Bachelor of Engineering in Mechanical Engineering Examination, 2019

(2nd Year, 1st Semester)

Mathematics - III

Time : Three hours Full Marks : 100

(50 marks for each Part)

Use a separate Answer-Script for each Part

PART - I (50 Marks)

Answer ***any five*** questions. 5×10=50

1. a) Show that the equation :

$$(e^{2y} - y\cos xy)dx + (2xe^{2y} - x\cos xy + 2y)dy = 0$$

is exact and find the general solution.

b) Solve : $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x\cos x$ 4+6

2. a) Find the series solution of the equation

$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - xy = 0$ about the point $x = 0$.

b) Show that $Pn(1) = 1$.

3. a) Show that

$$\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$$

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b) Solve : $(1+y^2)dx-(\tan^{-1}y-x)dy=0$ 5+5

4. a) Solve : $\dfrac{d^2y}{dx^2}-5\dfrac{dy}{dx}+4y=8e^x$

b) Show that,

$$nP_n = xP_n'(x)-P_{n-1}'(x)$$ 5+5

5. a) Solve : $x^3\dfrac{d^3y}{dx^3}+2x^2\dfrac{d^2y}{dx^2}+2y=10\left(x+\dfrac{1}{x}\right)$ 5

b) Solve by method of variation of parameter

$$\frac{d^2y}{dx^2}+y=\operatorname{cosec}x$$ 5

6. a) Eliminate arbitary constants from $z=(x-a)^2+(y-b)^2$ to form the partial differential equation.

b) Solve : $y^2(x-y)p+x^2(y-x)q=z(x^2+y^2)$ 4+6

7. Let u be harmonic function in the interior of a rectangle $0\le x\le a,\ 0\le y\le b$ in the xy-plane satisfying Laplace's equation $\dfrac{\partial^2u}{\partial x^2}+\dfrac{\partial^2u}{\partial y^2}=0$

with u(0, y), u(a, y), u(x, b) = 0 and u(x, 0) = f (x),

Determine u. 10

b) Find the Fourier transform of $e^{-a|x|}$ and evaluate

$$\int_0^\infty \frac{1}{(x^2+a^2)^2}dx,\ a>0.$$ (2+3)+5

12. a) Define Laplace transformation and state its existence condition.

b) Find Laplace transform of $\displaystyle\int_0^t \frac{e^{-t}\sin t}{t}dt.$ 5+5

13. a) Find $L^{-1}\left[\ln\dfrac{(s+1)}{(s-1)}\right].$

b) Using Convolution theorem, find $L^{-1}\left[\dfrac{1}{s(s^2-a^2)}\right].$ 5+5

14. a) Show that $Z[n^p]=-z\dfrac{d}{dz}\{Z[n^{p-1}]\}$, p being a positive integer.

b) If $Z[u_n]=U(z)$, show that

$Z[u_{n-k}]=z^{-k}U(z),\ k>0.$ 6+4

15. a) Show that

$$Z[\cos n\theta]=\frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}.$$

b) Find $Z[n\sin n\theta]$. 5+5

PART - II (50 Marks)

Answer ***any five*** questions.

9. a) Calculate the Fourier sine series of $f(x) = x(\pi - x)$ on $(0, \pi)$. Hence derive the value of the infinite series

$$1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots$$

b) Let h be a given number in the interval $\left(0, \frac{\pi}{2}\right)$. Find the Fourier cosine series representation of

$$f(x) = \begin{cases} 1 & \text{if} \quad x = 0, \\ \dfrac{2h - x}{2h} & \text{if} \quad 0 < x < 2h, \\ 0 & \text{if} \quad 2h < x < \pi, \end{cases}$$

where f is of period 2π. 5+5

10. a) Find the Fourier sine transform of $\dfrac{x}{x^2 + a^2}$.

b) If $F_c[f(x)] = \sqrt{\dfrac{2}{\pi}}, \dfrac{\sin as}{s}, \; a > 0, \; s \neq 0$, then find $f(x)$.

5+5

11. a) Write down Parseval's Identity for Fourier transforms. Find the Fourier transform of

$$f(x) = \begin{cases} a^2 - x^2, & |x| < a \\ 0 \quad , & |x| > a > 0. \end{cases}$$

8. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a velocity $ux(l - x)$, find the displacement of the string at any distance x from one end at any time t. 10