S.No. 6541 **P 16 MA 21**

(For candidates admitted from 2016-2017 onwards)

M.Sc. DEGREE EXAMINATION, APRIL 2021.

Mathematics

COMPLEX ANALYSIS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20)

Answer ALL questions.

1. Define Jordan arc.

2. Define indirectly conformal.

3. Define rectifiable arc.

4. Consider $\int_C \frac{dz}{z-3}$ where C is the circle $|z-2| = 5$.

5. Define isolated singularity.

6. State maximum principle theorem.

7. Define simply connected.

8. Find the residue of $\frac{1}{(z^2 + a^2)}$ at $z = ai$.

9. Write down the Laplace's equation in polar coordinates.

10. Write down conjugate differential equation of du.

SECTION B — (5 × 5 = 25)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every set has a unique decomposition in components.

Or

(b) Prove that under a continuous mapping the image of every compact set is compact and consequently closed.

12. (a) Evaluate $\int\limits_{|z|=5} \frac{2z+3}{z^2-2z-3}\,dz$ using Cauchy's integral formula.

Or

(b) State and prove Morera's theorem.

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity.

Or

(b) State and prove generalized S Chwartz lemma.

14. (a) Evaluate $\int\limits_{|z-1|=2} \frac{2z+3}{(z-1)(z-2)}\,dz$ using Cauchy's Residue theorem.

Or

(b) Prove that $pdx + qdy$ is locally exact in Ω if and only if $\int\limits_{\partial R} pdx + qdy = 0$ for every rectangle $RC\Omega$ with sides parallel to the axes.

15. (a) State and prove the mean value property for harmonic functions.

Or

(b) Expand $\frac{1}{z(z^2+3z+2)}$ in $0 < |z| < 1$, $1 < |z| < 2$.

SECTION C — (3 × 10 = 30)

Answer any THREE questions.

16. Prove that every linear fractional transformation maps circles and straight lines to circles and straight lines.

17. State and prove Cauchy's theorem for Rectangle with exceptional point.

18. State and prove local correspondence theorem.

19. State and prove general form of Cauchy's theorem.

20. State and prove Laurent's series.

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