PG – 632

IV Semester M.Sc. Degree Examination, June 2017
(RNS Repeaters)
(2011-12 and Onwards)
MATHEMATICS
M 405 C : Computational Fluid Dynamics

Time : 3 Hours   Max. Marks : 80

Instructions : *1)* ***All*** *questions have* ***equal*** *marks.*
2) Answer ***any five*** *questions.*

1. a) Obtain the forward-time and forward-space finite difference scheme for $u_t + au_x = 0, a > 0$.
Also discuss its stability.

   b) Derive the explicit and implicit finite differences for the non linear Burger's equation. **(8+8)**

2. a) Solve the problem $u_t + u_x = 0$ with conditions

$$u(x, 0) = \begin{cases} \sin 2\pi x, & 0 \le x \le 1 \\ 0, & 1 \le x \le 2 \end{cases}$$

$u(0, t) = 0 = u(2, t)$

using finite difference scheme with $\Delta x = 0.5$ and $\Delta t = 0.2$.

   b) Solve the initial boundary value problem $u_t = x\, u_{xx}$, $0 \le x \le 1$

with

$u(x, 0) = 1$

$u_x(0, t) = u_x(1, t) = 0, t > 0$

using the implicit finite difference scheme with $\Delta x = \frac{1}{3}$, $\lambda = \frac{1}{3}$. **(8+8)**

P.T.O.

3. a) Obtain the solution of $U_{xx} + U_{yy} = 0$ with conditions.
$u(x, 0) = 200$, $u(x, 2) = u(0, y) = u(2, y) = 0$.

Take $\Delta x = \Delta y = \frac{2}{3}$.

b) Solve the wave equation $U_{tt} = U_{xx}$, $0 \leq x \leq 1$, $t \geq 0$ with conditions.

$$\left.\begin{array}{r} U(x, 0) = \sin \pi x, \\ U_t(x, 0) = 0 \end{array}\right\} 0 \leq x \leq 1$$

$U(0, t) = 0 = U(1, t)$, $t \geq 0$.

Using the finite difference scheme with $\Delta x = \frac{1}{4}$, $\Delta t = 0.01$. Obtain the solution at second-time level. **(8+8)**

4. a) Discuss the stability of the Crank-Nicolson finite difference scheme.

b) Explain the artificial compressibility method for the Navier-Stokes equation. **(8+8)**

5. Consider the highly viscous flow between two parallel surfaces in which the lower surface is at rest and the upper surface moves with velocity K. The magnetic field is applied normal to the flow. Derive the lubrication model for magnetic flow between two parallel surfaces. **16**

6. Obtain the quasi linearized three-dimensional Euler's equations for inviscid flow. **16**

7. a) Discuss the main steps involved in the finite element method. Also detail out the advantages of it.

b) Solve the ordinary differential equation $y''(x) + 6y'(x) + 5y(x) = x$, $y(0) = 0$, $y(1) = 1$ using finite element method. **(8+8)**

8. Solve the elliptic partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$, $0 \leq x, y \leq 1$ with $u = 0$ on the boundary using the finite-element method. **16**
