S.No. 9548 D **DOR 5**

(For candidates admitted from 2006 – 2007 onwards)

DIPLOMA EXAMINATION, NOVEMBER 2020.

Operations Research

OPERATION RESEARCH –IV

Time : Three hours Maximum : 100 marks

SECTION A — ($10 \times 2 = 20$)

Answer ALL the questions.

1. What is Processing Order?
2. What is "No passing" rule in a sequence algorithm?
3. What is compound amount factor?
4. Explain Mortality Tables.
5. Define Simulation.
6. What is Monte – Carlo Simulation?
7. Write down the formula for average queue length.
8. Explain simulation in budgeting.
9. Define Lagrangian function.
10. A company faces a responsive price – volume relationship for its products, the lower a product's price–the greater is the sales quantity, even if face of resultant price decreases by competitors. If the sales – revenue does not vary proportionately with price, reflect this phenomenon in the non – linear objective function of the price. Write the mathematical formulation.

SECTION B — ($5 \times 6 = 30$)

Answer ALL questions, choosing either (a) or (b).

11. (a) In a factory, there are six jobs to perform, each of which should go through two machine A and B in the order A, B/ The processing timings (in hours) for the jobs are given here. You are required to determine the sequence for performing the jobs that would minimize the total elapsed time T. What is the value of T?

Or

(b) We have five jobs, each of which must go through the two machines A and B in the order AB. Processing times in hours are given in the table below :

Job(j)	1	2	3	4	5
Machine A :	5	1	9	3	10
Machine B :	2	6	7	8	4

Determine a sequence for the five jobs that will minimize the elapsed time.

12. (a) A pipeline is due for repairs. It will cost Rs. 10,000 and last for 3 years. Alternatively, a new pipeline can he laid at a cost of Rs. 30,000 and lasts for 10 years. Assuming cost of capital to be 10% and ignoring salvage value, which alternative should be chosen?

Or

(b) Machine A costs Rs. 9,000. Annual operating costs are Rs. 200 for the first year, and then increase by Rs. 2,000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy is followed, what will be the average yearly cost of owning and operating the machine?

13. (a) Explain simulation models.

Or

(b) Customers arrive at a milk booth for the required service. Assume that inter-arrival and service times are constant and given by 1.8 and 4 time units, respectively. Stimulate the system by hand computations for 14 time of the facility? (Assume that the system starts at t = o)

14. (a) An assembly line has three works stations. The time required for each station to complete its operation is as follows :

Time (Minutes)	Probabilities Station 1	Station 2	Station 3
4	0.25	0.10	0.05
5	0.25	0.30	0.25
6	0.25	0.40	0.25
7	0.25	0.20	0.45

The times given are the only values the operation times taken on. Stimulate the flow of 20 items through the assembly line. What is the average time that an item takes to go through all the operations?

Or

(b) A company has a single service station which has following characteristics : The mean arrival rate of customers and the mean service time are 6.2 minutes and 5.5 minutes respectively. The time between an arrival and its services varies from one minute to seven minutes. The arrival and service time distributions are given below :

Time (minutes)	Arrival (Probability)	Service (Probability)
1 – 2	0.05	0.10
2 – 3	0.20	0.20
3 – 4	0.35	0.40
4 – 5	0.25	0.20
5 – 6	0.10	0.10
6 –7	0.05	–

The queueing process starts at 11 A.M. and closes at 12 P.M. An arrival moves immediately into the service facility if it empty. On the other hand, if the service station is busy, the arrival will wait in the queue. Customers are served on the first come, first served basis.

If the clerk's wages are Rs. 6 per hour and the customer's waiting line costs Rs. 5 per hour, would it be economical for the Manager to engage the second clerk? Use Monte – Carlo simulation technique.

15. (a) Obtain the necessary and sufficient conditions for the optimum solution of the following NLPP :

Minimize $z = f(x_1, x_2) = 3e^{2x_1+1} + 2e^{x_2+5}$

Subject to the constraints :

$x_1 + x_2 = 7, \ x_1, x_2 \geq 0.$

Or

(b) Solve the non – linear programming problem :

Minimize $z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$

Subject to the constraints :

$x_1 + x_2 + x_3 = 11. \ x_1, x_2, x_3 \geq 0.$

SECTION C — ($5 \times 10 = 50$)

Answer ALL the questions, choosing either (a) or (b).

16. (a) Use graphical method to minimize the time added to process the following jobs on the machines shown, i.e, for each machine find the job which should be done first. Also calculate the total time elapsed to complete both the jobs.

Job 1 sequence	A	B	C	D	E
Time	3	4	2	6	2
Job 2 sequence time:	C	B	A	D	E
Time	5	4	3	2	6

Or

(b) We have five jobs, each of which must go through machines A, B and C in the order A B C. Processing time (in hours) are given in the following table :

Job :	1	2	3	4	5
Machine (A) :	8	10	6	7	11
Machine (B) :	5	6	2	3	4
Machine (C):	4	9	8	6	5

17. (a) At time zero all items in a system are new. Each item has a probability P of failing immediately before the end of the first month of life, and a probability $q = 1 - p$ of failing immediately before the end of the second month (i.e., all items fail by the end of the second month). If all items are replaced as they fail, show that the expected number of failures $f(x)$ at the end of month x is given by $f(x) = \frac{N}{1+q}\left[1 - (-q)^{x+1}\right]$, Where N is the number of items in the system. If the cost per item of individual replacement is C_1, and the cost per item of group replacement is C_2. Find the conditions under which.

(i) A group replacement policy at the end of each month is most profitable.

(ii) No group replacement policy is better than a policy of pure individual replacement.

Or

(b) A manufacturer is offered two machines A and B. A is priced at Rs. 5,000 and running costs are estimated at Rs. 800 for each of the first five years, increasing by Rs. 200 per year in the sixth and subsequent years. Machine B, which has the same capacity as A, cost Rs. 2,500 but will have running costs of Rs. 1,200 per year for six years, increasing by Rs. 200 per year thereafter.

If money is worth 10% per year, which machine should be purchased? (Assuming that the machine will eventually be sold for scrap at a negligible price).

18. (a) The occurrence of rain in a city on a day is dependent upon whether or not in rained on the previous day. If it rained on the previous day, the rain distribution is :

Event :	No rain	1 cm rain	2 cm rain	3 cm rain	4 cm rain	5 cm rain
Probability :	0.50	0.25	0.15	0.05	0.03	0.02

If it did not rain on the previous day. The rain distribution is

Event :	No rain	1 cm rain	2 cm rain	3 cm rain
Probability :	0.75	0.15	0.06	0.04

Stimulate the city's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period. Use the following random numbers :

67 63 39 55 29 78 70 06 78 76 simulation.

Assume that for the first day of simulation it had not rained the day before.

Or

(b) Consider the setting up a 'Q' system of the inventory control for a vital spare part. Though a demand pattern has not been established, yet the quantities demanded during the past 50 weeks are known. Similarly, the suppliers' delivery lead – time has been found to vary between 1 and 4 weeks with no established pattern. We know that the carrying cost is 30% per annum and the ordering cost is Rs. 60 per order. The stock – out cost in this case is around Rs. 75 per unit per week while the inventory carrying cost works out to Rs. 15 per unit per week. Simulate the demand for 20 weeks and obtain an optimal solution.

19. (a) Observations of past data show the following patterns in respect of inter – arrival duration and service duration in a single channel queueing system. Using the random number table below, simulate the queue behaviour for a period of 60 minutes and estimate the probability of the service being idle and then mean time spent by a customer waiting for service.

Inter-arrival time		Service time	
Minutes	Probability	Minutes	Probability
2	0.15	1	0.10
4	0.23	3	0.22
6	0.35	5	0.35
8	0.17	7	0.23
10	0.10	9	0.10

Random numbers (start at NW corner and proceed along the row).

9371	1463	7214	1053	2164
8142	8707	9054	3866	1053
2924	1725	1185	6885	9980
5119	4086	3083	5217	7150

Or

(b) The M.C. Company is evaluating an investment proposal which has uncertainty associated with the three important aspects : the original cost, the useful life, and the annual net cash flows. The three probability distributions for these variables are shown below :

Original cost		Useful life		Annual net cash inflows	
Value	Probability	Period	Probability	Value	Probability
Rs. 60,000	0.3	5 years	0.4	Rs. 10,000	0.1
Rs. 70,000	0.6	6 years	0.4	Rs. 15,000	0.3
Rs. 90,000	0.1	7 years	0.2	Rs. 20,000	0.4
				Rs. 25,000	0.2

The firm wants to perform five simulation runs of this project's life. The firm's cost of capital is 15% and the risk – free rate is 6%; for simplicity it is assumed that these two values are known for certain and will remain constant over the life of the project.

To simulate the probability distributions of original cost, useful life and annual net cash inflows, use the following sets of random numbers :

09, 84, 41, 92, 65; 24,38,73,07,04; and 07,48,57,64,72 respectively each of the five simulation runs.

20. (a) Explain Bordered Hession matrix. Also write sufficient conditions for maximum and minimum stationary points.

Or

(b) Solve the following non – linear programming problems, using the method of Lagrangian multipliers.

Minimize $z = 6x_1^2 + 5x_2^2$

Subject to the constraints :

$x_1 + 5x_2 = 3, x_1, x_2 \geq 0$.

———————