S.No. 6539 **P16MA14**

(For candidates admitted from 2016 -2017 onwards)

M.Sc. DEGREE EXAMINATION, APRIL 2021.

MATHEMATICS

GRAPH THEORY

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$)

Answer ALL questions.

1. Define complete graph.
2. Prove that the number of vertices of odd degree is even in any graph.
3. Define cut edge.
4. Define edge contraction.
5. Define covering.
6. Define matching.
7. Define k-colorable graph.
8. Define achromatic number.
9. Define maximal planar graph.
10. Define homeomorph of a graph.

SECTION B – ($5 \times 5 = 25$)

Answer ALL questions.

11. (a) Prove that if a simple graph G is not connected, then G^c is connected.

Or

(b) Prove that the number of edges of a simple graph of order n having ω components cannot exceed $\frac{(n-\omega)(n-\omega+1)}{2}$.

12. (a) Prove that a simple cubic connected graph G has a cut vertex if and only if it has a cut edge.

Or

(b) Prove that for any loopless connected graph G, $\kappa(G) \leq \lambda(G) \leq \delta(G)$.

13. (a) Prove that a matching M of a graph G is maximum if and only if G has no M-augmenting path.

Or

(b) Prove that if G is Hamiltonian, then for every nonempty proper subset S of V, $\omega(G-S) \leq |S|$.

14. (a) Prove that no vertex cut of a critical graph is a clique.

Or

(b) Prove that a simple graph G on n vertices is a tree if and only if $f(G;\lambda) = \lambda(\lambda-1)^{n-1}$.

15. (a) Prove that a graph is planar if and only if it is embeddable on a sphere.

Or

(b) Prove that K_5 is nonplanar.

SECTION C – $(3 \times 10 = 30)$

Answer any THREE questions.

16. Prove that every vertex of a disconnected tournament T on n vertices with $n \geq 3$ is contained in a directed k-cycle, $3 \leq k \leq n$.
17. Prove that $\tau(K_n) = n^{n-2}$, where K_n is a labeled complete graph on n vertices, $n \geq 2$.
18. Prove that a graph G is Eulerian if and only if each edge e of G belongs to an odd number of cycles of G.
19. Prove that for any simple graph $G, \Delta(G) \leq \chi'(G) \leq 1 + \Delta(G)$.
20. State and prove Kuratowski theorem.

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