(6 pages)

DECEMBER 2018 **U/ID 32351/UCMA**

Time : Three hours Maximum : 100 marks

PART A — (10 × 3 = 30 marks)

Answer any TEN questions.

1. Form the 4th equation whose one root is $\sqrt{5}+\sqrt{3}$.

2. If α, β, γ are the roots of $x^3 - 3ax + b = 0$ prove that $\Sigma(\alpha - \beta)(\alpha - \beta) = 9a$.

3. Show that the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$.

4. Expand $(1 - x)^{-1}$.

5. Prove that $2 < e < 3$.

6. If $-1 < x < 1$, prove that

$$\log\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\infty\right).$$

7. If $x = \cos\theta + i\sin\theta$, find the value of $x^n \frac{1}{x^n}$.

8. Define a skew symmetric matrix.

9. Find the eigen values of $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

10. Define a cyclic group.

11. State Fermat's theorem.

12. State Wilson's theorem.

PART B — (5 × 6 = 30 marks)

Answer any FIVE questions.

13. Solve the equation $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$ whose roots are in A.P.

14. Solve : $x^4 - 4x^2 + 8x + 35 = 0$, if $2 + i\sqrt{2}$ in a root.

15. Sum to infinity

$$1 + \frac{2}{6} + \frac{2 \cdot 15}{6 \cdot 12} + \frac{2 \cdot 5}{6 \cdot 12 \cdot 18} + \ldots\infty.$$

16. Prove that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1 .$$

17. Show that $\begin{bmatrix} 3 & 1+2i \\ 1-2i & 2 \end{bmatrix}$ is a hermitian matrix.

18. Prove that $\phi \neq H \subset G$ is a subgroup of $G \Leftrightarrow ab^{-1} \in H$.

19. Find the value of $\log(4+3i)$.

PART C — (4 × 10 = 40 marks)

Answer any FOUR questions.

20. Solve : $2x^6 - 9x^5 + 10x^4 - 3x^3 + 10x^2 - 9x + 2 = 0$.

21. Prove that

$$1+\left(\frac{1}{2}+\frac{1}{3}\right)\frac{1}{4}+\left(\frac{1}{4}+\frac{1}{5}\right)\frac{1}{4^2}+\left(\frac{1}{6}+\frac{1}{7}\right)\frac{1}{4^3}+\ldots\infty=\log\sqrt{12}\,.$$

22. Prove that

$$64(\cos^8\theta+\sin^8\theta)=\cos 8\theta+28\cos 4\theta+35\,.$$

23. Find all the characteristic roots and vectors of

$$A=\begin{bmatrix}1 & 2 & 3\\ 0 & 2 & 3\\ 0 & 0 & 2\end{bmatrix}.$$

24. State and prove Lagrange's theorem on groups.

25. Show that $i\log\left(\frac{x-1}{x+1}\right)=\pi-2\tan^{-1}x$.

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