



I Year M.Sc. (DCC) Degree Examination, January 2018
(Y2K13 Scheme) (Fresh and Repeaters)

MATHEMATICS

M101 : Algebra

Time : 3 Hours

Max. Marks : 80

Instructions : 1) Answer **any 5** questions, choosing **at least two** from **each Part**.

2) **All** questions carry **equal** marks.

PART – A

1. a) Let $\phi : G \rightarrow \bar{G}$ be a homomorphism with Kernel K and let N be a normal subgroup of G . Then show that $\frac{G}{N} \approx \frac{\bar{G}}{\bar{N}}$. 5
- b) Prove that $\text{Inn}(G) \approx \frac{G}{Z(G)}$, where $\text{Inn}(G)$ is a group of inner automorphisms of G and $Z(G)$ in the centre of G . 5
- c) Show that every group is isomorphic to a subgroup of $A(S)$, for some appropriate S . 6
2. a) Verify the class equation for symmetric group S_3 , by using generator-relation form. 5
- b) Prove that any two p -Sylow subgroups are conjugate to each other. 6
- c) Let G be a group of order pq , where p and q are distinct primes with $p < q$ and $q \not\equiv 1 \pmod{p}$, then prove that G is abelian. 5
3. a) Let R be a commutative ring with unity whose ideals are $\{0\}$ and R only. Prove that R is a field. 5
- b) Let U be the left ideal of a ring R and $\lambda(U) = \{x \in R : x u = 0 \text{ for all } u \in U\}$. Prove that $\lambda(U)$ is an ideal of R . 4
- c) Define a maximal ideal of a ring R . If R is a commutative ring with unity and M is an ideal of R , then show that M is a maximal ideal of R if and only if R/M is a field. 7



4. a) Define Euclidean ring. Prove that the ring $Z[i]$ of Gaussian integers is an Euclidean ring. 5
- b) State and prove unique factorization theorem. 5
- c) State and prove Eisenstein criterion for irreducibility of a polynomial. 6

PART – B

5. a) Let K be an extension of a field F and $a \in K$ be algebraic over F and of degree n . Prove that $[F(a) : F] = n$. 6
- b) Let $f(x) \in F[x]$ be degree $n \geq 1$. Then prove that there is an extension E of F of degree at most $n!$ in which $f(x)$ has n -roots. 5
- c) Define splitting field of a polynomial over a field F . Determine the splitting field of $x^3 - 2$ over the field Q . 5
6. a) If F is a field of characteristic zero and a, b are algebraic over F , then prove that $F(a, b)$ is a simple extension of F . 5
- b) Define a perfect field. Show that any field of characteristic zero is perfect field. 6
- c) If K is a finite Galois extension of a field F and if $G(K, F)$ is a group of all F automorphisms of K , then prove that $O(G(K, F)) = [K : F]$. 5
7. a) Let V be finite-dimensional vector space over F , prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0. 5
- b) Define the range and rank of a linear transformation T . If V is finite dimensional vector space over F , then show that $T \in A(V)$ is regular if and only if T maps V onto V . 6
- c) If V is n -dimensional vector space over F and if $T \in A(V)$ has all its characteristic roots in F , then show that T satisfies a polynomial of degree n over F . 5
8. a) Prove that two nilpotent linear transformations are similar if and only if they have the same invariants. 5
- b) Define a unitary transformation T . Prove that linear transformation, T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V . 4
- c) State and prove Sylvester's law of inertia for real quadratic form. 7