic elastic solid, obtain the generalized Hooke's law in its standard form. Further show that this relation is equivalent to the following relations taken together

   $$\text{tr}\ \underset{\sim}{T} = (3\lambda + 2\mu)\,\text{tr}\ \underset{\sim}{E}$$

   $$\underset{\sim}{T}^{(d)} = 2\mu\, \underset{\sim}{E}^{(d)}.$$

   b) Derive Navier's equation of equilibrium in its standard form. **(10+6)**

8. a) Prove that every motion of an elastic fluid under conservative body force is circulation preserving.

   b) For a certain flow of an inviscid fluid of a constant density under the earth's gravitational field, the velocity distribution is given by $\vec{v} = \nabla\phi$, where $\phi = x^3 - 3xy^2$. Find the pressure distribution.

   c) Obtain the Navier-Stokes equation for a compressible fluid in its standard form. **(5+5+6)**
