



Second Semester M.Sc. Degree Examination, July 2017
(RNS – Repeaters) (2011-12 and Onwards)
MATHEMATICS
M – 201 : Algebra – II

Time : 3 Hours

Max. Marks : 80

Instructions: 1) Answer **any five full** questions. Choosing **atleast two** from **each Part**.
2) **All** questions carry **equal** marks.

PART – A

1. a) Let K be an extension of a field F and $a \in K$ be algebraic over F and of degree n . Prove that $[F(a) : F] = n$.
b) Prove that every finite extension K of a field F is algebraic and may be obtained from F by the adjunction of finitely many algebraic elements.
c) Let $a = \sqrt{2}$, $b = \sqrt[4]{2}$ in R , where R is an extension of Q . Verify that $a + b$ and ab are algebraic of degree atmost $(\deg a)(\deg b)$. **(6+6+4)**
2. a) Prove that a polynomial of degree n over a field F can have atmost n roots in any extension field K .
Is the result true, when extension field K is not a field F ? Justify with an example.
b) Define the splitting field of a polynomial over a field F .
Determine the splitting field of
 - i) $x^3 - 2$
 - ii) $x^2 + x + 1$, over the field Q . **(8+8)**
3. a) Show that the polynomial $f(x) \in F[x]$ has multiple roots if and only if $f(x)$ and $f'(x)$ have a non-trivial common factors.
b) Prove that a regular pentagon is constructible by using edge and compass.
c) Show that any field of characteristic zero is perfect field. **(6+5+5)**

P.T.O.



4. a) Define a fixed field. Let G be a subgroup of the group of all automorphisms of a field K . Then show that fixed field of G is a subfield of K .
- b) State and prove the fundamental theorem of Galois theory. **(4+12)**

PART – B

5. a) Let V be a finite – dimensional vector space over F . Prove that $T \in A(V)$ is invertible iff and only iff the constant term of the minimal polynomial for T is not zero.
- b) Give an example to show that $ST \neq TS$ for $S, T \in A(V)$.
- c) Define a rank of a linear transformation T . If V is a finite dimensional vector space over F and $S, T \in A(V)$, then prove that $r(ST) \leq \min \{r(T), r(S)\}$. **(6+4+6)**
6. a) If $T, S \in A(V)$ and if S is regular, then show that T and STS^{-1} have the same minimal polynomial.
- b) Define a characteristic root of $T \in A(V)$. If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then for any $q(x) \in F[x]$, prove that $q(\lambda)$ is a characteristic root of $q(T)$.
- c) If V is n -dimensional vector space over a field F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $\{v_1, v_2, \dots, v_n\}$ and the matrix $m_2(T)$ in the basis $\{w_1, w_2, \dots, w_n\}$ of V over F , then prove that there is a matrix C in F_n such that $m_2(T) = C m_1(T) C^{-1}$. **(5+5+6)**
7. a) If V is n -dimensional vector space over F and if $T \in A(V)$ has all its characteristic roots in F , then prove that T satisfies a polynomial of degree n over F .
- b) Let $T \in A(V)$ and V_1 an n_1 -dimensional subspace of V and spanned by $\{v, vT, vT^2, \dots, vT^{n_1-1}\}$, where $v \neq 0$. If $u \in V_1$ is such that $uT^{n_1-k} = 0, 0 < k \leq n_1$, then show that $u_0 T^k = u$ for some $u_0 \in V_1$.
- c) Prove that two nil potent linear transformations are similar iff and only iff they have the same invariants. **(6+4+6)**
8. a) Let V be a finite dimensional complex inner product space. If $T \in A(V)$ is such that $\langle T(v) : v \rangle = 0$ for each $v \in V$, then prove that $T = 0$.
- b) If λ is a characteristic root of the normal transformation N and if $vN = \lambda v$, then show that $vN^* = \bar{\lambda}v$.
- c) State and prove the Sylvester's law of inertia for real quadratic form. **(4+4+8)**