S.No. 6547 **P 16 MAE 2 A**

(For candidates admitted from 2016-2017 onwards)

M.Sc. DEGREE EXAMINATION, APRIL 2021.

Mathematics – Elective

STOCHASTIC PROCESSES

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20)

Answer ALL questions.

1. Define : Markov Chain.
2. Explain the strong Markov property.
3. Define : Persistent.
4. Find the stationary distribution.
5. Give a Erlang's loss formula.
6. Define a loss system.
7. Define : Periodic.
8. What is a stopping time?
9. Define : Busy period.
10. Define a Little's formula.

PART B — (5 × 5 = 25)

Answer ALL questions, choosing either (a) or (b).

11. (a) Stat and prove that Markov process.

Or

(b) Let $\{x_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix $\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ and the initial distribution $P_r\{x_0 = i\} = 1/3,\ i = 0,1,2,.....$

12. (a) Prove that state j is persistent non-null, then as $n \to \infty$

(i) $P_{jj}^{(nt)} \to t / M_{jj}$ when state j is periodic with period t. and

(ii) $P_{jj}^{(n)} \to 1 / M_{jj}$ when state j is a periodic.

Or

(b) Let a_{ik} denote the probability that the chain starting with a transient state i eventually gets absorbed in an absorbing state k. If we denote the absorption probability matrix by $A = (a_{ik}), i \in T, K \in S - T$. Then $A = (I - Q)^{-1} R = NR, N = (I - Q)^{-1}$.

13. (a) If $M(t)$ denotes the total number of occurrence in an interval of length t under the conditions (i) and (ii) stated above, then the generating function of $M(t)$ is given by $G(P(S)) = \exp[\lambda t \{P(S) - 1\}]$.

Or

(b) Explain the application in inventory theory.

14. (a) Derive the Wald's equation.

Or

(b) Prove that renewal function M satisfies the equation $M(t) = F(t) + \int_0^1 M(t - x)\, dF(x)$.

15. (a) Explain the steady state distribution.

Or

(b) Derive the model $G_1 / M / 1$.

PART C — (3 × 10 = 30)

Answer any THREE questions.

16. Derive Chapman-Kolmogorov equation.

17. State and prove Ergodic theorem.

18. Explain the p.g.f. of a non-homogeneous process $\{N(t), t \geq 0\}$ is given by $Q(s,t) = \exp\{m(t)(s-1)\}$ where $m(t) = \int_0^t \lambda(x)dx$ is the expectation of $N(t)$.

19. Let $\{X_n, n = 1,2,....\}$ be a renewal process with distribution *F*, for which the mean $\mu = E(X_i)$ and variance $\sigma^2 = E\{(X_i - \mu)^2\}$ exist and are finite. Let $\{N(t), t \geq 0\}$ be the renewal process generated by *F*. Then $\lim_{t \to \infty} \left\{ \frac{N(t) - t/\mu}{\sqrt{t\sigma^2/\mu^3}} < x \right\} = \phi(x)$. Where $\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2}t^2\right)dt$ is the d.f. of the standard normal distribution.

20. Prove that Queues with poisson input model $M/G/1$.
