(6 pages)

S.No. 15605 T **RCCSMM 8**

(For candidates admitted from 2008-2015 batch)

B.Sc. DEGREE EXAMINATION, APRIL 2021.

Part III — Mathematics – Major

ABSTRACT ALGEBRA

Time : Three hours Maximum : 75 marks

SECTION A — $(10 \times 2 = 20)$

Answer ALL the questions.

1. Define a cyclic group.

2. Define left coset.

3. When do you say that a subgroup is normal?

4. Define Kernel of a homomorphism.

5. Define a zero divisor.

6.

Give an example of an integral domain which is not a field.

7.

If $T: V \to W$ is a linear transformation, prove that $T(V)$ is a substance of W.

8.

Define a linearly independent set.

9.

Define basis.

10.

Find the rank and nullity of identity linear transformation.

SECTION B — $(5 \times 5 = 25)$

Answer ALL the questions.

11.

a) Prove that the intersection of two subgroups of a group is a subgroup.

Or

b) State and prove Euler's theorem.

12.

a) Prove that N is a normal subgroup of G iff $gNg^{-1} = N$ for every $g \in G$.

Or

b) Prove that any finite cyclic group of order n is isomorphic to $(Zn, \oplus)$.

13.

a) Prove that a field has no proper ideals.

Or

b) Prove that every field is an integral domain.

14.

a) Prove that the intersection of two subspaces of a vector space is a subspace.

Or

b) Let V be a vector space over a field F and $S \subseteq V$ be a non-empty subset of V. Then prove that

(i) L (S) is a subspace of V

(ii) $S \subseteq L(S)$.

15.

a) Let $T: U \rightarrow V$ be a linear transformation, prove that rank T + nullity T = dim (v).

Or

b) If V, W are any two vector spaces over F, that are isomorphic, then show that the isomorphism maps a basis of V onto a basis of W.

SECTION C — ($3 \times 10 = 30$)

Answer any THREE questions.

16.

If A, B are subgroups of an abelian group G, then prove that AB is also a subgroup of G.

17.

State and prove fundamental theorem of homomorphism.

18.

Let R be a commutative ring with identity. Prove that an ideal M of R is an maximal ideal iff $\frac{R}{M}$ is a field.

19. If V is a vector space over F and S, T $\subseteq$ V. Prove that the following :

(a) $S \subset T \Rightarrow L(S) \subset L(T)$

(b) $L(S \cup T) = L(S) + L(T)$

(c) $L(S) = S \Leftrightarrow S$ is a subspace of V.

20.

Let V be a finite dimensional vector space over a field F. Let W be a subspace of V.

Prove that, $\dim\left(V/W\right) = \dim V - \dim W$.