S.No. 18493 D **P 8 MA 4**

(For candidates admitted from 2008-2015 batch)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2020.

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — $(10 \times 2 = 20)$

Answer ALL questions.

1. Show that $y = c_1 x + c_2 x^2$ is the general solution of $x^2 y'' - 2xy' + 2y = 0$.

2. Define Wronskian of $y_1(x)$ and $y_2(x)$ and give an example.

3. Define Gamma function.

4. Write the orthogonal property of Legendre polynomial.

5. Find the Linear system of the form $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

6. State Pichards theorem.

7. Let $g(x)$ be an non-trivial solution of $y'' + g(x)y = 0$ on $[a,b]$. Prove that $y(x)$ has atmost a finite number of zeros in this interval.

8. Write down the Fourier coefficients of Eulers formula.

9. Define positive definite and negative definite for the function $ax^2 + bxy + cy^2$.

10. Define simple critical point.

SECTION B — $(5 \times 5 = 25)$

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that $y = c_1 \sin x + c_2 \cos x$ is the general solution of $y'' + y = 0$ on any interval and find the particular solution for which $y(0) = 2$ and $y'(0) = 3$.

Or

(b) Find the particular solution of $y'' + y = \operatorname{cosec} x$.

12. (a) Show that $\frac{d}{dx}\left[x^{+p} J_p(x)\right] = x^p J_{p-1}(x)$.

Or

(b) Explain Least square approximation for the Legendre polynomial.

13. (a) Find the general solution of the system $\frac{dx}{dt} = x + y$, $\frac{dy}{dt} = 4x - 2y$.

Or

(b) Find the exact solution of the initial value problem $y' = y$, $y(0) = 1$ starting with $y_0(x) = 1$, apply pichard's method to calculate $y_1(x), y_2(x), y_3(x)$.

14. (a) If $\rho(x) < 0$ and $u(n)$ is a non-trivial solution of $u'' + \Sigma(x)u = 0$ then prove that $u(n)$ has atmost one zero.

Or

(b) Find the eigen values and eigen functions for the equation $y'' + \lambda y = 0$, $y(0) = 0$, $y(\pi) = 0$.

15. (a) Show that (0, 0) is an unstable critical point of the system

$$\begin{cases} \frac{dx}{dt} &= -2x + 3y \\ \frac{dy}{dt} &= -x + y \end{cases}.$$

Or

(b) Prove that the function $E(x, y) = ax^2 + bxy + cy^2$ is positive definite if and only if $a > 0$ and $b^2 - 4ac < 0$

SECTION C — (3 × 10 = 30)

Answer any THREE questions.

16. Explain the method of variation of parameters for the equation $y'' + p(x)y' + q(x)y = R(x)$.

17. Explain general solution of Bessel's equation.

18. Find the general solution of the system $\frac{dx}{dt} = 3x - 4y$, $\frac{dy}{dt} = x - y$.

19. Derive Bernoulli's solution of the one dimensional wave equation.

20. State and prove Liapunox's basic discovery theorem.

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